Edexcel Maths Core 4

Mark Scheme Pack

2006-2013

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GCSE

Edexcel GCE

Core Mathematics C4 (6666)

Summer 2005

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Mark Scheme (Results)

Edexcel GCE
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Final Version

June 2005
6666 Core C4
Mark Scheme

Question Number	Scheme	Marks
1.	$(4-9x)^{\frac{1}{2}} = 2\left(1-\frac{9x}{4}\right)^{\frac{1}{2}}$	B1
	$= 2\left(1+\frac{\frac{1}{2}}{1}\left(-\frac{9x}{4}\right)+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{1.2}\left(-\frac{9x}{4}\right)^2+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1.2.3}\left(-\frac{9x}{4}\right)^3+\ ...\right)$	M1
	$= 2\left(1-\frac{9}{8}x-\frac{81}{128}x^2-\frac{729}{1024}x^3+\ ...\right)$	
	$= 2-\frac{9}{4}x, -\frac{81}{64}x^2, -\frac{729}{512}x^3+\ ...$	A1, A1, A1
		[5]
	Note The M1 is gained for $\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{1.2}(\ ...\ )^2$ or $\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1.2.3}(\ ...\ )^3$	
	Special Case	
	If the candidate reaches $= 2\left(1-\frac{9}{8}x-\frac{81}{128}x^2-\frac{729}{1024}x^3+\ ...\right)$ and goes no further	
	allow A1 A0 A0	

Question Number	Scheme	Marks
2.	$2x+\left(2x\frac{dy}{dx}+2y\right)-6y\frac{dy}{dx}=0$	M1 (A1) A1
	$\frac{dy}{dx}=0 \Rightarrow x+y=0$ or equivalent	M1
	Eliminating either variable and solving for at least one value of x or y.	M1
	$y^2-2y^2-3y^2+16=0$ or the same equation in x	
	$y=\pm 2$ or $x=\pm 2$	A1
	$(2,-2),(-2,2)$	A1
		[7]
	Note: $\frac{dy}{dx}=\frac{x+y}{3y-x}$	
	Alternative	
	$3y^2-2xy-\left(x^2+16\right)=0$	
	$y=\frac{2x\pm\sqrt{\left(16x^2+192\right)}}{6}$	
	$\frac{dy}{dx}=\frac{1}{3}\pm\frac{1}{3}.\frac{8x}{\sqrt{\left(16x^2+192\right)}}$	M1 A1± A1
	$\frac{dy}{dx}=0 \Rightarrow \frac{8x}{\sqrt{\left(16x^2+192\right)}}=\pm 1$	M1
	$64x^2=16x^2+192$	
	$x=\pm 2$	M1 A1
	$(2,-2),(-2,2)$	A1
		[7]

Question Number	Scheme	Marks
3.	(a) $\dfrac{5x+3}{(2x-3)(x+2)}=\dfrac{A}{2x-3}+\dfrac{B}{x+2}$	
	$5x+3=A(x+2)+B(2x-3)$	
	Substituting $x=-2$ or $x=\frac{3}{2}$ and obtaining *A* or *B*; or equating coefficients and solving a pair of simultaneous equations to obtain *A* or *B*.	M1
	$A=3,\ B=1$	A1, A1 **(3)**
	If the cover-up rule is used, give M1 A1 for the first of *A* or *B* found, A1 for the second.	
	(b) $\displaystyle\int\frac{5x+3}{(2x-3)(x+2)}\,\mathrm{d}x=\frac{3}{2}\ln(2x-3)+\ln(x+2)$	M1 A1ft
	$\left[\ \dots\ \right]_2^6=\dfrac{3}{2}\ln 9+\ln 2$	M1 A1
	$=\ln 54$ cao	A1 **(5)** **[8]**

Question Number	Scheme	Marks
4.	$\int \frac{1}{(1-x^2)^{\frac{1}{2}}}\,dx = \int \frac{1}{(1-\sin^2\theta)^{\frac{3}{2}}}\cos\theta\,d\theta$ Use of $x=\sin\theta$ and $\frac{dx}{d\theta}=\cos\theta$	M1
	$= \int \frac{1}{\cos^2\theta}\,d\theta$	M1 A1
	$= \int \sec^2\theta\,d\theta = \tan\theta$	M1 A1
	Using the limits 0 and $\frac{\pi}{6}$ to evaluate integral	M1
	$[\tan\theta]_0^{\frac{\pi}{6}} = \frac{1}{\sqrt{3}} \quad \left(=\frac{\sqrt{3}}{3}\right)$ cao	A1
		[7]
	Alternative for final M1 A1	
	Returning to the variable x and using the limits 0 and $\frac{1}{2}$ to evaluate integral	M1
	$\left[\frac{x}{\sqrt{(1-x^2)}}\right]_0^{\frac{1}{2}} = \frac{1}{\sqrt{3}} \quad \left(=\frac{\sqrt{3}}{3}\right)$ cao	A1

Question Number	Scheme	Marks
5.	(a) $\int x\mathrm{e}^{2x}\mathrm{d}x = \frac{1}{2}x\mathrm{e}^{2x} - \frac{1}{2}\int \mathrm{e}^{2x}\,\mathrm{d}x$ Attempting parts in the right direction	M1 A1
	$= \frac{1}{2}x\mathrm{e}^{2x} - \frac{1}{4}\mathrm{e}^{2x}$	A1
	$\left[\frac{1}{2}x\mathrm{e}^{2x} - \frac{1}{4}\mathrm{e}^{2x}\right]_0^1 = \frac{1}{4} + \frac{1}{4}\mathrm{e}^2$	M1 A1
		(5)
	(b) $x = 0.4 \Rightarrow y \approx 0.890\,22$	
	$x = 0.8 \Rightarrow y \approx 3.962\,43$ Both are required to 5 d.p.	B1
		(1)
	(c) $I \approx \frac{1}{2} \times 0.2 \times [\ \ldots\ ]$	B1
	$\approx \ \ldots \times \left[0 + 7.389\,06 + 2\left(0.298\,36 + .890\,22 + 1.992\,07 + 3.962\,43\right)\right]$	M1 A1ft
	ft their answers to (b)	
	$\approx 0.1 \times 21.675\,22$	
	≈ 2.168 cao	A1 **(4)**
		[10]
	Note $\frac{1}{4} + \frac{1}{4}\mathrm{e}^2 \approx 2.097 \ \ldots$	

Question Number	Scheme		Marks
6.	(a) $\dfrac{\mathrm{d}x}{\mathrm{d}t} = -2\operatorname{cosec}^2 t,\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = 4\sin t\cos t$	both	M1 A1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{-2\sin t\cos t}{\operatorname{cosec}^2 t} \quad \left(= -2\sin^3 t\cos t\right)$		M1 A1
			(4)
	(b) At $t = \frac{\pi}{4}$, $x = 2,\ y = 1$	both x and y	B1
	Substitutes $t = \frac{\pi}{4}$ into an attempt at $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ to obtain gradient $\left(-\frac{1}{2}\right)$		M1
	Equation of tangent is $y - 1 = -\frac{1}{2}(x - 2)$		M1 A1
	Accept $x + 2y = 4$ or any correct equivalent		**(4)**
	(c) Uses $1 + \cot^2 t = \operatorname{cosec}^2 t$, or equivalent, to eliminate t		M1
	$1 + \left(\dfrac{x}{2}\right)^2 = \dfrac{2}{y}$	correctly eliminates t	A1
	$y = \dfrac{8}{4 + x^2}$	cao	A1
	The domain is $x \ldots 0$		B1 **(4)** **[12]**
	An alternative in (c)		
	$\sin t = \left(\dfrac{y}{2}\right)^{\frac{1}{2}};\ \cos t = \dfrac{x}{2}\sin t = \dfrac{x}{2}\left(\dfrac{y}{2}\right)^{\frac{1}{2}}$		
	$\sin^2 t + \cos^2 t = 1 \quad \Rightarrow \quad \dfrac{y}{2} + \dfrac{x^2}{4} \times \dfrac{y}{2} = 1$		M1 A1
	Leading to $y = \dfrac{8}{4 + x^2}$		A1

Question Number	Scheme	Marks
7.	(a) **k** component $2+4\lambda=-2 \quad \Rightarrow \quad \lambda=-1$	M1 A1
	Note $\mu=2$	
	Substituting their λ (or μ) into equation of line and obtaining B	M1
	B: $(2, 2, -2)$ Accept vector forms	A1
		(4)
	(b) $\left\lvert\begin{pmatrix}1\\-1\\4\end{pmatrix}\right\rvert=\sqrt{18};\quad \left\lvert\begin{pmatrix}1\\-1\\0\end{pmatrix}\right\rvert=\sqrt{2}$ both	B1
	$\begin{pmatrix}1\\-1\\4\end{pmatrix}\cdot\begin{pmatrix}1\\-1\\0\end{pmatrix}=1+1+0\ (=2)$	B1
	$\cos\theta=\dfrac{2}{\sqrt{18}\sqrt{2}}=\dfrac{1}{3}$ cao	M1 A1
		(4)
	(c) $\overrightarrow{AB}=-\mathbf{i}+\mathbf{j}-4\mathbf{k}\Rightarrow\left\lvert\overrightarrow{AB}\right\rvert^2=18$ or $\left\lvert\overrightarrow{AB}\right\rvert=\sqrt{18}$ ignore direction of vector	M1
	$\overrightarrow{BC}=3\mathbf{i}-3\mathbf{j}\Rightarrow\left\lvert\overrightarrow{BC}\right\rvert^2=18$ or $\left\lvert\overrightarrow{BC}\right\rvert=\sqrt{18}$ ignore direction of vector	M1
	Hence $\left\lvert\overrightarrow{AB}\right\rvert=\left\lvert\overrightarrow{BC}\right\rvert$ ∗	A1 **(3)**
	(d) $\overrightarrow{OD}=6\mathbf{i}-2\mathbf{j}+2\mathbf{k}$ Allow first B1 for any two correct	B1 B1
	Accept column form or coordinates	**(2)**
		[13]

Question Number	Scheme	Marks
8.	(a) $\frac{dV}{dt}$ is the rate of increase of volume (with respect to time)	B1
	$-kV$: k is constant of proportionality and the negative shows decrease (or loss)	
	giving $\frac{dV}{dt} = 20 - kV$ ∗ These Bs are to be awarded independently	B1
		(2)
	(b) $\int \frac{1}{20-kV} dV = \int 1 dt$ separating variables	M1
	$-\frac{1}{k}\ln(20-kV) = t \quad (+C)$	M1 A1
	Using $V = 0, t = 0$ to evaluate the constant of integration	M1
	$c = -\frac{1}{k}\ln 20$	
	$t = \frac{1}{k}\ln\left(\frac{20}{20-kV}\right)$	
	Obtaining answer in the form $V = A + Be^{-kt}$	M1
	$V = \frac{20}{k} - \frac{20}{k}e^{-kt}$ Accept $\frac{20}{k}\left(1-e^{-kt}\right)$	A1 **(6)**
	(c) $\frac{dV}{dt} = 20e^{-kt}$ Can be implied	M1
	$\frac{dV}{dt} = 10, t = 5 \Rightarrow 10 = 20e^{-kt} \Rightarrow k = \frac{1}{5}\ln 2 \approx 0.139$	M1 A1
	At $t = 10$, $V = \frac{75}{\ln 2}$ awrt 108	M1 A1 **(5)**
		[13]
	Alternative to (b)	
	Using printed answer and differentiating $\frac{dV}{dt} = -kBe^{-kt}$	M1
	Substituting into differential equation	
	$-kBe^{-kt} = 20 - kA - kBe^{-kt}$	M1
	$A = \frac{20}{k}$	M1 A1
	Using $V = 0, t = 0$ in printed answer to obtain $A + B = 0$	M1
	$B = -\frac{20}{k}$	A1 **(6)**

GCE

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Mark Scheme

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>1.</td><td>Differentiates

to obtain : $6x+8y\frac{dy}{dx}-2,$
$\ldots\ldots\ldots\ldots\ldots\ldots\ldots+(6x\frac{dy}{dx}+6y)=0$
$\left[\frac{dy}{dx}=\frac{2-6x-6y}{6x+8y}\right]$

Substitutes $x = 1$, $y = -2$ into expression involving $\frac{dy}{dx}$, to give $\frac{dy}{dx}=-\frac{8}{10}$

Uses line equation with numerical 'gradient' $y-(-2)=(\text{their gradient})(x-1)$
or finds c and uses $y=(\textit{their gradient})\,x+\text{"}c\text{"}$

To give $5y+4x+6=0$ (or equivalent = 0)</td><td>M1

A1,

+(B1)

M1, A1

M1

A1√ **[7]**</td></tr>
<tr><td>2.
(a)</td><td>
<table>
<tr><td>x</td><td>0</td><td>$\frac{\pi}{16}$</td><td>$\frac{\pi}{8}$</td><td>$\frac{3\pi}{16}$</td><td>$\frac{\pi}{4}$</td></tr>
<tr><td>y</td><td>1</td><td>**1.01959**</td><td>**1.08239**</td><td>1.20269</td><td>**1.41421**</td></tr>
</table>

M1 for one correct, A1 for all correct</td><td>M1 A1

(2)</td></tr>
<tr><td>(b)</td><td>Integral = $\frac{1}{2}\times\frac{\pi}{16}\times\{1+1.4142+2(1.01959+\ldots+1.20269)\}$
$\left(=\frac{\pi}{32}\,x\,9.02355\right)=0.8859$</td><td>M1 A1√

A1 cao
(3)</td></tr>
<tr><td>(c)</td><td>Percentage error = $\frac{approx-0.88137}{0.88137}\times 100$ = 0.51 % (allow 0.5% to 0.54% for A1)

M1 gained for $(\pm)\ \frac{approx-\ln(1+\sqrt{2})}{\ln(1+\sqrt{2})}$</td><td>M1 A1 (2)

[7]</td></tr>
</table>

Question Number	Scheme	Marks
3.	Uses substitution to obtain x = f(u) $\left[\dfrac{u^2+1}{2}\right]$,	M1
	and to obtain $u\dfrac{du}{dx} =$ const. or equiv.	M1
	Reaches $\int \dfrac{3(u^2+1)}{2u} u du$ or equivalent	A1
	Simplifies integrand to $\int \left(3u^2 + \dfrac{3}{2}\right) \mathrm{d}u$ or equiv.	M1
	Integrates to $\frac{1}{2}u^3 + \frac{3}{2}u$	M1 A1√
	A1√ dependent on all previous Ms	
	Uses new limits 3 and 1 substituting and subtracting (or returning to function of x with old limits)	M1
	To give 16 cso	A1 **[8]**
	"By Parts" Attempt at " right direction" by parts M1 $[\ 3x\left(2x-1\right)^{\frac{1}{2}}\ - \{\int 3\left(2x-1\right)^{\frac{1}{2}} \mathrm{d}x\}]$ M1{M1A1} ……………. $-\ \left(2x-1\right)^{\frac{3}{2}}$ M1A1√ Uses limits 5 and 1 correctly; [42 – 26] 16 M1A1	

4.	Attempts $V = \pi\int x^2 e^{2x}\mathrm{d}x$	M1
	$= \pi\left[\frac{x^2e^{2x}}{2} - \int xe^{2x}dx\right]$ (M1 needs parts in the correct direction)	M1 A1
	$= \pi\left[\frac{x^2e^{2x}}{2} - \left(\frac{xe^{2x}}{2} - \int\frac{e^{2x}}{2}dx\right)\right]$ (M1 needs second application of parts)	M1 A1√
	M1A1√ refers to candidates $\int xe^{2x}\,\mathrm{d}x$, but dependent on prev. M1	
	$= \pi\left[\frac{x^2e^{2x}}{2} - \left(\frac{xe^{2x}}{2} - \frac{e^{2x}}{4}\right)\right]$	A1 cao
	Substitutes limits 3 and 1 and subtracts to give… [dep. on second and third Ms]	dM1
	$= \pi\left[\frac{13}{4}e^6 - \frac{1}{4}e^2\right]$ or any correct exact equivalent.	A1
		[8]
	[Omission of π loses first and last marks only]	

Question Number	Scheme	Marks
5. (a)	Considers $3x^2+16 = A(2+x)^2 + B(1-3x)(2+x) + C(1-3x)$	
	and substitutes x = –2 , or x = 1/3 ,	M1
	or compares coefficients and solves simultaneous equations	
	To obtain A = 3, and C = 4	A1, A1
	Compares coefficients or uses simultaneous equation to show B = 0.	B1 (4)
(b)	Writes $3(1-3x)^{-1} + 4(2+x)^{-2}$	M1
	$= 3(1+3x, +9x^2+27x^3+......)$ +	(M1, A1)
	$\frac{4}{4}(1+\frac{(-2)}{1}\left(\frac{x}{2}\right)+\frac{(-2)(-3)}{1.2}\left(\frac{x}{2}\right)^2+\frac{(-2)(-3)(-4)}{1.2.3}\left(\frac{x}{2}\right)^3+....)$	(M1 A1)
	$= 4+8x,\ +27\tfrac{3}{4}x^2+80\tfrac{1}{2}x^3+...$	A1, A1 (7)
	Or uses $(3x^2+16)(1-3x)^{-1}(2+x)^{-2}$	M1
	$(3x^2+16)\ (1+3x,+9x^2+27x^3+)\ \times$	(M1A1)×
	$¼(1+\frac{(-2)}{1}\left(\frac{x}{2}\right)+\frac{(-2)(-3)}{1.2}\left(\frac{x}{2}\right)^2+\frac{(-2)(-3)(-4)}{1.2.3}\left(\frac{x}{2}\right)^3)$	(M1A1)
	$= 4+8x, +27\tfrac{3}{4}x^2+80\tfrac{1}{2}x^3+...$	A1, A1 (7)
		[11]

6. (a)	$\lambda=-4\rightarrow a=18,\qquad \mu=1\rightarrow b=9$	M1 A1, A1 (3)
(b)	$\begin{pmatrix}8+\lambda\\12+\lambda\\14-\lambda\end{pmatrix}\bullet\begin{pmatrix}1\\1\\-1\end{pmatrix}=0$	M1
	$\therefore 8+\lambda+12+\lambda-14+\lambda=0$	A1
	Solves to obtain λ $(\lambda=-2)$	dM1
	Then substitutes value for λ to give P at the point (6, 10, 16) (any form)	M1, A1 (5)
(c)	OP = $\sqrt{36+100+256}$	M1
	(= $\sqrt{392}$) $=14\sqrt{2}$	A1 cao (2) **[10]**
7. (a)	$\dfrac{dV}{dr}=4\pi r^2$	B1 (1)
(b)	Uses $\dfrac{dr}{dt}=\dfrac{dV}{dt}.\dfrac{dr}{dV}$ in any form, $=\dfrac{1000}{4\pi r^2(2t+1)^2}$	M1,A1 (2)
(c)	$V=\int 1000(2t+1)^{-2}dt$ and integrate to $p\ (2t+1)^{-1}$, $=-500(2t+1)^{-1}(+c)$	M1, A1
	Using V=0 when t=0 to find c , (c = 500 , or equivalent)	M1
	$\therefore V=500(1-\dfrac{1}{2t+1})$ (any form)	A1 (4)
(d)	(i) Substitute t = 5 to give V,	M1,
	then use $r=\sqrt[3]{\left(\dfrac{3V}{4\pi}\right)}$ to give *r* , = *4.77*	M1, A1 (3)
	(ii) Substitutes t = 5 and r = 'their value' into 'their' part (b)	M1
	$\dfrac{\mathrm{d}r}{\mathrm{d}t}=0.0289$ $(\approx 2.90x10^{-2})$ (cm/s) $*$ AG	A1 (2) **[12]**

8. (a)	Solves y = 0 $\Rightarrow$ $\cos t = \frac{1}{2}$ to obtain $t = \frac{\pi}{3}$ *or* $\frac{5\pi}{3}$ (need both for A1)	M1 A1
	Or substitutes **both** values of *t* and shows that $y = 0$	(2)
(b)	$\frac{dx}{dt} = 1 - 2\cos t$	M1 A1
	Area= $\int y\,dx = \int_{\pi/3}^{5\pi/3} (1-2\cos t)(1-2\cos t)dt = \int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1-2\cos t)^2 dt$ * AG	B1
		(3)
(c)	Area $= \int 1 - 4\cos t + 4\cos^2 t\,dt$ 3 terms	M1
	$= \int 1 - 4\cos t + 2(\cos 2t + 1)dt$ (use of correct double angle formula)	M1
	$= \int 3 - 4\cos t + 2\cos 2t\,dt$	
	$= \left[3t - 4\sin t + \sin 2t\right]$	M1 A1
	Substitutes the two correct limits $t = \frac{5\pi}{3}$ and $\frac{\pi}{3}$ and subtracts.	M1
	$= 4\pi + 3\sqrt{3}$	A1A1
		(7)
		[12]

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June 2006
6666 Pure Mathematics C4
Mark Scheme

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1.	$\left\{\frac{\cancel{d}y}{\cancel{d}x}\cancel{\times}\right\}$ $6x - 4y\frac{dy}{dx} + 2 - 3\frac{dy}{dx} = 0$	Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $\pm 3\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$.) Correct equation.	M1 A1
	$\left\{\frac{dy}{dx} = \frac{6x+2}{4y+3}\right\}$	*not necessarily required.*	
	At (0, 1), $\frac{dy}{dx} = \frac{0+2}{4+3} = \frac{2}{7}$	Substituting x = 0 & y = 1 into an *equation* involving $\frac{dy}{dx}$; to give $\frac{2}{7}$ or $\frac{-2}{-7}$	dM1; A1 **cso**
	Hence m(**N**) = $-\frac{7}{2}$ or $\frac{-1}{\frac{2}{7}}$	Uses m(**T**) to 'correctly' find m(**N**). Can be ft from "their tangent gradient".	A1$\sqrt{}$ oe.
	Either **N**: $y - 1 = -\frac{7}{2}(x - 0)$ or **N**: $y = -\frac{7}{2}x + 1$	$y - 1 = m(x - 0)$ with 'their tangent or normal gradient'; or uses $y = mx + 1$ with 'their tangent or normal gradient';	M1;
	N: 7x + 2y – 2 = 0	Correct equation in the form '$ax + by + c = 0$', where a, b and c are integers.	A1 oe **cso** **[7]**
			7 marks

Beware: $\frac{dy}{dx} = \frac{2}{7}$ does not necessarily imply the award of all the first four marks in this question. So please ensure that you check candidates' initial differentiation before awarding the first A1 mark.

Beware: The final accuracy mark is for completely correct solutions. If a candidate flukes the final line then they must be awarded A0.

Beware: A candidate finding an m(**T**) = 0 can obtain A1ft for m(**N**) = ∞, but obtains M0 if they write $y - 1 = \infty(x - 0)$. If they write, however, **N**: x = 0, then can score M1.

Beware: A candidate finding an m(**T**) = ∞ can obtain A1ft for m(**N**) = 0, and also obtains M1 if they write $y - 1 = 0(x - 0)$ or y = 1.

Beware: The final **cso** refers to the whole question.

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Question Number	Scheme		Marks
Aliter			
1.	$\left\{\frac{\cancel{d}\cancel{x}}{\cancel{d}\cancel{y}} \cancel{\times}\right\}$ $6x\frac{dx}{dy} - 4y + 2\frac{dx}{dy} - 3 = 0$	Differentiates implicitly to include either $\pm kx\frac{dx}{dy}$ or $\pm 2\frac{dx}{dy}$. (Ignore $\left(\frac{dx}{dy} = \right)$.) Correct equation.	M1 A1
Way 2	$\left\{\frac{dx}{dy} = \frac{4y+3}{6x+2}\right\}$	*not necessarily required.*	
	At (0, 1), $\frac{dx}{dy} = \frac{4+3}{0+2} = \frac{7}{2}$	Substituting x = 0 & y = 1 into an *equation* involving $\frac{dx}{dy}$; to give $\frac{7}{2}$	dM1; A1 **cso**
	Hence m(**N**) = $-\frac{7}{2}$ or $\frac{-1}{\frac{2}{7}}$	Uses m(**T**) or $\frac{dx}{dy}$ to 'correctly' find m(**N**). Can be ft using "$-1.\frac{dx}{dy}$".	A1$\sqrt{}$ oe.
	Either **N**: $y - 1 = -\frac{7}{2}(x - 0)$ or **N**: $y = -\frac{7}{2}x + 1$	$y - 1 = m(x - 0)$ with 'their tangent, $\frac{dx}{dy}$ or normal gradient'; or uses $y = mx + 1$ with 'their tangent, $\frac{dx}{dy}$ or normal gradient' ;	M1;
	N: 7x + 2y – 2 = 0	Correct equation in the form '$ax + by + c = 0$', where a, b and c are integers.	A1 oe **cso**
			7 marks

Question Number	Scheme		Marks
Aliter **1. Way 3**	$2y^2+3y-3x^2-2x-5=0$		
	$\left(y+\frac{3}{4}\right)^2-\frac{9}{16}=\frac{3x^2}{2}+x+\frac{5}{2}$		
	$y=\sqrt{\left(\frac{3x^2}{2}+x+\frac{49}{16}\right)}-\frac{3}{4}$		
	$\frac{dy}{dx}=\frac{1}{2}\left(\frac{3x^2}{2}+x+\frac{49}{16}\right)^{-\frac{1}{2}}(3x+1)$	Differentiates using the chain rule; Correct expression for $\frac{dy}{dx}$.	M1; A1 oe
	At (0, 1), $\frac{dy}{dx}=\frac{1}{2}\left(\frac{49}{16}\right)^{-\frac{1}{2}}=\frac{1}{2}\left(\frac{4}{7}\right)=\frac{2}{7}$	Substituting x = 0 into an *equation* involving $\frac{dy}{dx}$; to give $\frac{2}{7}$ or $\frac{-2}{-7}$	dM1 A1 **cso**
	Hence m(**N**) = $-\frac{7}{2}$	Uses m(**T**) to 'correctly' find m(**N**). Can be ft from "their tangent gradient".	A1$\sqrt{}$
	Either **N**: $y-1=-\frac{7}{2}(x-0)$ or **N**: $y=-\frac{2}{7}x+1$	$y-1=m(x-0)$ with 'their tangent or normal gradient'; or uses $y=mx+1$ with 'their tangent or normal gradient'	M1
	N: 7x + 2y – 2 = 0	Correct equation in the form '$ax+by+c=0$', where a, b and c are integers.	A1 oe **[7]**
			7 marks

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Question Number	Scheme		Marks
2. (a)	$3x - 1 \equiv A(1-2x) + B$	Considers this identity and either substitutes $x = \frac{1}{2}$, equates coefficients or solves simultaneous equations	*complete* M1
	Let $x = \frac{1}{2}$; $\frac{3}{2} - 1 = B \Rightarrow B = \frac{1}{2}$		
	Equate x terms; $3 = -2A \Rightarrow A = -\frac{3}{2}$	$A = -\frac{3}{2}$; $B = \frac{1}{2}$	A1;A1
	(**No working seen**, but A and B correctly stated $\Rightarrow$ award all three marks. If one of A or B correctly stated give two out of the three marks available for this part.)		**[3]**
(b)	$f(x) = -\frac{3}{2}(1-2x)^{-1} + \frac{1}{2}(1-2x)^{-2}$	Moving powers to top on any one of the two expressions	M1
	$= -\frac{3}{2}\left\{\underline{1 + (-1)(-2x); + \frac{(-1)(-2)}{2!}(-2x)^2 + \frac{(-1)(-2)(-3)}{3!}(-2x)^3 + \ldots}\right\}$	Either $1 \pm 2x$ or $1 \pm 4x$ from either first or second expansions respectively	dM1;
	$+\frac{1}{2}\left\{\underline{1 + (-2)(-2x); + \frac{(-2)(-3)}{2!}(-2x)^2 + \frac{(-2)(-3)(-4)}{3!}(-2x)^3 + \ldots}\right\}$	Ignoring $-\frac{3}{2}$ and $\frac{1}{2}$, any one correct $\{\underline{\ldots\ldots\ldots\ldots}\}$ expansion.	A1
		Both $\{\underline{\ldots\ldots\ldots\ldots}\}$ correct.	A1
	$= -\frac{3}{2}\left\{1 + 2x + 4x^2 + 8x^3 + \ldots\right\} + \frac{1}{2}\left\{1 + 4x + 12x^2 + 32x^3 + \ldots\right\}$		
	$= -1 - x\ ; + 0x^2 + 4x^3$	$-1 - x$; $(0x^2) + 4x^3$	A1; A1 **[6]**
			9 marks

Beware: In part (a) take care to spot that $A = -\frac{3}{2}$ and $B = \frac{1}{2}$ are the right way around.

Beware: In ePEN, make sure you aware the marks correctly in part (a). The first A1 is for $A = -\frac{3}{2}$ and the second A1 is for $B = \frac{1}{2}$.

Beware: If a candidate uses a method of long division please escalate this to you team leader.

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Question Number	Scheme		Marks
Aliter **2.** (b) **Way 2**	$f(x) = (3x-1)(1-2x)^{-2}$	Moving power to top	M1
	$= (3x-1) \times \left(1 + (-2)(-2x)\ ;\ + \frac{(-2)(-3)}{2!}(-2x)^2 + \frac{(-2)(-3)(-4)}{3!}(-2x)^3 + \ldots\right)$	$1 \pm 4x$; Ignoring $(3x-1)$, correct $(\ldots\ldots\ldots\ldots)$ expansion	dM1; A1
	$= (3x-1)(1 + 4x + 12x^2 + 32x^3 + \ldots)$		
	$= \underline{3x + 12x^2 + 36x^3 - 1 - 4x - 12x^2 - 32x^3} + \ldots$	<u>Correct expansion</u>	A1
	$= -1 - x\ ; + 0x^2 + 4x^3$	$-1 - x$; $(0x^2) + 4x^3$	A1; A1 **[6]**
Aliter **2.** (b) **Way 3**	Maclaurin expansion		
	$f(x) = -\frac{3}{2}(1-2x)^{-1} + \frac{1}{2}(1-2x)^{-2}$	Bringing both powers to top	M1
	$f'(x) = -3(1-2x)^{-2} + 2(1-2x)^{-3}$	Differentiates to give $a(1-2x)^{-2} \pm b(1-2x)^{-3}$; $-3(1-2x)^{-2} + 2(1-2x)^{-3}$	M1; A1 oe
	$f''(x) = -12(1-2x)^{-3} + 12(1-2x)^{-4}$		
	$f'''(x) = -72(1-2x)^{-4} + 96(1-2x)^{-5}$	Correct $f''(x)$ and $f'''(x)$	A1
	$\therefore f(0) = -1$, $f'(0) = -1$, $f''(0) = 0$ and $f'''(0) = 24$		
	gives $f(x) = -1 - x; + 0x^2 + 4x^3 + \ldots$	$-1 - x$; $(0x^2) + 4x^3$	A1; A1 **[6]**

Question Number	Scheme		Marks
Aliter **2.** (b) **Way 4**	$f(x) = -3(2-4x)^{-1} + \frac{1}{2}(1-2x)^{-2}$	Moving powers to top on any one of the two expressions	M1
	$= -3\left\{\underline{(2)^{-1} + (-1)(2)^{-2}(-4x); + \frac{(-1)(-2)}{2!}(2)^{-3}(-4x)^2 + \frac{(-1)(-2)(-3)}{3!}(2)^{-4}(-4x)^3 + \ldots}\right\}$	Either $\frac{1}{2} \pm x$ or $1 \pm 4x$ from either first or second expansions respectively	dM1;
	$+\frac{1}{2}\left\{\underline{1 + (-2)(-2x); + \frac{(-2)(-3)}{2!}(-2x)^2 + \frac{(-2)(-3)(-4)}{3!}(-2x)^3 + \ldots}\right\}$	Ignoring -3 and $\frac{1}{2}$, any one correct $\{\underline{\ldots\ldots\ldots\ldots}\}$ expansion.	A1
		Both $\{\underline{\ldots\ldots\ldots\ldots}\}$ correct.	A1
	$= -3\left\{\frac{1}{2} + x + 2x^2 + 4x^3 + \ldots\right\} + \frac{1}{2}\left\{1 + 4x + 12x^2 + 32x^3 + \ldots\right\}$		
	$= -1 - x\ ; + 0x^2 + 4x^3$	$-1 - x\ ;\quad (0x^2) + 4x^3$	A1; A1 **[6]**

Question Number	Scheme		Marks
3. (a)	Area Shaded $= \int_0^{2\pi} 3\sin\left(\frac{x}{2}\right)\,dx$		
	$= \left[\dfrac{-3\cos\left(\frac{x}{2}\right)}{\frac{1}{2}}\right]_0^{2\pi}$	Integrating $3\sin\left(\frac{x}{2}\right)$ to give $k\cos\left(\frac{x}{2}\right)$ with $k \neq 1$. Ignore limits.	M1
	$= \left[-6\cos\left(\frac{x}{2}\right)\right]_0^{2\pi}$	$-6\cos\left(\frac{x}{2}\right)$ or $\frac{-3}{\frac{1}{2}}\cos\left(\frac{x}{2}\right)$	A1 oe.
	$= [-6(-1)] - [-6(1)] = 6 + 6 = \underline{12}$	$\underline{12}$	A1 cao **[3]**
	(Answer of 12 with no working scores M0A0A0.)		
(b)	Volume $= \underline{\pi\int_0^{2\pi}\left(3\sin\left(\frac{x}{2}\right)\right)^2}\,dx = 9\pi\int_0^{2\pi}\sin^2\left(\frac{x}{2}\right)\,dx$	**Use** of $\underline{V = \pi\int y^2}\,dx$. Can be implied. Ignore limits.	M1
	$\left[\text{NB}: \underline{\cos 2x = \pm 1 \pm 2\sin^2 x} \text{ gives } \sin^2 x = \frac{1-\cos 2x}{2}\right]$ $\left[\text{NB}: \underline{\cos x = \pm 1 \pm 2\sin^2\left(\frac{x}{2}\right)} \text{ gives } \sin^2\left(\frac{x}{2}\right) = \frac{1-\cos x}{2}\right]$	Consideration of the Half Angle Formula for $\sin^2\left(\frac{x}{2}\right)$ or the Double Angle Formula for $\sin^2 x$	M1$*$
	$\therefore$ Volume $= 9(\pi)\int_0^{2\pi}\left(\dfrac{1-\cos x}{2}\right)dx$	Correct expression for Volume Ignore limits and π.	A1
	$= \dfrac{9(\pi)}{2}\int_0^{2\pi}\underline{(1-\cos x)}\,dx$		
	$= \dfrac{9(\pi)}{2}\underline{[x - \sin x]}_0^{2\pi}$	$\underline{\text{Integrating to give } \pm ax \pm b\sin x}$; $\underline{\text{Correct integration}}$ $\underline{k - k\cos x \to kx - k\sin x}$	depM1$*$; A1
	$= \dfrac{9\pi}{2}[(2\pi - 0) - (0 - 0)]$		
	$= \dfrac{9\pi}{2}(2\pi) = \underline{9\pi^2}$ or $\underline{88.8264\ldots}$	Use of limits to give either $9\pi^2$ or awrt 88.8 Solution must be completely correct. No flukes allowed.	A1 **cso** **[6]**
			9 marks

Question 3

Note: π is not needed for the middle four marks of question 3(b).

Beware: Owing to the symmetry of the curve between x = 0 and x = 2π candidates can find:

- Area = $2\int_0^{\pi} 3\sin\left(\frac{x}{2}\right) dx$ in part (a).

- Volume = $2\pi\int_0^{\pi} \left(3\sin\left(\frac{x}{2}\right)\right)^2 dx$

Beware: If a candidate gives the correct answer to part (b) with no working please escalate this response up to your team leader.

Question Number	Scheme		Marks
4. (a)	$x = \sin t,\quad y = \sin\left(t + \frac{\pi}{6}\right)$		
	$\frac{dx}{dt} = \cos t,\quad \frac{dy}{dt} = \cos\left(t + \frac{\pi}{6}\right)$	Attempt to differentiate both x and y wrt t to give two terms in cos	M1
		Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$	A1
	When $t = \frac{\pi}{6}$, $\frac{dy}{dx} = \underline{\frac{\cos\left(\frac{\pi}{6}+\frac{\pi}{6}\right)}{\cos\left(\frac{\pi}{6}\right)} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58}$	Divides in correct way and substitutes for t to give any of the four underlined oe: Ignore the double negative if candidate has differentiated $\sin \to -\cos$	A1
	When $t = \frac{\pi}{6}$, $x = \frac{1}{2},\ y = \frac{\sqrt{3}}{2}$	The point $\underline{\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)}$ or $\underline{\left(\frac{1}{2}, \text{awrt } 0.87\right)}$	B1
	T: $\underline{y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(x - \frac{1}{2}\right)}$	Finding an equation of a tangent with their point and their tangent gradient or finds c and uses $y = (\text{their gradient})x + \text{"c"}$.	dM1
		Correct <u>EXACT</u> equation of <u>tangent</u> oe.	<u>A1</u> oe
	or $\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(\frac{1}{2}\right) + c \Rightarrow c = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$		
	or **T**: $\left[\underline{y = \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}}\right]$		
			[6]
(b)	$y = \sin\left(t + \frac{\pi}{6}\right) = \sin t \cos\frac{\pi}{6} + \cos t \sin\frac{\pi}{6}$	Use of compound angle formula for sine.	M1
	Nb: $\sin^2 t + \cos^2 t \equiv 1 \Rightarrow \cos^2 t \equiv 1 - \sin^2 t$		
	$\therefore x = \sin t$ gives $\cos t = \sqrt{(1 - x^2)}$	Use of trig identity to find $\cos t$ in terms of x or $\cos^2 t$ in terms of x.	M1
	$\therefore y = \frac{\sqrt{3}}{2}\sin t + \frac{1}{2}\cos t$		
	gives $y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{(1 - x^2)}$ **AG**	Substitutes for $\sin t,\ \cos\frac{\pi}{6},\ \cos t$ and $\sin\frac{\pi}{6}$ to give y in terms of x.	A1 cso
			[3]
			9 marks

Question Number	Scheme		Marks
Aliter **4.** (a) **Way 2**	$x = \sin t, \quad y = \sin\left(t + \frac{\pi}{6}\right) = \sin t \cos\frac{\pi}{6} + \cos t \sin\frac{\pi}{6}$	(Do not give this for part (b))	
		Attempt to differentiate x and y wrt t to give $\frac{dx}{dt}$ in terms of cos and $\frac{dy}{dt}$ in the form $\pm a\cos t \pm b\sin t$	M1
	$\frac{dx}{dt} = \cos t, \quad \frac{dy}{dt} = \cos t \cos\frac{\pi}{6} - \sin t \sin\frac{\pi}{6}$	Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$	A1
	When $t = \frac{\pi}{6}, \ \frac{dy}{dx} = \underline{\frac{\cos\frac{\pi}{6}\cos\frac{\pi}{6} - \sin\frac{\pi}{6}\sin\frac{\pi}{6}}{\cos\left(\frac{\pi}{6}\right)}}$ $= \underline{\frac{\frac{3}{4} - \frac{1}{4}}{\frac{\sqrt{3}}{2}} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58}$	Divides in correct way and substitutes for t to give any of the four underlined oe:	A1
	When $t = \frac{\pi}{6}, \quad x = \frac{1}{2}, \ y = \frac{\sqrt{3}}{2}$	The point $\underline{\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)}$ or $\underline{\left(\frac{1}{2}, \text{awrt } 0.87\right)}$	B1
	T: $\underline{y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(x - \frac{1}{2}\right)}$	Finding an equation of a tangent with their point and their tangent gradient or finds c and uses $y = (\text{their gradient})x + "c"$.	dM1
		Correct EXACT equation of <u>tangent</u> oe.	<u>A1</u> oe
	or $\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(\frac{1}{2}\right) + c \ \Rightarrow \ c = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$		
	or **T**: $\left[\underline{y = \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}}\right]$		
			[6]

Question Number	Scheme		Marks
Aliter **4.** (a) **Way 3**	$y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{(1-x^2)}$		
	$\frac{dy}{dx} = \frac{\sqrt{3}}{2} + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(1-x^2)^{-\frac{1}{2}}(-2x)$	Attempt to differentiate two terms using the chain rule for the second term. Correct $\frac{dy}{dx}$	M1 A1
	$\frac{dy}{dx} = \underline{\frac{\sqrt{3}}{2} + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(1-(0.5)^2)^{-\frac{1}{2}}(-2(0.5)) = \frac{1}{\sqrt{3}}}$	Correct substitution of $x = \frac{1}{2}$ into a correct $\frac{dy}{dx}$	A1
	When $t = \frac{\pi}{6}$, $x = \frac{1}{2}$, $y = \frac{\sqrt{3}}{2}$	The point $\underline{\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)}$ or $\underline{\left(\frac{1}{2}, \text{awrt } 0.87\right)}$	B1
	T: $\underline{y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(x - \frac{1}{2}\right)}$	Finding an equation of a tangent with their point and their tangent gradient or finds c and uses $y = \text{(their gradient)}x + "c"$. Correct <u>EXACT</u> equation of <u>tangent</u> oe.	dM1 <u>A1</u> oe
	or $\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(\frac{1}{2}\right) + c \Rightarrow c = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$		
	or **T**: $\left[\underline{y = \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}}\right]$		
			[6]
Aliter **4.** (b) **Way 2**	$x = \sin t$ gives $y = \frac{\sqrt{3}}{2}\sin t + \frac{1}{2}\sqrt{(1-\sin^2 t)}$	Substitutes $x = \sin t$ into the equation give in y.	M1
	Nb: $\sin^2 t + \cos^2 t \equiv 1 \Rightarrow \cos^2 t \equiv 1 - \sin^2 t$		
	$\cos t = \sqrt{(1-\sin^2 t)}$	Use of trig identity to deduce that $\cos t = \sqrt{(1-\sin^2 t)}$.	M1
	gives $y = \frac{\sqrt{3}}{2}\sin t + \frac{1}{2}\cos t$		
	Hence $y = \sin t \cos\frac{\pi}{6} + \cos t \sin\frac{\pi}{6} = \sin\left(t + \frac{\pi}{6}\right)$	Using the compound angle formula to prove $y = \sin\left(t + \frac{\pi}{6}\right)$	A1 cso
			[3]
			9 marks

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Question Number	Scheme		Marks
5. (a)	Equating **i** ; $0 = 6 + \lambda \Rightarrow \lambda = -6$	$\underline{\lambda = -6}$ Can be implied	B1 $\Rightarrow$ d
	Using $\lambda = -6$ and		
	equating **j** ; $a = 19 + 4(-6) = -5$	For inserting **their stated** λ into either a correct **j** or **k** component Can be implied.	M1 $\Rightarrow$ d
	equating **k** ; $b = -1 - 2(-6) = 11$	$a = -5$ and $b = 11$	A1
			[3]
	With no working… … only one of a or b stated correctly gains the first 2 marks. … both a and b stated correctly gains 3 marks.		
(b)	$\overrightarrow{OP} = (6+\lambda)\mathbf{i} + (19+4\lambda)\mathbf{j} + (-1-2\lambda)\mathbf{k}$		
	direction vector or $l_1 = \mathbf{d} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$		
	$\overrightarrow{OP} \perp l_1 \Rightarrow \underline{\overrightarrow{OP} \bullet \mathbf{d} = 0}$	Allow <u>this statement</u> for M1 if $\overrightarrow{OP}$ and **d** are defined as above.	
	ie. $\underline{\begin{pmatrix} 6+\lambda \\ 19+4\lambda \\ -1-2\lambda \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = 0}$ (or $\underline{x + 4y - 2z = 0}$)	Allow either of these two <u>underlined statements</u>	M1
	$\therefore\ 6 + \lambda + 4(19 + 4\lambda) - 2(-1 - 2\lambda) = 0$	Correct equation	A1 oe
	$6 + \lambda + 76 + 16\lambda + 2 + 4\lambda = 0$	Attempt to solve the equation in λ	dM1
	$21\lambda + 84 = 0 \Rightarrow \lambda = -4$	$\lambda = -4$	A1
	$\overrightarrow{OP} = (6-4)\mathbf{i} + (19+4(-4))\mathbf{j} + (-1-2(-4))\mathbf{k}$	Substitutes their λ into an expression for $\overrightarrow{OP}$	M1
	$\overrightarrow{OP} = 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$	$2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$ or P(2, 3, 7)	A1
			[6]

Note: A similar method may be used by using $\overrightarrow{OP} = (0+\lambda)\mathbf{i} + (-5+4\lambda)\mathbf{j} + (11-2\lambda)\mathbf{k}$ and $\mathbf{d} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$

$\overrightarrow{OP} \bullet \mathbf{d} = 0$ yields $6 + \lambda + 4(-5 + 4\lambda) - 2(11 - 2\lambda) = 0$

This simplifies to $21\lambda - 42 = 0 \Rightarrow \lambda = 2$.

$\overrightarrow{OP} = (0+2)\mathbf{i} + (-5+4(2))\mathbf{j} + (11-2(2))\mathbf{k}$

$\overrightarrow{OP} = 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$

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Question Number	Scheme		Marks
Aliter (b) **Way 2**	$\overrightarrow{OP} = (6+\lambda)\mathbf{i} + (19+4\lambda)\mathbf{j} + (-1-2\lambda)\mathbf{k}$		
	$\overrightarrow{AP} = (6+\lambda-0)\mathbf{i} + (19+4\lambda+5)\mathbf{j} + (-1-2\lambda-11)\mathbf{k}$		
	direction vector or $l_1 = \mathbf{d} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$		
	$\overrightarrow{AP} \perp \overrightarrow{OP} \Rightarrow \underline{\overrightarrow{AP} \bullet \overrightarrow{OP} = 0}$	Allow <u>this statement</u> for M1 if $\overrightarrow{AP}$ and $\overrightarrow{OP}$ are defined as above.	
	ie. $\underline{\begin{pmatrix} 6+\lambda \\ 24+4\lambda \\ -12-2\lambda \end{pmatrix} \bullet \begin{pmatrix} 6+\lambda \\ 19+4\lambda \\ -1-2\lambda \end{pmatrix} = 0}$	<u>underlined statement</u>	M1
	$\therefore (6+\lambda)(6+\lambda) + (24+4\lambda)(19+4\lambda) + (-12-2\lambda)(-1-2\lambda) = 0$	Correct equation	A1 oe
	$36+12\lambda+\lambda^2+456+96\lambda+76\lambda+16\lambda^2+12+24\lambda+2\lambda+4\lambda^2 = 0$	Attempt to solve the equation in λ	dM1
	$21\lambda^2+210\lambda+504=0$		
	$\lambda^2+10\lambda+24=0 \Rightarrow (\lambda=-6)\ \underline{\lambda=-4}$	$\lambda = -4$	A1
	$\overrightarrow{OP} = (6-4)\mathbf{i} + (19+4(-4))\mathbf{j} + (-1-2(-4))\mathbf{k}$	Substitutes their λ into an expression for $\overrightarrow{OP}$	M1
	$\overrightarrow{OP} = 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$	$2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$ or P(2, 3, 7)	A1
			[6]

Note: A similar method to way 2 may be used by using $\overrightarrow{OP} = (5+\lambda)\mathbf{i} + (15+4\lambda)\mathbf{j} + (1-2\lambda)\mathbf{k}$
and $\overrightarrow{AP} = (5+\lambda-0)\mathbf{i} + (15+4\lambda+5)\mathbf{j} + (1-2\lambda-11)\mathbf{k}$
$\overrightarrow{AP} \bullet \overrightarrow{OP} = 0$ yields $(5+\lambda)(5+\lambda) + (20+4\lambda)(15+4\lambda) + (-10-2\lambda)(1-2\lambda) = 0$
This simplifies to $21\lambda^2+168\lambda+315=0$. $\lambda^2+8\lambda+15=0 \Rightarrow (\lambda=-5)\ \underline{\lambda=-3}$
$\overrightarrow{OP} = (5-3)\mathbf{i} + (15+4(-3))\mathbf{j} + (1-2(-3))\mathbf{k}$
$\overrightarrow{OP} = 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$

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Question Number	Scheme		Marks
5. (c)	$\overrightarrow{OP} = 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$		
	$\overrightarrow{OA} = 0\mathbf{i} - 5\mathbf{j} + 11\mathbf{k}$ and $\overrightarrow{OB} = 5\mathbf{i} + 15\mathbf{j} + \mathbf{k}$		
	$\overrightarrow{AP} = \pm(2\mathbf{i} + 8\mathbf{j} - 4\mathbf{k})$, $\overrightarrow{PB} = \pm(3\mathbf{i} + 12\mathbf{j} - 6\mathbf{k})$ $\overrightarrow{AB} = \pm(5\mathbf{i} + 20\mathbf{j} - 10\mathbf{k})$	Subtracting vectors to find any two of $\overrightarrow{AP}$, $\overrightarrow{PB}$ or $\overrightarrow{AB}$; and both are correctly ft using candidate's $\overrightarrow{OA}$ and $\overrightarrow{OP}$ found in parts (a) and (b) respectively.	M1; A1$\sqrt{\ }$ $\pm$
	As $\overrightarrow{AP} = \frac{2}{3}(3\mathbf{i} + 12\mathbf{j} - 6\mathbf{k}) = \frac{2}{3}\overrightarrow{PB}$ or $\overrightarrow{AB} = \frac{5}{2}(2\mathbf{i} + 8\mathbf{j} - 4\mathbf{k}) = \frac{5}{2}\overrightarrow{AP}$ or $\overrightarrow{AB} = \frac{5}{3}(3\mathbf{i} + 12\mathbf{j} - 6\mathbf{k}) = \frac{5}{3}\overrightarrow{PB}$ or $\overrightarrow{PB} = \frac{3}{2}(2\mathbf{i} + 8\mathbf{j} - 4\mathbf{k}) = \frac{3}{2}\overrightarrow{AP}$ or $\overrightarrow{AP} = \frac{2}{5}(5\mathbf{i} + 20\mathbf{j} - 10\mathbf{k}) = \frac{2}{5}\overrightarrow{AB}$ or $\overrightarrow{PB} = \frac{3}{5}(5\mathbf{i} + 20\mathbf{j} - 10\mathbf{k}) = \frac{3}{5}\overrightarrow{AB}$ etc…	$\overrightarrow{AP} = \frac{2}{3}\overrightarrow{PB}$ or $\overrightarrow{AB} = \frac{5}{2}\overrightarrow{AP}$ or $\overrightarrow{AB} = \frac{5}{3}\overrightarrow{PB}$ or $\overrightarrow{PB} = \frac{3}{2}\overrightarrow{AP}$ or $\overrightarrow{AP} = \frac{2}{5}\overrightarrow{AB}$ or $\overrightarrow{PB} = \frac{3}{5}\overrightarrow{AB}$	
	alternatively candidates could say for example that $\overrightarrow{AP} = 2(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k})$ $\overrightarrow{PB} = 3(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k})$		
	then <u>the points A, P and B are collinear</u>.	<u>A, P and B are collinear</u> Completely correct proof.	A1
	$\therefore \overrightarrow{AP} : \overrightarrow{PB} = 2:3$	2:3 or 1: $\frac{3}{2}$ or $\sqrt{84} : \sqrt{189}$ aef allow SC $\frac{2}{3}$	B1 oe **[4]**
Aliter **5.** (c) Way 2	At B; $\underline{5 = 6 + \lambda}$, $\underline{15 = 19 + 4\lambda}$ or $\underline{1 = -1 - 2\lambda}$ or at B; $\lambda = -1$	Writing down any of the three <u>underlined equations</u>.	M1
	gives $\lambda = -1$ for all three equations. or when $\lambda = -1$, this gives $\mathbf{r} = 5\mathbf{i} + 15\mathbf{j} + \mathbf{k}$	$\lambda = -1$for all three equations or $\lambda = -1$ gives $\mathbf{r} = 5\mathbf{i} + 15\mathbf{j} + \mathbf{k}$	A1
	<u>Hence B lies on l_1</u>. As stated in the question both A and P lie on l_1. $\therefore$ <u>A, P and B are collinear</u>.	<u>Must state B lies on l_1</u> $\Rightarrow$ A, P and B are collinear	A1
	$\therefore \overrightarrow{AP} : \overrightarrow{PB} = 2:3$	2:3 or aef	B1 oe
			[4]
			13 marks

Beware of candidates who will try to fudge that one vector is multiple of another for the final A mark in part (c).

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Question Number	Scheme		Marks
6. (a)			

x	1	1.5	2	2.5	3
y	0	0.5 ln 1.5	ln 2	1.5 ln 2.5	2 ln 3
or y	0	0.2027325541…	ln2	1.374436098…	2 ln 3

Question Number	Scheme		Marks
		Either 0.5 ln 1.5 and 1.5 ln 2.5 or awrt 0.20 and 1.37 (or mixture of decimals and ln's)	B1 **[1]**
(b)(i)	$I_1 \approx \frac{1}{2} \times 1 \times \underline{\{0 + 2(\ln 2) + 2\ln 3\}}$	<u>For structure of trapezium rule</u> $\underline{\{\ldots\ldots\ldots\ldots\}}$;	M1;
	$= \frac{1}{2} \times 3.583518938... = 1.791759... = 1.792$ (4sf)	1.792	A1 cao
(ii)	$I_2 \approx \frac{1}{2} \times 0.5\ ; \times \underline{\{0 + 2(0.5\ln 1.5 + \ln 2 + 1.5\ln 2.5) + 2\ln 3\}}$	Outside brackets $\frac{1}{2} \times 0.5$	B1;
		<u>For structure of trapezium rule</u> $\underline{\{\ldots\ldots\ldots\ldots\}}$;	M1 $\sqrt{}$
	$= \frac{1}{4} \times 6.737856242... = 1.684464...$	awrt 1.684	A1 **[5]**
(c)	With increasing ordinates, <u>the line segments at the top of the trapezia are closer to the curve.</u>	<u>Reason</u> or an appropriate diagram elaborating the correct reason.	B1 **[1]**

Beware: In part (b) candidate can add up the individual trapezia:

(b)(i) $I_1 \approx \frac{1}{2}(\underline{0 + \ln 2}) + \frac{1}{2}(\underline{\ln 2 + \ln 3})$

(ii) $I_2 \approx \frac{1}{2}.\frac{1}{2}(\underline{0 + 0.5\ln 1.5}) + \frac{1}{2}.\frac{1}{2}(\underline{0.5\ln 1.5 + \ln 2}) + \frac{1}{2}.\frac{1}{2}(\underline{\ln 2 + 1.5\ln 2.5}) + \frac{1}{2}.\frac{1}{2}(\underline{1.5\ln 2.5 + 2\ln 3})$

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Question Number	Scheme		Marks
6. (d)	$\begin{Bmatrix} u = \ln x & \Rightarrow & \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = x - 1 & \Rightarrow & v = \frac{x^2}{2} - x \end{Bmatrix}$	Use of 'integration by parts' formula in the correct direction	M1
	$I = \left(\frac{x^2}{2} - x\right)\ln x - \int \frac{1}{x}\left(\frac{x^2}{2} - x\right) dx$	Correct expression	A1
	$= \left(\frac{x^2}{2} - x\right)\ln x - \underline{\int\left(\frac{x}{2} - 1\right) dx}$	An attempt to multiply at least one term through by $\frac{1}{x}$ and an attempt to ...	
	$= \left(\frac{x^2}{2} - x\right)\ln x - \underline{\left(\frac{x^2}{4} - x\right)}\quad (+c)$	… integrate; <u>correct integration</u>	M1; A1
	$\therefore I = \left[\left(\frac{x^2}{2} - x\right)\ln x - \frac{x^2}{4} + x\right]_1^3$		
	$= \left(\frac{3}{2}\ln 3 - \frac{9}{4} + 3\right) - \left(-\frac{1}{2}\ln 1 - \frac{1}{4} + 1\right)$	Substitutes limits of 3 and 1 and subtracts.	ddM1
	$= \frac{3}{2}\ln 3 + \frac{3}{4} + 0 - \frac{3}{4} \quad = \underline{\frac{3}{2}\ln 3}$ **AG**	$\frac{3}{2}\ln 3$	A1 cso
			[6]
Aliter **6.** (d) **Way 2**	$\int (x-1)\ln x\,dx = \int x\ln x\,dx - \int \ln x\,dx$		
	$\int x\ln x\,dx = \frac{x^2}{2}\ln x - \int \frac{x^2}{2}.\left(\frac{1}{x}\right) dx$	Correct application of 'by parts'	M1
	$= \frac{x^2}{2}\ln x - \frac{x^2}{4}\quad (+c)$	Correct integration	A1
	$\int \ln x\,dx = x\ln x - \int x.\left(\frac{1}{x}\right) dx$	Correct application of 'by parts'	M1
	$= x\ln x - x\quad (+c)$	Correct integration	A1
	$\therefore \int_1^3 (x-1)\ln x\,dx = \left(\frac{9}{2}\ln 3 - 2\right) - \left(3\ln 3 - 2\right) = \frac{3}{2}\ln 3$ **AG**	Substitutes limits of 3 and 1 into both integrands and subtracts. $\frac{3}{2}\ln 3$	ddM1 A1 cso
			[6]

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Question Number	Scheme		Marks
Aliter **6.** (d) **Way 3**	$\left\{\begin{array}{ll} u = \ln x & \Rightarrow \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = (x-1) & \Rightarrow v = \frac{(x-1)^2}{2} \end{array}\right\}$	Use of 'integration by parts' formula in the correct direction	M1
	$I = \frac{(x-1)^2}{2}\ln x - \int \frac{(x-1)^2}{2x}\,dx$	Correct expression	A1
	$= \frac{(x-1)^2}{2}\ln x - \int \frac{x^2-2x+1}{2x}\,dx$	Candidate multiplies out numerator to obtain three terms…	
	$= \frac{(x-1)^2}{2}\ln x - \int \underline{\left(\frac{1}{2}x - 1 + \frac{1}{2x}\right)}\,dx$	… multiplies at least one term through by $\frac{1}{x}$ and then attempts to ...	
	$= \frac{(x-1)^2}{2}\ln x - \underline{\left(\frac{x^2}{4} - x + \frac{1}{2}\ln x\right)}\quad (+c)$	… integrate the result; <u>correct integration</u>	M1; A1
	$\therefore I = \left[\frac{(x-1)^2}{2}\ln x - \frac{x^2}{4} + x - \frac{1}{2}\ln x\right]_1^3$		
	$= \left(2\ln 3 - \frac{9}{4} + 3 - \frac{1}{2}\ln 3\right) - \left(0 - \frac{1}{4} + 1 - 0\right)$	Substitutes limits of 3 and 1 and subtracts.	ddM1
	$= 2\ln 3 - \frac{1}{2}\ln 3 + \frac{3}{4} + \frac{1}{4} - 1 \quad = \underline{\frac{3}{2}\ln 3}$ **AG**	$\frac{3}{2}\ln 3$	A1 cso
			[6]

Beware: $\int \frac{1}{2x}\,dx$ can also integrate to $\frac{1}{2}\ln 2x$

Beware: If you are marking using WAY 2 please make sure that you allocate the marks in the order they appear on the mark scheme. For example if a candidate only integrated lnx correctly then they would be awarded M0A0M1A1M0A0 on ePEN.

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Question Number	Scheme		Marks
Aliter **6.** (d) **Way 4**	By substitution $u = \ln x \quad \Rightarrow \quad \frac{du}{dx} = \frac{1}{x}$		
	$I = \int (e^u - 1).ue^u \ du$	Correct expression	
	$= \int u(e^{2u} - e^u)du$	Use of 'integration by parts' formula in the correct direction	M1
	$= u\left(\frac{1}{2}e^{2u} - e^u\right) - \int \underline{\left(\frac{1}{2}e^{2u} - e^u\right)} dx$	Correct expression	A1
	$= u\left(\frac{1}{2}e^{2u} - e^u\right) - \underline{\left(\frac{1}{4}e^{2u} - e^u\right)} \ (+c)$	Attempt <u>to integrate</u>; <u>correct integration</u>	M1; A1
	$\therefore I = \left[\frac{1}{2}ue^{2u} - ue^u - \frac{1}{4}e^{2u} + e^u\right]_{\ln 1}^{\ln 3}$		
	$= \left(\frac{9}{2}\ln 3 - 3\ln 3 - \frac{9}{4} + 3\right) - \left(0 - 0 - \frac{1}{4} + 1\right)$	Substitutes limits of ln3 and ln1 and subtracts.	ddM1
	$= \frac{3}{2}\ln 3 + \frac{3}{4} + \frac{1}{4} - 1 \quad = \underline{\frac{3}{2}\ln 3}$ **AG**	$\frac{3}{2}\ln 3$	A1 cso
			[6]
			13 marks

Question Number	Scheme		Marks
7. (a)	From question, $\frac{dS}{dt} = 8$	$\frac{dS}{dt} = 8$	B1
	$S = 6x^2 \Rightarrow \frac{dS}{dx} = 12x$	$\frac{dS}{dx} = 12x$	B1
	$\frac{dx}{dt} = \frac{dS}{dt} \div \frac{dS}{dx} = \underline{\frac{8}{12x}}$; $= \frac{\frac{2}{3}}{x} \Rightarrow \left(k = \frac{2}{3}\right)$	Candidate's $\frac{dS}{dt} \div \frac{dS}{dx}$; $\underline{\frac{8}{12x}}$	M1; <u>A1</u>oe
			[4]
(b)	$V = x^3 \Rightarrow \frac{dV}{dx} = 3x^2$	$\frac{dV}{dx} = 3x^2$	B1
	$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} = 3x^2.\left(\frac{2}{3x}\right)$; $= 2x$	Candidate's $\frac{dV}{dx} \times \frac{dx}{dt}$; λx	M1; A1 $\sqrt{}$
	As $x = V^{\frac{1}{3}}$, then $\frac{dV}{dt} = 2V^{\frac{1}{3}}$ **AG**	Use of $x = V^{\frac{1}{3}}$, to give $\frac{dV}{dt} = 2V^{\frac{1}{3}}$	A1
			[4]
(c)	$\int \frac{dV}{V^{\frac{1}{3}}} = \int 2\ dt$	Separates the variables with $\int \frac{dV}{V^{\frac{1}{3}}}$ or $\int V^{-\frac{1}{3}} dV$ on one side and $\int 2\ dt$ on the other side. integral signs not necessary.	B1
	$\int V^{-\frac{1}{3}}\ dV = \int 2\ dt$		
	$\frac{3}{2}V^{\frac{2}{3}} = 2t \ \ (+c)$	Attempts to integrate and … … must see $V^{\frac{2}{3}}$ and 2t; Correct equation with/without + c.	M1; A1
	$\frac{3}{2}(8)^{\frac{2}{3}} = 2(0) + c \Rightarrow c = 6$	Use of V = 8 and t = 0 in a changed equation containing c ; $c = 6$	M1∗; A1
	Hence: $\frac{3}{2}V^{\frac{2}{3}} = 2t + 6$		
	$\frac{3}{2}\left(16\sqrt{2}\right)^{\frac{2}{3}} = 2t + 6 \Rightarrow 12 = 2t + 6$	Having found their "c" candidate … … substitutes $V = 16\sqrt{2}$ into an equation involving V, t and "c".	depM1∗
	giving $t = 3$.	$t = 3$	A1 cao **[7]**
			15 marks

Question Number	Scheme		Marks
Aliter **7.** (b) **Way 2**	$x = V^{\frac{1}{3}}$ & $S = 6x^2 \Rightarrow S = 6V^{\frac{2}{3}}$	$S = 6V^{\frac{2}{3}}$	B1$\sqrt{}$
	$\frac{dS}{dV} = 4V^{-\frac{1}{3}}$ or $\frac{dV}{dS} = \frac{1}{4}V^{\frac{1}{3}}$	$\frac{dS}{dV} = 4V^{-\frac{1}{3}}$ or $\frac{dV}{dS} = \frac{1}{4}V^{\frac{1}{3}}$	B1
	$\frac{dV}{dt} = \frac{dS}{dt} \times \frac{dV}{dS} = 8.\left(\frac{1}{4V^{-\frac{1}{3}}}\right); = \frac{2}{V^{-\frac{1}{3}}} = 2V^{\frac{1}{3}}$ **AG**	Candidate's $\frac{dS}{dt} \times \frac{dV}{dS}$; $2V^{\frac{1}{3}}$	M1; A1
		In ePEN, award Marks for Way 2 in the order they appear on this mark scheme.	
			[4]
Aliter			
7. (c)	$\int \frac{dV}{2V^{\frac{1}{3}}} = \int 1\, dt$	Separates the variables with $\int \frac{dV}{2V^{\frac{1}{3}}}$ or $\int \frac{1}{2}V^{-\frac{1}{3}}dV$ oe on one side and $\int 1\, dt$ on the other side. integral signs not necessary.	B1
Way 2	$\frac{1}{2}\int V^{-\frac{1}{3}}\, dV = \int 1\, dt$		
	$\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)V^{\frac{2}{3}} = t \;\; (+c)$	Attempts to integrate and … … must see $V^{\frac{2}{3}}$ and t; Correct equation with/without + c.	M1; A1
	$\frac{3}{4}(8)^{\frac{2}{3}} = (0) + c \Rightarrow c = 3$	Use of V = 8 and t = 0 in a changed equation containing c ; $c = 3$	M1$*$; A1
	Hence: $\frac{3}{4}V^{\frac{2}{3}} = t + 3$		
	$\frac{3}{4}\left(16\sqrt{2}\right)^{\frac{2}{3}} = t + 3 \quad \Rightarrow \quad 6 = t + 3$	Having found their "c" candidate … … substitutes $V = 16\sqrt{2}$ into an equation involving V, t and "c".	depM1$*$
	giving t = 3.	t = 3	A1 cao **[7]**

Beware: On ePEN award the marks in part (c) in the order they appear on the mark scheme.

Question Number	Scheme		Marks
Aliter (b) **Way 3**	*similar to way 1.* $V = x^3 \Rightarrow \frac{dV}{dx} = 3x^2$	$\frac{dV}{dx} = 3x^2$	B1
	$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dS}{dt} \times \frac{dx}{dS} = 3x^2.8.\left(\frac{1}{12x}\right); = 2x$	Candidate's $\frac{dV}{dx} \times \frac{dS}{dt} \times \frac{dx}{dS}$; λx	M1; A1 $\sqrt{\ }$
	As $x = V^{\frac{1}{3}}$, then $\frac{dV}{dt} = 2V^{\frac{1}{3}}$ **AG**	Use of $x = V^{\frac{1}{3}}$, to give $\frac{dV}{dt} = 2V^{\frac{1}{3}}$	A1 **[4]**
Aliter (c) **Way 3**	$\int \frac{dV}{V^{\frac{1}{3}}} = \int 2\ dt$	Separates the variables with $\int \frac{dV}{V^{\frac{1}{3}}}$ or $\int V^{-\frac{1}{3}} dV$ on one side and $\int 2\ dt$ on the other side. integral signs not necessary.	B1
	$\int V^{-\frac{1}{3}}\ dV = \int 2\ dt$		
	$V^{\frac{2}{3}} = \frac{4}{3}t\ \ (+c)$	Attempts to integrate and … … must see $V^{\frac{2}{3}}$ and $\frac{4}{3}t$; Correct equation with/without + c.	M1; A1
	$(8)^{\frac{2}{3}} = \frac{4}{3}(0) + c \Rightarrow c = 4$	Use of V = 8 and t = 0 in a changed equation containing c ; $c = 4$	M1∗; A1
	Hence: $V^{\frac{2}{3}} = \frac{4}{3}t + 4$		
	$\left(16\sqrt{2}\right)^{\frac{2}{3}} = \frac{4}{3}t + 6 \quad \Rightarrow \quad 8 = \frac{4}{3}t + 4$	Having found their "c" candidate … … substitutes $V = 16\sqrt{2}$ into an equation involving V, t and "c".	depM1∗
	giving $t = 3$.	$t = 3$	A1 cao **[7]**

- **Beware** when marking question 7(c). There are a variety of valid ways that a candidate can use to find the constant "c".
- In questions 7(b) and 7(c) there may be "Ways" that I have not listed. Please use the mark scheme as a guide of how the mark the students' responses.
- In 7(c), if a candidate instead tries to solve the differential equation in part (a) escalate the response to your team leader.
- IF YOU ARE UNSURE ON HOW TO APPLY THE MARK SCHEME PLEASE ESCALATE THE RESPONSE UP TO YOUR TEAM LEADER VIA THE REVIEW SYSTEM.
- Note: dM1 denotes a method mark which is dependent upon the award of the previous method mark.
  ddM1 denotes a method mark which is dependent upon the award of the previous two method marks.
  depM1∗ denotes a method mark which is dependent upon the award of M1∗.

Mark Scheme (Results)

January 2007

GCE

GCE Mathematics

Core Mathematics C4 (6666)

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January 2007
6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme		Marks
	** represents a constant		
1.	$f(x) = (2-5x)^{-2} = \underline{(2)^{-2}}\left(1-\frac{5x}{2}\right)^{-2} = \underline{\frac{1}{4}}\left(1-\frac{5x}{2}\right)^{-2}$	Takes 2 outside the bracket to give any of $(2)^{-2}$ or $\frac{1}{4}$.	B1
	$= \frac{1}{4}\left\{\underline{1+(-2)(**x);+\frac{(-2)(-3)}{2!}(**x)^2+\frac{(-2)(-3)(-4)}{3!}(**x)^3+...}\right\}$	Expands $(1+**x)^{-2}$ to give an unsimplified $1+(-2)(**x)$;	M1
		A correct unsimplified $\{\underline{..........}\}$ expansion with candidate's $(**x)$	A1
	$= \frac{1}{4}\left\{\underline{1+(-2)(\tfrac{-5x}{2});+\frac{(-2)(-3)}{2!}(\tfrac{-5x}{2})^2+\frac{(-2)(-3)(-4)}{3!}(\tfrac{-5x}{2})^3+...}\right\}$		
	$= \frac{1}{4}\left\{1+5x;+\frac{75x^2}{4}+\frac{125x^3}{2}+...\right\}$		
	$= \frac{1}{4}+\frac{5x}{4};+\frac{75x^2}{16}+\frac{125x^3}{8}+...$	Anything that cancels to $\frac{1}{4}+\frac{5x}{4}$;	A1;
		Simplified $\frac{75x^2}{16}+\frac{125x^3}{8}$	A1
	$= \frac{1}{4}+1\frac{1}{4}x;+4\frac{11}{16}x^2+15\frac{5}{8}x^3+...$		
			[5]
			5 marks

Question Number	Scheme		Marks
Aliter 1. **Way 2**	$f(x) = (2-5x)^{-2}$		
		$\frac{1}{4}$ or $(2)^{-2}$	B1
	$= \left\{ (2)^{-2} + (-2)(2)^{-3}(**x); + \frac{(-2)(-3)}{2!}(2)^{-4}(**x)^2 + \frac{(-2)(-3)(-4)}{3!}(2)^{-5}(**x)^3 + ... \right\}$	Expands $(2-5x)^{-2}$ to give an unsimplifed $(2)^{-2} + (-2)(2)^{-3}(**x)$;	M1
		A correct unsimplified $\{\underline{..........}\}$ expansion with candidate's $(**x)$	A1
	$= \left\{ (2)^{-2} + (-2)(2)^{-3}(-5x); + \frac{(-2)(-3)}{2!}(2)^{-4}(-5x)^2 + \frac{(-2)(-3)(-4)}{3!}(2)^{-5}(-5x)^3 + ... \right\}$		
	$= \left\{ \frac{1}{4} + (-2)(\frac{1}{8})(-5x); + (3)(\frac{1}{16})(25x^2) + (-4)(\frac{1}{16})(-125x^3) + ... \right\}$		
	$= \frac{1}{4} + \frac{5x}{4}; + \frac{75x^2}{16} + \frac{125x^3}{8} + ...$	Anything that cancels to $\frac{1}{4} + \frac{5x}{4}$;	A1;
		Simplified $\frac{75x^2}{16} + \frac{125x^3}{8}$	A1
	$= \frac{1}{4} + 1\frac{1}{4}x; + 4\frac{11}{16}x^2 + 15\frac{5}{8}x^3 + ...$		
			[5]
			5 marks

Attempts using Maclaurin expansions need to be referred to your team leader.

Question Number	Scheme		Marks
2. (a)	Volume $=\pi\int_{-\frac{1}{4}}^{\frac{1}{2}}\left(\frac{1}{3(1+2x)}\right)^2 dx = \frac{\pi}{9}\int_{-\frac{1}{4}}^{\frac{1}{2}}\frac{1}{(1+2x)^2}dx$	**Use** of $V=\pi\int y^2\,dx$. Can be implied. Ignore limits.	B1
	$=\left(\frac{\pi}{9}\right)\int_{-\frac{1}{4}}^{\frac{1}{2}}(1+2x)^{-2}\,dx$	Moving their power to the top. **(Do not allow power of -1.)** Can be implied. Ignore limits and $\frac{\pi}{9}$	M1
	$=\left(\frac{\pi}{9}\right)\left[\frac{(1+2x)^{-1}}{(-1)(2)}\right]_{-\frac{1}{4}}^{\frac{1}{2}}$	Integrating to give $\pm p(1+2x)^{-1}$	M1
		$-\frac{1}{2}(1+2x)^{-1}$	A1
	$=\left(\frac{\pi}{9}\right)\left[-\frac{1}{2}(1+2x)^{-1}\right]_{-\frac{1}{4}}^{\frac{1}{2}}$		
	$=\left(\frac{\pi}{9}\right)\left[\left(\frac{-1}{2(2)}\right)-\left(\frac{-1}{2(\frac{1}{2})}\right)\right]$		
	$=\left(\frac{\pi}{9}\right)\left[-\frac{1}{4}-(-1)\right]$		
	$=\frac{\pi}{12}$	Use of limits to give exact values of $\frac{\pi}{12}$ or $\frac{3\pi}{36}$ or $\frac{2\pi}{24}$ or aef	A1 aef
			[5]
(b)	From Fig.1, AB $=\frac{1}{2}-\left(-\frac{1}{4}\right)=\frac{3}{4}$ units		
	As $\frac{3}{4}$ units $\equiv$ 3cm		
	then scale factor $k=\frac{3}{\left(\frac{3}{4}\right)}=4$.		
	Hence Volume of paperweight $=(4)^3\left(\frac{\pi}{12}\right)$	$(4)^3\times$(their answer to part (a))	M1
	$V=\frac{16\pi}{3}\text{ cm}^3=16.75516...\text{ cm}^3$	$\frac{16\pi}{3}$ or awrt 16.8 or $\frac{64\pi}{12}$ or aef	A1
			[2]
			7 marks

Note: $\frac{\pi}{9}$ (or implied) is not needed for the middle three marks of question 2(a).

Question Number	Scheme		Marks
Aliter 2. (a) **Way 2**	Volume $= \underline{\pi \int_{-\frac{1}{4}}^{\frac{1}{2}} \left(\frac{1}{3(1+2x)}\right)^2 \mathrm{d}x} = \pi \int_{-\frac{1}{4}}^{\frac{1}{2}} \frac{1}{(3+6x)^2} \mathrm{d}x$	**Use** of $V = \underline{\pi\int y^2 \, \mathrm{d}x}$. Can be implied. Ignore limits.	B1
	$= (\pi)\int_{-\frac{1}{4}}^{\frac{1}{2}} (3+6x)^{-2} \, \mathrm{d}x$	Moving their power to the top. **(Do not allow power of -1.)** Can be implied. Ignore limits and π	M1
	$= (\pi)\left[\frac{(3+6x)^{-1}}{(-1)(6)}\right]_{-\frac{1}{4}}^{\frac{1}{2}}$	Integrating to give $\underline{\pm p(3+6x)^{-1}}$ $\underline{-\frac{1}{6}(3+6x)^{-1}}$	M1 A1
	$= (\pi)\left[\ \underline{-\frac{1}{6}(3+6x)^{-1}}\ \right]_{-\frac{1}{4}}^{\frac{1}{2}}$		
	$= (\pi)\left[\left(\frac{-1}{6(6)}\right) - \left(\frac{-1}{6(\frac{3}{2})}\right)\right]$		
	$= (\pi)\left[-\frac{1}{36} - (-\frac{1}{9})\right]$		
	$= \frac{\pi}{12}$	Use of limits to give exact values of $\frac{\pi}{12}$ or $\frac{3\pi}{36}$ or $\frac{2\pi}{24}$ or aef	A1 aef **[5]**

Note: π is not needed for the middle three marks of question 2(a).

Question Number	Scheme		Marks
3. (a)	$x = 7\cos t - \cos 7t\,,\ \ y = 7\sin t - \sin 7t\,,$		
	$\frac{dx}{dt} = -7\sin t + 7\sin 7t\,,\ \ \frac{dy}{dt} = 7\cos t - 7\cos 7t$	Attempt to differentiate x **and** y with respect to t to give $\frac{dx}{dt}$ in the form $\pm A\sin t \pm B\sin 7t$; $\frac{dy}{dt}$ in the form $\pm C\cos t \pm D\cos 7t$	M1
		Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$	A1
	$\therefore \frac{dy}{dx} = \frac{7\cos t - 7\cos 7t}{-7\sin t + 7\sin 7t}$	Candidate's $\frac{dy/dt}{dx/dt}$	B1 $\sqrt{}$
			[3]
(b)	When $t = \frac{\pi}{6}$, m(**T**) $= \frac{dy}{dx} = \frac{7\cos\frac{\pi}{6} - 7\cos\frac{7\pi}{6}}{-7\sin\frac{\pi}{6} + 7\sin\frac{7\pi}{6}}$;	Substitutes $t = \frac{\pi}{6}$ or 30° into their $\frac{dy}{dx}$ expression;	M1
	$= \underline{\frac{\frac{7\sqrt{3}}{2} - \left(-\frac{7\sqrt{3}}{2}\right)}{-\frac{7}{2} - \frac{7}{2}}} = \underline{\frac{7\sqrt{3}}{-7}} = \underline{-\sqrt{3}} = \underline{\text{awrt } -1.73}$	to give any of the four underlined expressions oe **(must be correct solution only)**	A1 **cso**
	Hence m(**N**) $= \frac{-1}{-\sqrt{3}}$ or $\frac{1}{\sqrt{3}}$ $=$ awrt 0.58	Uses m(**T**) to 'correctly' find m(**N**). Can be ft from "their tangent gradient".	A1$\sqrt{}$ oe.
	When $t = \frac{\pi}{6}$, $x = 7\cos\frac{\pi}{6} - \cos\frac{7\pi}{6} = \frac{7\sqrt{3}}{2} - \left(-\frac{\sqrt{3}}{2}\right) = \frac{8\sqrt{3}}{2} = 4\sqrt{3}$ $y = 7\sin\frac{\pi}{6} - \sin\frac{7\pi}{6} = \frac{7}{2} - \left(-\frac{1}{2}\right) = \frac{8}{2} = 4$	The point $\underline{\left(4\sqrt{3},\ 4\right)}$ or $\underline{\left(\text{awrt } 6.9,\ 4\right)}$	B1
	N: $y - 4 = \frac{1}{\sqrt{3}}\left(x - 4\sqrt{3}\right)$	Finding an equation of a normal with their point and their normal gradient or finds c by using $y = (\text{their gradient})x + "c"$.	M1
	N: $\underline{y = \frac{1}{\sqrt{3}}x}$ or $\underline{y = \frac{\sqrt{3}}{3}x}$ or $\underline{3y = \sqrt{3}x}$	Correct simplified EXACT equation of <u>normal</u>. This is dependent on candidate using correct $\left(4\sqrt{3},\ 4\right)$	<u>A1</u> oe
	or $4 = \frac{1}{\sqrt{3}}\left(4\sqrt{3}\right) + c \Rightarrow c = 4 - 4 = 0$		
	Hence **N**: $\underline{y = \frac{1}{\sqrt{3}}x}$ or $\underline{y = \frac{\sqrt{3}}{3}x}$ or $\underline{3y = \sqrt{3}x}$		
			[6] **9 marks**

Question Number	Scheme		Marks
Aliter 3. (a) **Way 2**	$x = 7\cos t - \cos 7t\,,\ \ y = 7\sin t - \sin 7t\,,$		
	$\frac{dx}{dt} = -7\sin t + 7\sin 7t\,,\ \ \frac{dy}{dt} = 7\cos t - 7\cos 7t$	Attempt to differentiate x **and** y with respect to t to give $\frac{dx}{dt}$ in the form $\pm A\sin t \pm B\sin 7t$ $\frac{dy}{dt}$ in the form $\pm C\cos t \pm D\cos 7t$	M1
		Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$	A1
	$\frac{dy}{dx} = \frac{7\cos t - 7\cos 7t}{-7\sin t + 7\sin 7t} = \frac{-7(-2\sin 4t\sin 3t)}{-7(2\cos 4t\sin 3t)} = \tan 4t$	Candidate's $\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$	B1 $\sqrt{}$
			[3]
(b)	When $t = \frac{\pi}{6}$, m(**T**) $= \frac{dy}{dx} = \tan\frac{4\pi}{6}$;	Substitutes $t = \frac{\pi}{6}$ or 30° into their $\frac{dy}{dx}$ expression;	M1
	$= \underline{\frac{2\left(\frac{\sqrt{3}}{2}\right)(1)}{2\left(-\frac{1}{2}\right)(1)}} = \underline{-\sqrt{3}} = \underline{\text{awrt } -1.73}$	to give any of the three underlined expressions oe **(must be correct solution only)**	A1 **cso**
	Hence m(**N**) $= \frac{-1}{-\sqrt{3}}$ or $\frac{1}{\sqrt{3}}$ = awrt 0.58	Uses m(**T**) to 'correctly' find m(**N**). Can be ft from "their tangent gradient".	A1$\sqrt{}$ oe.
	When $t = \frac{\pi}{6}$, $x = 7\cos\frac{\pi}{6} - \cos\frac{7\pi}{6} = \frac{7\sqrt{3}}{2} - \left(-\frac{\sqrt{3}}{2}\right) = \frac{8\sqrt{3}}{2} = 4\sqrt{3}$ $y = 7\sin\frac{\pi}{6} - \sin\frac{7\pi}{6} = \frac{7}{2} - \left(-\frac{1}{2}\right) = \frac{8}{2} = 4$	The point $\underline{\left(4\sqrt{3},\ 4\right)}$ or $\underline{\left(\text{awrt } 6.9,\ 4\right)}$	B1
	N: $y - 4 = \frac{1}{\sqrt{3}}\left(x - 4\sqrt{3}\right)$	Finding an equation of a normal with their point and their normal gradient or finds c by using $y = (\text{their gradient})x + \text{"c"}$.	M1
	N: $\underline{y = \frac{1}{\sqrt{3}}x}$ or $\underline{y = \frac{\sqrt{3}}{3}x}$ or $\underline{3y = \sqrt{3}x}$	Correct simplified EXACT equation of <u>normal</u>. This is dependent on candidate using correct $\left(4\sqrt{3},\ 4\right)$	<u>A1</u> oe
	or $4 = \frac{1}{\sqrt{3}}\left(4\sqrt{3}\right) + c \Rightarrow c = 4 - 4 = 0$		
	Hence **N**: $\underline{y = \frac{1}{\sqrt{3}}x}$ or $\underline{y = \frac{\sqrt{3}}{3}x}$ or $\underline{3y = \sqrt{3}x}$		
			[6]
			9 marks

Beware: A candidate finding an m(**T**) = 0 can obtain A1ft for m(**N**) $\rightarrow \infty$, but obtains M0 if they write $y - 4 = \infty(x - 4\sqrt{3})$. If they write, however, **N**: $x = 4\sqrt{3}$, then they can score M1.

Beware: A candidate finding an m(**T**) = ∞ can obtain A1ft for m(**N**) = 0, and also obtains M1 if they write $y - 4 = 0(x - 4\sqrt{3})$ or y = 4.

Question Number	Scheme		Marks
4. (a)	$\frac{2x-1}{(x-1)(2x-3)} \equiv \frac{A}{(x-1)} + \frac{B}{(2x-3)}$		
	$2x-1 \equiv A(2x-3)+B(x-1)$	Forming this identity. **NB**: A & B are not assigned in this question	M1
	Let $x=\frac{3}{2}$, $2 = B\left(\frac{1}{2}\right) \Rightarrow B = 4$		
	Let $x=1$, $1 = A(-1) \Rightarrow A = -1$	either one of $A = -1$ or $B = 4$. both correct for their A, B.	A1 A1
	giving $\frac{-1}{(x-1)} + \frac{4}{(2x-3)}$		**[3]**
(b) & (c)	$\int \frac{dy}{y} = \int \frac{(2x-1)}{(2x-3)(x-1)}\,dx$	Separates variables as shown Can be implied	B1
	$= \int \frac{-1}{(x-1)} + \frac{4}{(2x-3)}\,dx$	Replaces RHS with their partial fraction to be integrated.	M1 $\sqrt{}$
	$\therefore \ln y = -\ln(x-1) + 2\ln(2x-3) + c$	***At least*** two terms in ln's ***At least*** two ln terms correct All three terms correct and '+ c'	M1 A1 $\sqrt{}$ A1 **[5]**
	$y=10, x=2$ gives $c=\ln 10$	$c = \ln 10$	B1
	$\therefore \ln y = -\ln(x-1) + 2\ln(2x-3) + \ln 10$		
	$\ln y = -\ln(x-1) + \ln(2x-3)^2 + \ln 10$	Using the power law for logarithms	M1
	$\ln y = \ln\left(\frac{(2x-3)^2}{(x-1)}\right) + \ln 10$ or $\ln y = \ln\left(\frac{10(2x-3)^2}{(x-1)}\right)$	Using the product and/or quotient laws for logarithms to obtain a single RHS logarithmic term with/without constant c.	M1
	$\underline{y = \frac{10(2x-3)^2}{(x-1)}}$	$\underline{y = \frac{10(2x-3)^2}{(x-1)}}$ or aef. isw	A1 aef **[4]**
			12 marks

Question Number	Scheme		Marks
Aliter 4. (b) & (c) **Way 2**	$\int \frac{dy}{y} = \int \frac{(2x-1)}{(2x-3)(x-1)}\,dx$	Separates variables as shown Can be implied	B1
	$= \int \frac{-1}{(x-1)} + \frac{4}{(2x-3)}\,dx$	Replaces RHS with their partial fraction to be integrated.	M1 $\sqrt{}$
	$\therefore \ln y = -\ln(x-1) + 2\ln(2x-3) + c$	***At least*** two terms in ln's ***At least*** two ln terms correct All three terms correct and '+ c'	M1 A1 $\sqrt{}$ A1
	See below for the award of B1	***decide to award B1 here!!***	B1
	$\ln y = -\ln(x-1) + \ln(2x-3)^2 + c$	Using the power law for logarithms	M1
	$\ln y = \ln\left(\frac{(2x-3)^2}{x-1}\right) + c$	Using the product and/or quotient laws for logarithms to obtain a single RHS logarithmic term with/without constant c.	M1
	$\ln y = \ln\left(\frac{A(2x-3)^2}{x-1}\right)$ where $c = \ln A$		
	or $e^{\ln y} = e^{\ln\left(\frac{(2x-3)^2}{x-1}\right)+c} = e^{\ln\left(\frac{(2x-3)^2}{x-1}\right)}e^{c}$		
	$y = \frac{A(2x-3)^2}{(x-1)}$		
	$y = 10,\ x = 2$ gives $A = 10$	$A = 10$ for B1	award above
	$\underline{y = \frac{10(2x-3)^2}{(x-1)}}$	$\underline{y = \frac{10(2x-3)^2}{(x-1)}}$ or aef & isw	A1 aef
			[5] & [4]

Note: The B1 mark (part (c)) should be awarded in the same place on ePEN as in the Way 1 approach.

Question Number	Scheme		Marks
Aliter (b) & (c) **Way 3**	$\int \frac{dy}{y} = \int \frac{(2x-1)}{(2x-3)(x-1)}\,dx$	Separates variables as shown Can be implied	B1
	$= \int \frac{-1}{(x-1)} + \frac{2}{(x-\frac{3}{2})}\,dx$	Replaces RHS with their partial fraction to be integrated.	M1 $\sqrt{\ }$
	$\therefore \ln y = -\ln(x-1) + 2\ln(x-\frac{3}{2}) + c$	***At least*** two terms in ln's ***At least*** two ln terms correct All three terms correct and '+ c'	M1 A1 $\sqrt{\ }$ A1 **[5]**
	$y = 10,\ x = 2$ gives $c = \underline{\ln 10 - 2\ln\left(\frac{1}{2}\right)} = \underline{\ln 40}$	$c = \ln 10 - 2\ln\left(\frac{1}{2}\right)$ or $c = \ln 40$	B1 oe
	$\therefore \ln y = -\ln(x-1) + 2\ln(x-\frac{3}{2}) + \ln 40$		
	$\ln y = -\ln(x-1) + \ln(x-\frac{3}{2})^2 + \ln 10$	Using the power law for logarithms	M1
	$\ln y = \ln\left(\frac{(x-\frac{3}{2})^2}{(x-1)}\right) + \ln 40$ or $\ln y = \ln\left(\frac{40(x-\frac{3}{2})^2}{(x-1)}\right)$	Using the product and/or quotient laws for logarithms to obtain a single RHS logarithmic term with/without constant c.	M1
	$\underline{y = \frac{40(x-\frac{3}{2})^2}{(x-1)}}$	$\underline{y = \frac{40(x-\frac{3}{2})^2}{(x-1)}}$ or aef. isw	A1 aef **[4]**

Note: Please mark parts (b) and (c) together for any of the three ways.

Question Number	Scheme		Marks
5. (a)	$\sin x + \cos y = 0.5 \qquad (\text{ eqn } *)$		
	$\left\{\frac{\cancel{d}y}{\cancel{d}x} \cancel{=}\right\} \quad \cos x - \sin y\frac{dy}{dx} = 0 \qquad (\text{ eqn \# })$	Differentiates implicitly to include $\pm\sin y\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$.)	M1
	$\frac{dy}{dx} = \underline{\frac{\cos x}{\sin y}}$	$\underline{\frac{\cos x}{\sin y}}$	A1 **cso**
			[2]
(b)	$\frac{dy}{dx} = 0 \Rightarrow \frac{\cos x}{\sin y} = 0 \Rightarrow \cos x = 0$	Candidate realises that they need to solve 'their numerator' = 0 …or candidate sets $\frac{dy}{dx} = 0$ in their (eqn #) and attempts to solve the resulting equation.	M1 $\sqrt{}$
	giving $\underline{x = -\frac{\pi}{2}}$ or $\underline{x = \frac{\pi}{2}}$	both $\underline{x = -\frac{\pi}{2}, \frac{\pi}{2}}$ or $\underline{x = \pm 90^\circ}$ or awrt $\underline{x = \pm 1.57}$ required here	A1
	When $x = -\frac{\pi}{2}$, $\sin\left(-\frac{\pi}{2}\right) + \cos y = 0.5$ When $x = \frac{\pi}{2}$, $\sin\left(\frac{\pi}{2}\right) + \cos y = 0.5$	Substitutes either their $x = \frac{\pi}{2}$ or $x = -\frac{\pi}{2}$ into eqn $*$	M1
	$\Rightarrow \cos y = 1.5 \Rightarrow$ y has no solutions $\Rightarrow \cos y = -0.5 \Rightarrow y = \frac{2\pi}{3}$ or $-\frac{2\pi}{3}$	Only one of $y = \underline{\frac{2\pi}{3}}$ or $\underline{-\frac{2\pi}{3}}$ or $\underline{120^\circ}$ or $\underline{-120^\circ}$ or awrt $\underline{-2.09}$ or awrt $\underline{2.09}$	A1
	In specified range $(x, y) = \underline{\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)}$ and $\underline{\left(\frac{\pi}{2}, -\frac{2\pi}{3}\right)}$	Only exact coordinates of $\underline{\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)}$ and $\underline{\left(\frac{\pi}{2}, -\frac{2\pi}{3}\right)}$ **Do not award this mark if candidate states other coordinates inside the required range.**	A1
			[5]
			7 marks

Question Number	Scheme		Marks
6.	$y = 2^x = e^{x\ln 2}$		
(a) **Way 1**	$\frac{dy}{dx} = \ln 2 . e^{x\ln 2}$	$\frac{dy}{dx} = \ln 2 . e^{x\ln 2}$	M1
	Hence $\frac{dy}{dx} = \ln 2 . (2^x) = 2^x \ln 2$ **AG**	$2^x \ln 2$ **AG**	A1 **cso**
			[2]
Aliter (a) **Way 2**	$\ln y = \ln\left(2^x\right)$ leads to $\ln y = x \ln 2$ $\frac{1}{y}\frac{dy}{dx} = \ln 2$	Takes logs of both sides, then uses the power law of logarithms… … and differentiates implicitly to give $\frac{1}{y}\frac{dy}{dx} = \ln 2$	M1
	Hence $\frac{dy}{dx} = y \ln 2 = 2^x \ln 2$ **AG**	$2^x \ln 2$ **AG**	A1 **cso**
			[2]
(b)	$y = 2^{(x^2)} \Rightarrow \frac{dy}{dx} = 2x . 2^{(x^2)} . \ln 2$	$Ax\ 2^{(x^2)}$ $2x . 2^{(x^2)} . \ln 2$ or $2x . y . \ln 2$ if y is defined	M1 A1
	When x = 2, $\frac{dy}{dx} = 2(2)2^4 \ln 2$	Substitutes x = 2 into their $\frac{dy}{dx}$ which is of the form $\pm k\, 2^{(x^2)}$ or $Ax\ 2^{(x^2)}$	M1
	$\frac{dy}{dx} = \underline{64 \ln 2} = 44.3614...$	$\underline{64 \ln 2}$ or awrt 44.4	A1
			[4]
			6 marks

Question Number	Scheme		Marks
Aliter 6. (b) **Way 2**	$\ln y = \ln\left(2^{x^2}\right)$ leads to $\ln y = x^2 \ln 2$		
	$\frac{1}{y}\frac{dy}{dx} = 2x.\ln 2$	$\frac{1}{y}\frac{dy}{dx} = Ax.\ln 2$	M1
		$\frac{1}{y}\frac{dy}{dx} = 2x.\ln 2$	A1
	When x = 2, $\frac{dy}{dx} = 2(2)2^4 \ln 2$	Substitutes x = 2 into their $\frac{dy}{dx}$ which is of the form $\pm k\,2^{(x^2)}$ or $Ax\ 2^{(x^2)}$	M1
	$\frac{dy}{dx} = \underline{64\ln 2} = 44.3614...$	$\underline{64\ln 2}$ or awrt 44.4	A1
			[4]

Question Number	Scheme		Marks
7.	$\mathbf{a} = \overrightarrow{OA} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k} \Rightarrow \lvert\overrightarrow{OA}\rvert = 3$ $\mathbf{b} = \overrightarrow{OB} = \mathbf{i} + \mathbf{j} - 4\mathbf{k} \Rightarrow \lvert\overrightarrow{OB}\rvert = \sqrt{18}$ $\overrightarrow{BC} = \pm(2\mathbf{i} + 2\mathbf{j} + \mathbf{k}) \Rightarrow \lvert\overrightarrow{BC}\rvert = 3$ $\overrightarrow{AC} = \pm(\mathbf{i} + \mathbf{j} - 4\mathbf{k}) \Rightarrow \lvert\overrightarrow{AC}\rvert = \sqrt{18}$		
(a)	$\mathbf{c} = \overrightarrow{OC} = \underline{3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}}$	$\underline{3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}}$	B1 **cao** **[1]**
(b)	$\overrightarrow{OA} \bullet \overrightarrow{OB} = \begin{pmatrix}2\\2\\1\end{pmatrix} \bullet \begin{pmatrix}1\\1\\-4\end{pmatrix} = \underline{2+2-4} = 0$ or… $\overrightarrow{BO} \bullet \overrightarrow{BC} = \begin{pmatrix}-1\\-1\\4\end{pmatrix} \bullet \begin{pmatrix}2\\2\\1\end{pmatrix} = \underline{-2-2+4} = 0$ or… $\overrightarrow{AC} \bullet \overrightarrow{BC} = \begin{pmatrix}1\\1\\-4\end{pmatrix} \bullet \begin{pmatrix}2\\2\\1\end{pmatrix} = \underline{2+2-4} = 0$ or… $\overrightarrow{AO} \bullet \overrightarrow{AC} = \begin{pmatrix}-2\\-2\\-1\end{pmatrix} \bullet \begin{pmatrix}1\\1\\-4\end{pmatrix} = \underline{-2-2+4} = 0$	An attempt to take the dot product between either $\overrightarrow{OA}$ and $\overrightarrow{OB}$, $\overrightarrow{OA}$ and $\overrightarrow{AC}$, $\overrightarrow{AC}$ and $\overrightarrow{BC}$ or $\overrightarrow{OB}$ and $\overrightarrow{BC}$ Showing the result is equal to zero.	<u>M1</u> A1
	and therefore OA is perpendicular to OB and hence OACB is a rectangle.	<u>perpendicular</u> and <u>OACB is a rectangle</u>	A1 **cso**
		Using distance formula to find either the correct height or width.	M1
	Area $= 3 \times \sqrt{18} = 3\sqrt{18} = 9\sqrt{2}$	Multiplying the rectangle's height by its width.	M1
		exact value of $3\sqrt{18}$, $9\sqrt{2}$, $\sqrt{162}$ or aef	A1 **[6]**
(c)	$\overrightarrow{OD} = \mathbf{d} = \frac{1}{2}(3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k})$	$\underline{\frac{1}{2}(3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k})}$	B1 **[1]**

Question Number	Scheme		Marks
	using dot product formula		
(d)	$\overrightarrow{DA} = \pm\left(\frac{1}{2}\mathbf{i} + \frac{1}{2}\mathbf{j} + \frac{5}{2}\mathbf{k}\right)$ & $\overrightarrow{DC} = \pm\left(\frac{3}{2}\mathbf{i} + \frac{3}{2}\mathbf{j} - \frac{3}{2}\mathbf{k}\right)$	Identifies a set of two relevant vectors	M1
Way 1	or $\overrightarrow{BA} = \pm(\mathbf{i} + \mathbf{j} + 5\mathbf{k})$ & $\overrightarrow{OC} = \pm(3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k})$	Correct vectors $\pm$	A1
	$\cos D = (\pm)\dfrac{\begin{pmatrix}0.5\\0.5\\2.5\end{pmatrix}\bullet\begin{pmatrix}1.5\\1.5\\-1.5\end{pmatrix}}{\frac{\sqrt{27}}{2}.\frac{\sqrt{27}}{2}} = (\pm)\dfrac{\frac{3}{4}+\frac{3}{4}-\frac{15}{4}}{\frac{27}{4}} = (\pm)\dfrac{1}{3}$	Applies dot product formula on multiples of these vectors. <u>Correct ft. application of dot product formula</u>	
	$D = \cos^{-1}\left(-\frac{1}{3}\right)$	Attempts to find the correct angle D rather than $180^\circ - D$.	ddM1 $\sqrt{}$
	$D = 109.47122...^\circ$	109.5° or awrt 109° or 1.91^c	A1
Aliter	*using dot product formula and direction vectors*		**[6]**
(d)	$\mathrm{d}\overrightarrow{BA} = \pm(\mathbf{i} + \mathbf{j} + 5\mathbf{k})$ & $\mathrm{d}\overrightarrow{OC} = \pm(\mathbf{i} + \mathbf{j} - \mathbf{k})$	Identifies a set of two direction vectors	M1
Way 2		Correct vectors $\pm$	A1
	$\cos D = (\pm)\dfrac{\begin{pmatrix}1\\1\\-1\end{pmatrix}\bullet\begin{pmatrix}1\\1\\5\end{pmatrix}}{\sqrt{3}.\sqrt{27}} = (\pm)\dfrac{1+1-5}{\sqrt{3}.\sqrt{27}} = (\pm)\dfrac{1}{3}$	Applies dot product formula on multiples of these vectors. <u>Correct ft. application of dot product formula.</u>	dM1 A1 $\sqrt{}$
	$D = \cos^{-1}\left(-\frac{1}{3}\right)$	Attempts to find the correct angle D rather than $180^\circ - D$.	ddM1 $\sqrt{}$
	$D = 109.47122...^\circ$	109.5° or awrt 109° or 1.91^c	A1 **[6]**

Question Number	Scheme		Marks
Aliter	*using dot product formula and similar triangles*		
(d)	$\mathbf{d}\overrightarrow{OA} = (2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ & $\mathbf{d}\overrightarrow{OC} = (\mathbf{i} + \mathbf{j} - \mathbf{k})$	Identifies a set of two direction vectors	M1
		Correct vectors	A1
Way 3			
	$\cos\left(\tfrac{1}{2}D\right) = \dfrac{\begin{pmatrix}2\\2\\1\end{pmatrix}\bullet\begin{pmatrix}1\\1\\-1\end{pmatrix}}{\sqrt{9}.\sqrt{3}} = \underline{\dfrac{2+2-1}{\sqrt{9}.\sqrt{3}}} = \underline{\dfrac{1}{\sqrt{3}}}$	Applies dot product formula on multiples of these vectors.	dM1
		<u>Correct ft. application of dot product formula.</u>	A1 $\sqrt{}$
	$D = 2\cos^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$	Attempts to find the correct angle D by doubling their angle for $\tfrac{1}{2}D$.	ddM1 $\sqrt{}$
	$D = 109.47122...^{\circ}$	109.5° or awrt 109° or 1.91^{c}	A1
			[6]
Aliter	*using cosine rule*		
(d)	$\overrightarrow{DA} = \tfrac{1}{2}\mathbf{i} + \tfrac{1}{2}\mathbf{j} + \tfrac{5}{2}\mathbf{k}$, $\overrightarrow{DC} = \tfrac{3}{2}\mathbf{i} + \tfrac{3}{2}\mathbf{j} - \tfrac{3}{2}\mathbf{k}$, $\overrightarrow{AC} = \mathbf{i} + \mathbf{j} - 4\mathbf{k}$		
Way 4			
	$\lvert\overrightarrow{DA}\rvert = \dfrac{\sqrt{27}}{2}$, $\lvert\overrightarrow{DC}\rvert = \dfrac{\sqrt{27}}{2}$, $\lvert\overrightarrow{AC}\rvert = \sqrt{18}$	Attempts to find all the lengths of all three edges of ΔADC	M1
		All Correct	A1
	$\cos D = \underline{\dfrac{\left(\frac{\sqrt{27}}{2}\right)^2 + \left(\frac{\sqrt{27}}{2}\right)^2 - \left(\sqrt{18}\right)^2}{2\left(\frac{\sqrt{27}}{2}\right)\left(\frac{\sqrt{27}}{2}\right)}} = \underline{-\dfrac{1}{3}}$	Using the cosine rule formula with correct 'subtraction'.	dM1
		<u>Correct ft application of the cosine rule formula</u>	A1 $\sqrt{}$
	$D = \cos^{-1}\left(-\dfrac{1}{3}\right)$	Attempts to find the correct angle D rather than $180^{\circ} - D$.	ddM1 $\sqrt{}$
	$D = 109.47122...^{\circ}$	109.5° or awrt 109° or 1.91^{c}	A1
			[6]

Question Number	Scheme		Marks
Aliter (d) **Way 5**	*using trigonometry on a right angled triangle* $\overrightarrow{DA} = \frac{1}{2}\mathbf{i} + \frac{1}{2}\mathbf{j} + \frac{5}{2}\mathbf{k}$ $\overrightarrow{OA} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ $\overrightarrow{AC} = \mathbf{i} + \mathbf{j} - 4\mathbf{k}$		
	Let X be the midpoint of AC	Attempts to find two out of the three lengths in ΔADX	M1
	$\lvert\overrightarrow{DA}\rvert = \dfrac{\sqrt{27}}{2}$ (hypotenuse), $\lvert\overrightarrow{DX}\rvert = \frac{1}{2}\lvert\overrightarrow{OA}\rvert = \dfrac{3}{2}$ (adjacent), $\lvert\overrightarrow{AX}\rvert = \frac{1}{2}\lvert\overrightarrow{AC}\rvert = \frac{1}{2}\sqrt{18}$ (opposite)	Any two correct	A1
	$\sin(\frac{1}{2}D) = \dfrac{\frac{\sqrt{18}}{2}}{\frac{\sqrt{27}}{2}}$, $\cos(\frac{1}{2}D) = \dfrac{\frac{3}{2}}{\frac{\sqrt{27}}{2}}$ or $\tan(\frac{1}{2}D) = \dfrac{\frac{\sqrt{18}}{2}}{\frac{3}{2}}$	Uses correct sohcahtoa to find $\frac{1}{2}D$ Correct ft application of sohcahtoa	dM1 A1 $\sqrt{}$
	eg. $D = 2\tan^{-1}\left(\dfrac{\frac{\sqrt{18}}{2}}{\frac{3}{2}}\right)$	Attempts to find the correct angle D by doubling their angle for $\frac{1}{2}D$.	ddM1 $\sqrt{}$
	$D = 109.47122...^{\circ}$	109.5° or awrt 109° or 1.91^{c}	A1
			[6]
Aliter (d) **Way 6**	*using trigonometry on a right angled similar triangle OAC* $\overrightarrow{OC} = 3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$ $\overrightarrow{OA} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ $\overrightarrow{AC} = \mathbf{i} + \mathbf{j} - 4\mathbf{k}$		
	$\lvert\overrightarrow{OC}\rvert = \sqrt{27}$ (hypotenuse), $\lvert\overrightarrow{OA}\rvert = 3$ (adjacent), $\lvert\overrightarrow{AC}\rvert = \sqrt{18}$ (opposite)	Attempts to find two out of the three lengths in ΔOAC	M1
		Any two correct	A1
	$\sin(\frac{1}{2}D) = \dfrac{\sqrt{18}}{\sqrt{27}}$, $\cos(\frac{1}{2}D) = \dfrac{3}{\sqrt{27}}$ or $\tan(\frac{1}{2}D) = \dfrac{\sqrt{18}}{3}$	Uses correct sohcahtoa to find $\frac{1}{2}D$ Correct ft application of sohcahtoa	dM1 A1 $\sqrt{}$
	eg. $D = 2\tan^{-1}\left(\dfrac{\sqrt{18}}{3}\right)$	Attempts to find the correct angle D by doubling their angle for $\frac{1}{2}D$.	ddM1 $\sqrt{}$
	$D = 109.47122...^{\circ}$	109.5° or awrt 109° or 1.91^{c}	A1
			[6]

Question Number	Scheme		Marks
Aliter 7. (b) (i) Way 2	$\mathbf{c} = \overrightarrow{OC} = \pm(3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k})$ $\overrightarrow{AB} = \pm(-\mathbf{i} - \mathbf{j} - 5\mathbf{k})$		
	$\lvert\overrightarrow{OC}\rvert = \sqrt{(3)^2 + (3)^2 + (-3)^2} = \sqrt{(1)^2 + (1)^2 + (-5)^2} = \lvert\overrightarrow{AB}\rvert$	A complete method of proving that the diagonals are equal.	**M1**
	As $\lvert\overrightarrow{OC}\rvert = \lvert\overrightarrow{AB}\rvert = \sqrt{27}$	Correct result.	**A1**
	then the diagonals are equal, and OACB is a rectangle.	diagonals are equal and OACB is a rectangle	**A1 cso** **[3]**
	$\mathbf{a} = \overrightarrow{OA} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k} \Rightarrow \lvert\overrightarrow{OA}\rvert = 3$ $\mathbf{b} = \overrightarrow{OB} = \mathbf{i} + \mathbf{j} - 4\mathbf{k} \Rightarrow \lvert\overrightarrow{OB}\rvert = \sqrt{18}$ $\overrightarrow{BC} = \pm(2\mathbf{i} + 2\mathbf{j} + \mathbf{k}) \Rightarrow \lvert\overrightarrow{BC}\rvert = 3$ $\overrightarrow{AC} = \pm(\mathbf{i} + \mathbf{j} - 4\mathbf{k}) \Rightarrow \lvert\overrightarrow{AC}\rvert = \sqrt{18}$ $\mathbf{c} = \overrightarrow{OC} = \pm(3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}) \Rightarrow \lvert\overrightarrow{OC}\rvert = \sqrt{27}$ $\overrightarrow{AB} = \pm(-\mathbf{i} - \mathbf{j} - 5\mathbf{k}) \Rightarrow \lvert\overrightarrow{AB}\rvert = \sqrt{27}$		
Aliter 7. (b) (i) Way 3	$(OA)^2 + (AC)^2 = (OC)^2$ or $(BC)^2 + (OB)^2 = (OC)^2$ or $(OA)^2 + (OB)^2 = (AB)^2$ or $(BC)^2 + (AC)^2 = (AB)^2$ or equivalent		
	$\Rightarrow \underline{(3)^2 + (\sqrt{18})^2 = (\sqrt{27})^2}$	A complete method of proving that Pythagoras holds using their values. Correct result	**M1** **A1**
	and therefore OA is perpendicular to OB or AC is perpendicular to BC and hence OACB is a rectangle.	perpendicular and OACB is a rectangle	**A1 cso** **[3]** **14marks**

Question Number	Scheme	Marks
8. (a)		
	x: 0, 1, 2, 3, 4, 5	
	y: e^1, e^2, $e^{\sqrt{7}}$, $e^{\sqrt{10}}$, $e^{\sqrt{13}}$, e^4	
	or y: 2.71828…, 7.38906…, 14.09403…, 23.62434…, 36.80197…, 54.59815…	
	Either $e^{\sqrt{7}}$, $e^{\sqrt{10}}$ and $e^{\sqrt{13}}$ or awrt 14.1, 23.6 and 36.8 or e to the power awrt 2.65, 3.16, 3.61 (or mixture of decimals and e's)	
	At least two correct	B1
	All three correct	B1
		[2]
(b)	$I \approx \frac{1}{2} \times 1 ; \times \underline{\left\{ e^1 + 2\left(e^2 + e^{\sqrt{7}} + e^{\sqrt{10}} + e^{\sqrt{13}}\right) + e^4 \right\}}$ Outside brackets $\frac{1}{2} \times 1$	B1;
	<u>For structure of trapezium rule</u> $\underline{\{............\}}$;	M1 $\sqrt{}$
	$= \frac{1}{2} \times 221.1352227... = 110.5676113... = \underline{110.6}$ (4sf) <u>110.6</u>	A1 **cao**
		[3]

Beware: In part (b) candidates can add up the individual trapezia:

(b) $I \approx \frac{1}{2}.1\left(\underline{e^1 + e^2}\right) + \frac{1}{2}.1\left(\underline{e^2 + e^{\sqrt{7}}}\right) + \frac{1}{2}.1\left(\underline{e^{\sqrt{7}} + e^{\sqrt{10}}}\right) + \frac{1}{2}.1\left(\underline{e^{\sqrt{10}} + e^{\sqrt{13}}}\right) + \frac{1}{2}.1\left(\underline{e^{\sqrt{13}} + e^4}\right)$

Question Number	Scheme		Marks
(c)	$t=(3x+1)^{\frac{1}{2}} \Rightarrow \frac{dt}{dx}=\frac{1}{2}.3.(3x+1)^{-\frac{1}{2}}$	$A(3x+1)^{-\frac{1}{2}}$ or $t\frac{dt}{dx}=A$	M1
	… or $t^2=3x+1 \Rightarrow \underline{2t\frac{dt}{dx}=3}$	$\underline{\frac{3}{2}(3x+1)^{-\frac{1}{2}}}$ or $\underline{2t\frac{dt}{dx}=3}$	A1
	so $\frac{dt}{dx}=\frac{3}{2.(3x+1)^{\frac{1}{2}}}=\frac{3}{2t} \Rightarrow \frac{dx}{dt}=\frac{2t}{3}$	Candidate obtains either $\frac{dt}{dx}$ or $\frac{dx}{dt}$ in terms of t …	
	$\therefore I=\int e^{\sqrt{(3x+1)}}\,dx=\int e^t\frac{dx}{dt}.dt=\int e^t.\frac{2t}{3}.dt$	… and moves on to substitute this into I to convert an integral wrt x to an integral wrt t.	dM1
	$\therefore I=\underline{\int \frac{2}{3}te^t}\,dt$	$\underline{\int \frac{2}{3}te^t}$	A1
	change limits: when x = 0, t = 1 & when x = 5, t = 4	changes limits $x\to t$ so that $0\to 1$ and $5\to 4$	B1
	Hence $I=\int_1^4 \frac{2}{3}te^t\,dt$; where $a=1,\ b=4,\ k=\frac{2}{3}$		
			[5]
(d)	$\left\{\begin{matrix} u=t & \Rightarrow & \frac{du}{dt}=1 \\ \frac{dv}{dt}=e^t & \Rightarrow & v=e^t \end{matrix}\right\}$	Let k be any constant for the first three marks of this part.	
	$k\int te^t\,dt = k\left(te^t-\int e^t.1\,dt\right)$	Use of 'integration by parts' formula in the correct direction. Correct expression with a constant factor k.	M1 A1
	$=k\left(\underline{te^t-e^t}\right)+c$	<u>Correct integration</u> with/without a constant factor k	A1
	$\therefore \int_1^4 \frac{2}{3}te^t\,dt=\frac{2}{3}\left\{\left(4e^4-e^4\right)-\left(e^1-e^1\right)\right\}$	Substitutes their changed limits into the integrand and subtracts oe.	dM1 **oe**
	$=\frac{2}{3}(3e^4)=\underline{2e^4}=109.1963...$	either $2e^4$ or awrt 109.2	A1
			[5] **15 marks**

- Note: dM1 denotes a method mark which is dependent upon the award of the previous method mark
- ddM1 denotes a method mark which is dependent upon the award of the previous two method marks.

Mark Scheme (Results)
Summer 2007

GCE

GCE Mathematics

Core Mathematics C4 (6666)

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June 2007
6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme		Marks
	** represents a constant		
1. (a)	$f(x)=(3+2x)^{-3}=\underline{(3)^{-3}}\left(1+\frac{2x}{3}\right)^{-3}=\underline{\frac{1}{27}}\left(1+\frac{2x}{3}\right)^{-3}$	Takes 3 outside the bracket to give any of $(3)^{-3}$ or $\frac{1}{27}$. See note below.	B1
	$=\frac{1}{27}\left\{\underline{1+(-3)(**x);+\frac{(-3)(-4)}{2!}(**x)^2+\frac{(-3)(-4)(-5)}{3!}(**x)^3+\ldots}\right\}$ with $**\neq 1$	Expands $(1+**x)^{-3}$ to give a simplified or an un-simplified $1+(-3)(**x)$;	M1;
		A correct simplified or an un-simplified $\underline{\{\ldots\ldots\ldots\}}$ expansion with candidate's followed thro' $(**x)$	A1 $\sqrt{}$
	$=\frac{1}{27}\left\{\underline{1+(-3)(\frac{2x}{3})+\frac{(-3)(-4)}{2!}(\frac{2x}{3})^2+\frac{(-3)(-4)(-5)}{3!}(\frac{2x}{3})^3+\ldots}\right\}$		
	$=\frac{1}{27}\left\{1-2x+\frac{8x^2}{3}-\frac{80}{27}x^3+\ldots\right\}$		
	$=\frac{1}{27}-\frac{2x}{27};+\frac{8x^2}{81}-\frac{80x^3}{729}+\ldots$	Anything that cancels to $\frac{1}{27}-\frac{2x}{27}$;	A1;
		Simplified $\frac{8x^2}{81}-\frac{80x^3}{729}$	A1
			[5]
			5 marks

Note: You would award: B1M1A0 for

$$=\frac{1}{27}\left\{\underline{1+(-3)(\tfrac{2x}{3})+\frac{(-3)(-4)}{2!}(2x)^2+\frac{(-3)(-4)(-5)}{3!}(2x)^3+\ldots}\right\}$$

because ** is not consistent.

Special Case: If you see the constant $\frac{1}{27}$ in a candidate's final binomial expression, then you can award B1

Question Number	Scheme		Marks
Aliter 1. Way 2	$f(x) = (3+2x)^{-3}$		
		$\frac{1}{27}$ or $(3)^{-3}$ (See note $\downarrow$)	B1
	$= \left\{ \begin{array}{l} (3)^{-3} + (-3)(3)^{-4}(**\,x); + \frac{(-3)(-4)}{2!}(3)^{-5}(**\,x)^2 \\ + \frac{(-3)(-4)(-5)}{3!}(3)^{-6}(**\,x)^3 + \ldots \end{array} \right\}$	Expands $(3+2x)^{-3}$ to give an un-simplified or simplified $(3)^{-3} + (-3)(3)^{-4}(**\,x)$;	M1
	with $** \neq 1$	A correct un-simplified or simplified $\{\ldots\ldots\ldots\}$ expansion with candidate's followed thro' $(**\,x)$	A1 $\sqrt{}$
	$= \left\{ \begin{array}{l} (3)^{-3} + (-3)(3)^{-4}(2x); + \frac{(-3)(-4)}{2!}(3)^{-5}(2x)^2 \\ + \frac{(-3)(-4)(-5)}{3!}(3)^{-6}(2x)^3 + \ldots \end{array} \right\}$		
	$= \left\{ \begin{array}{l} \frac{1}{27} + (-3)(\frac{1}{81})(2x); + (6)(\frac{1}{243})(4x^2) \\ + (-10)(\frac{1}{729})(8x^3) + \ldots \end{array} \right\}$		
	$= \frac{1}{27} - \frac{2x}{27}; + \frac{8x^2}{81} - \frac{80x^3}{729} + \ldots$	Anything that cancels to $\frac{1}{27} - \frac{2x}{27}$; Simplified $\frac{8x^2}{81} - \frac{80x^3}{729}$	A1; A1
			[5]
			5 marks

Attempts using Maclaurin expansions need to be escalated up to your team leader.

If you feel the mark scheme does not apply fairly to a candidate please escalate the response up to your team leader.

> **Special Case**: If you see the constant $\frac{1}{27}$ in a candidate's final binomial expression, then you can award B1

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Question Number	Scheme		Marks
2.	$\displaystyle\int_0^1 \frac{2^x}{(2^x+1)^2}\,\mathrm{d}x$, with substitution $u = 2^x$		
	$\dfrac{\mathrm{d}u}{\mathrm{d}x} = 2^x.\ln 2 \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}u} = \dfrac{1}{2^x.\ln 2}$	$\frac{\mathrm{d}u}{\mathrm{d}x} = 2^x.\ln 2$ or $\frac{\mathrm{d}u}{\mathrm{d}x} = u.\ln 2$ or $\left(\frac{1}{u}\right)\frac{\mathrm{d}u}{\mathrm{d}x} = \ln 2$	B1
	$\displaystyle\int \frac{2^x}{(2^x+1)^2}\,\mathrm{d}x = \left(\frac{1}{\ln 2}\right)\int \frac{1}{(u+1)^2}\,\mathrm{d}u$	$k\displaystyle\int \frac{1}{(u+1)^2}\,\mathrm{d}u$ where k is constant	M1 *
	$= \left(\dfrac{1}{\ln 2}\right)\left(\dfrac{-1}{(u+1)}\right) + c$	$(u+1)^{-2} \rightarrow a(u+1)^{-1}$	M1
		$(u+1)^{-2} \rightarrow -1.(u+1)^{-1}$	A1
	change limits: when $x = 0$ & $x = 1$ then $u = 1$ & $u = 2$		
	$\displaystyle\int_0^1 \frac{2^x}{(2^x+1)^2}\,\mathrm{d}x = \frac{1}{\ln 2}\left[\frac{-1}{(u+1)}\right]_1^2$		
	$= \dfrac{1}{\ln 2}\left[\left(-\dfrac{1}{3}\right) - \left(-\dfrac{1}{2}\right)\right]$	Correct use of limits $u = 1$ and $u = 2$	depM1 *
	$= \dfrac{1}{6\ln 2}$	$\frac{1}{6\ln 2}$ or $\frac{1}{\ln 4} - \frac{1}{\ln 8}$ or $\frac{1}{2\ln 2} - \frac{1}{3\ln 2}$	A1 aef
		Exact value only!	[6]
	Alternatively candidate can revert back to x …		
	$\displaystyle\int_0^1 \frac{2^x}{(2^x+1)^2}\,\mathrm{d}x = \frac{1}{\ln 2}\left[\frac{-1}{(2^x+1)}\right]_0^1$		
	$= \dfrac{1}{\ln 2}\left[\left(-\dfrac{1}{3}\right) - \left(-\dfrac{1}{2}\right)\right]$	Correct use of limits $x = 0$ and $x = 1$	depM1 *
	$= \underline{\dfrac{1}{6\ln 2}}$	$\frac{1}{6\ln 2}$ or $\frac{1}{\ln 4} - \frac{1}{\ln 8}$ or $\frac{1}{2\ln 2} - \frac{1}{3\ln 2}$	A1 aef
		Exact value only!	
			6 marks

If you see this **integration** applied anywhere in a candidate's working then you can award M1, A1

There are other acceptable answers for A1, eg: $\frac{1}{2\ln 8}$ or $\frac{1}{\ln 64}$

NB: Use your calculator to check eg. 0.240449…

Question Number	Scheme		Marks
3. (a)	$\left\{\begin{array}{lll} u = x & \Rightarrow & \frac{du}{dx} = 1 \\ \frac{dv}{dx} = \cos 2x & \Rightarrow & v = \frac{1}{2}\sin 2x \end{array}\right\}$		
		(see note below)	
	$\text{Int} = \int x\cos 2x \, dx = \frac{1}{2}x\sin 2x - \int \frac{1}{2}\sin 2x.1 \, dx$	Use of 'integration by parts' formula in the correct direction. Correct expression.	M1 A1
	$= \frac{1}{2}x\sin 2x - \frac{1}{2}\left(-\frac{1}{2}\cos 2x\right) + c$	$\sin 2x \to -\frac{1}{2}\cos 2x$ or $\sin kx \to -\frac{1}{k}\cos kx$ with $k \neq 1$, $k > 0$	dM1
	$= \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$	Correct expression with $+c$	A1
			[4]
(b)	$\int x\cos^2 x \, dx = \int x\left(\frac{\cos 2x + 1}{2}\right) dx$	Substitutes correctly for $\cos^2 x$ in the given integral	M1
	$= \frac{1}{2}\int x\cos 2x \, dx + \frac{1}{2}\int x \, dx$		
	$= \underline{\frac{1}{2}\left(\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x\right)}; + \frac{1}{2}\int x \, dx$	$\frac{1}{2}$(their answer to (a)); or <u>underlined expression</u>	A1; $\sqrt{}$
	$= \frac{1}{4}x\sin 2x + \frac{1}{8}\cos 2x + \frac{1}{4}x^2 \; (+c)$	Completely correct expression with/without $+c$	A1
			[3]
			7 marks

Notes:

(b)	$\text{Int} = \int x\cos 2x \, dx = \frac{1}{2}x\sin 2x \pm \int \frac{1}{2}\sin 2x.1 \, dx$	This is acceptable for M1	M1
	$\left\{\begin{array}{lll} u = x & \Rightarrow & \frac{du}{dx} = 1 \\ \frac{dv}{dx} = \cos 2x & \Rightarrow & v = \lambda\sin 2x \end{array}\right\}$		
	$\text{Int} = \int x\cos 2x \, dx = \lambda x\sin 2x \pm \int \lambda\sin 2x.1 \, dx$	This is also acceptable for M1	M1

Question Number	Scheme		Marks
Aliter 3. (b) Way 2	$\int x\cos^2 x\,dx = \int x\left(\frac{\cos 2x+1}{2}\right)dx$	Substitutes <u>correctly</u> for $\cos^2 x$ in the given integral …	M1
	$\begin{cases} u = x & \Rightarrow \frac{du}{dx} = 1 \\ \frac{dv}{dx} = \frac{1}{2}\cos 2x + \frac{1}{2} & \Rightarrow v = \frac{1}{4}\sin 2x + \frac{1}{2}x \end{cases}$	… or $u = x$ and $\frac{dv}{dx} = \frac{1}{2}\cos 2x + \frac{1}{2}$	
	$= \frac{1}{4}x\sin 2x + \frac{1}{2}x^2 - \int\left(\frac{1}{4}\sin 2x + \frac{1}{2}x\right)dx$		
	$= \underline{\frac{1}{4}x\sin 2x} + \frac{1}{2}x^2 + \underline{\frac{1}{8}\cos 2x} - \frac{1}{4}x^2 + c$	$\frac{1}{2}$(their answer to (a)); or <u>underlined expression</u>	A1 $\sqrt{}$
	$= \frac{1}{4}x\sin 2x + \frac{1}{8}\cos 2x + \frac{1}{4}x^2\ (+c)$	Completely correct expression with/without $+c$	A1
			[3]
Aliter (b) Way 3	$\int x\cos 2x\,dx = \int x\left(2\cos^2 x - 1\right)dx$	Substitutes <u>correctly</u> for $\cos 2x$ in $\int x\cos 2x\,dx$	M1
	$\Rightarrow 2\int x\cos^2 x\,dx - \int x\,dx = \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$		
	$\Rightarrow \int x\cos^2 x\,dx = \underline{\frac{1}{2}\left(\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x\right)}; + \frac{1}{2}\int x\,dx$	$\frac{1}{2}$(their answer to (a)); or <u>underlined expression</u>	A1; $\sqrt{}$
	$= \frac{1}{4}x\sin 2x + \frac{1}{8}\cos 2x + \frac{1}{4}x^2\ (+c)$	Completely correct expression with/without $+c$	A1
			[3]
			7 marks

Question Number	Scheme	Marks
4. (a) Way 1	A method of long division gives,	
	$\dfrac{2(4x^2+1)}{(2x+1)(2x-1)} \equiv 2 + \dfrac{4}{(2x+1)(2x-1)}$ $\qquad A = 2$	B1
	$\dfrac{4}{(2x+1)(2x-1)} \equiv \dfrac{B}{(2x+1)} + \dfrac{C}{(2x-1)}$	
	$4 \equiv B(2x-1) + C(2x+1)$ or their remainder, $Dx + E \equiv B(2x-1) + C(2x+1)$ — Forming any one of these two identities. Can be implied.	M1
	Let $x = -\frac{1}{2}$, $\quad 4 = -2B \;\Rightarrow\; B = -2$	
	Let $x = \frac{1}{2}$, $\quad 4 = 2C \;\Rightarrow\; C = 2$ — See note below; either one of $B = -2$ or $C = 2$	A1
	both B and C correct	A1 [4]
Aliter 4. (a) Way 2	$\dfrac{2(4x^2+1)}{(2x+1)(2x-1)} \equiv A + \dfrac{B}{(2x+1)} + \dfrac{C}{(2x-1)}$	
	See below for the award of B1 — *decide to award B1 here!! … … for* $A = 2$	B1
	$2(4x^2+1) \equiv A(2x+1)(2x-1) + B(2x-1) + C(2x+1)$ — Forming this identity. Can be implied.	M1
	Equate x^2, $\quad 8 = 4A \;\Rightarrow\; A = 2$	
	Let $x = -\frac{1}{2}$, $\quad 4 = -2B \;\Rightarrow\; B = -2$	
	Let $x = \frac{1}{2}$, $\quad 4 = 2C \;\Rightarrow\; C = 2$ — See note below; either one of $B = -2$ or $C = 2$	A1
	both B and C correct	A1 [4]

If a candidate states one of either B or C correctly then the method mark M1 can be implied.

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Question Number	Scheme		Marks
4. (b)	$\int \frac{2(4x^2+1)}{(2x+1)(2x-1)}\,dx = \int 2 - \frac{2}{(2x+1)} + \frac{2}{(2x-1)}\,dx$		
	$= 2x - \frac{2}{2}\ln(2x+1) + \frac{2}{2}\ln(2x-1)\ (+c)$	Either $p\ln(2x+1)$ or $q\ln(2x-1)$ or either $p\ln 2x+1$ or $q\ln 2x-1$	M1 *
		$A \to Ax$	B1 $\sqrt{}$
		$-\frac{2}{2}\ln(2x+1) + \frac{2}{2}\ln(2x-1)$ or $-\ln(2x+1) + \ln(2x-1)$ See note below.	A1 cso & aef
	$\int_1^2 \frac{2(4x^2+1)}{(2x+1)(2x-1)}\,dx = \left[2x - \ln(2x+1) + \ln(2x-1)\right]_1^2$		
	$= (4 - \ln 5 + \ln 3) - (2 - \ln 3 + \ln 1)$	Substitutes limits of 2 and 1 and subtracts the correct way round. (Invisible brackets okay.)	depM1 *
	$= 2 + \ln 3 + \ln 3 - \ln 5$		
	$= 2 + \ln\left(\frac{3(3)}{5}\right)$	Use of correct product (or power) and/or quotient laws for logarithms to obtain a single logarithmic term for *their numerical* expression.	M1
	$= 2 + \ln\left(\frac{9}{5}\right)$	$2 + \ln\left(\frac{9}{5}\right)$ Or $2 - \ln\left(\frac{5}{9}\right)$ and k stated as $\frac{9}{5}$.	A1 [6]
			10 marks

Some candidates may find rational values for *B* and *C*. They may combine the denominator of their *B* or C with (2x +1) or (2x – 1). Hence: Either $\frac{a}{b(2x-1)} \to k\ln(b(2x-1))$ or $\frac{a}{b(2x+1)} \to k\ln(b(2x+1))$ is okay for M1.

Candidates are not allowed to fluke $-\ln(2x+1) + \ln(2x-1)$ for A1. Hence **cso**. If they do fluke this, however, they can gain the final A1 mark for this part of the question.

To award this M1 mark, the candidate must use the appropriate law(s) of logarithms for their ln terms to give a **one single** logarithmic term. Any error ***in applying the laws of logarithms*** would then earn M0.

Note: This is not a dependent method mark.

Question Number	Scheme		Marks
5. (a)	If l_1 and l_2 intersect then:		
	$\begin{pmatrix}1\\0\\-1\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\0\end{pmatrix}=\begin{pmatrix}1\\3\\6\end{pmatrix}+\mu\begin{pmatrix}2\\1\\-1\end{pmatrix}$		
	Any two of $\mathbf{i}: \quad 1+\lambda=1+2\mu \quad (1)$ $\mathbf{j}: \quad \lambda=3+\mu \quad (2)$ $\mathbf{k}: \quad -1=6-\mu \quad (3)$	Writes down any two of these equations correctly.	M1
	(1) & (2) yields $\lambda=6$, $\mu=3$ (1) & (3) yields $\lambda=14$, $\mu=7$ (2) & (3) yields $\lambda=10$, $\mu=7$	Solves two of the above equations to find … either one of λ or μ correct both λ and μ correct	 A1 A1
	Either checking eqn (3), $-1 \neq 3$ checking eqn (2), $14 \neq 10$ checking eqn (1), $11 \neq 15$	Complete method of putting their values of λ and μ into a third equation to show a contradiction.	B1 $\sqrt{}$
	or for example: checking eqn (3), LHS = -1, RHS = 3 $\Rightarrow$ Lines l_1 and l_2 do not intersect	this type of explanation is also allowed for B1 $\sqrt{}$.	
			[4]
Aliter 5. (a) Way 2	$\mathbf{k}: \ -1=6-\mu \ \Rightarrow \ \mu=7$ $\mathbf{i}: \quad 1+\lambda=1+2\mu \ \Rightarrow \ 1+\lambda=1+2(7)$ $\mathbf{j}: \quad \lambda=3+\mu \ \Rightarrow \ \lambda=3+(7)$	Uses the k component to find μ and substitutes their value of μ into either one of the i or j component.	M1
	$\mathbf{i}: \quad \lambda=14$ $\mathbf{j}: \quad \lambda=10$	either one of the λ's correct both of the λ's correct	A1 A1
	Either: These equations are then inconsistent Or: $14 \neq 10$ Or: Lines l_1 and l_2 do not intersect	Complete method giving rise to any one of these three explanations.	B1 $\sqrt{}$
			[4]

Question Number	Scheme		Marks
Aliter 5. (a) Way 3	If l_1 and l_2 intersect then:		
	$\begin{pmatrix}1\\0\\-1\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\0\end{pmatrix}=\begin{pmatrix}1\\3\\6\end{pmatrix}+\mu\begin{pmatrix}2\\1\\-1\end{pmatrix}$		
	Any two of $\mathbf{i}:\ 1+\lambda=1+2\mu \quad (1)$ $\mathbf{j}:\ \lambda=3+\mu \quad (2)$ $\mathbf{k}:\ -1=6-\mu \quad (3)$	Writes down any two of these equations	M1
	(1) & (2) yields $\mu=3$	either one of the μ's correct	A1
	(3) yields $\mu=7$	both of the μ's correct	A1
	Either: These equations are then inconsistent Or: $3\neq 7$ Or: Lines l_1 and l_2 do not intersect	Complete method giving rise to any one of these three explanations.	B1 $\sqrt{}$
			[4]
Aliter 5. (a) Way 4	Any two of $\mathbf{i}:\ 1+\lambda=1+2\mu \quad (1)$ $\mathbf{j}:\ \lambda=3+\mu \quad (2)$ $\mathbf{k}:\ -1=6-\mu \quad (3)$	Writes down any two of these equations	M1
	(1) & (2) yields $\mu=3$	$\mu=3$	A1
	(3) RHS $=6-3=3$	RHS of (3) = 3	A1
	(3) yields $-1\neq 3$	Complete method giving rise to this explanation.	B1 $\sqrt{}$
			[4]

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Question Number	Scheme		Marks
5. (b)	$\lambda = 1 \Rightarrow \overrightarrow{OA} = \begin{pmatrix}2\\1\\-1\end{pmatrix}$ & $\mu = 2 \Rightarrow \overrightarrow{OB} = \begin{pmatrix}5\\5\\4\end{pmatrix}$	<u>Only one of either</u> $\overrightarrow{OA} = \begin{pmatrix}2\\1\\-1\end{pmatrix}$ or $\overrightarrow{OB} = \begin{pmatrix}5\\5\\4\end{pmatrix}$ or $A(2,1,-1)$ or $B(5,5,4)$. (can be implied)	B1
	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \underline{\begin{pmatrix}5\\5\\4\end{pmatrix} - \begin{pmatrix}2\\1\\-1\end{pmatrix}} = \begin{pmatrix}3\\4\\5\end{pmatrix}$ or $\overrightarrow{BA} = \underline{\begin{pmatrix}-3\\-4\\-5\end{pmatrix}}$	Finding the difference between their $\overrightarrow{OB}$ and $\overrightarrow{OA}$. (can be implied)	<u>M1</u> $\sqrt{}$
	$\overrightarrow{AB} = 3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$, $\mathbf{d}_1 = \mathbf{i} + \mathbf{j} + 0\mathbf{k}$ & θ is angle	Applying the dot product formula between "allowable" vectors. See notes below.	M1
	$\cos\theta = \dfrac{\overrightarrow{AB} \bullet \mathbf{d}_1}{\lvert\overrightarrow{AB}\rvert . \lvert\mathbf{d}_1\rvert} = \pm\left(\dfrac{3+4+0}{\sqrt{50}.\sqrt{2}}\right)$	Applies dot product formula between $\mathbf{d}_1$ and their $\pm\overrightarrow{AB}$.	M1 $\sqrt{}$
		Correct expression.	A1
	$\cos\theta = \frac{7}{10}$	$\frac{7}{10}$ or <u>0.7</u> or $\frac{7}{\sqrt{100}}$ but not $\frac{7}{\sqrt{50}\sqrt{2}}$	A1 cao [6]
			10 marks

Candidates can score this mark if there is a complete method for finding the dot product between their vectors in the following cases:

Case 1: their ft $\pm\overrightarrow{AB} = \pm(3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k})$ and $\mathbf{d}_1 = \mathbf{i} + \mathbf{j} + 0\mathbf{k}$
$\Rightarrow \cos\theta = \pm\left(\dfrac{3+4+0}{\sqrt{50}.\sqrt{2}}\right)$

Case 2: $\mathbf{d}_1 = \mathbf{i} + \mathbf{j} + 0\mathbf{k}$ and $\mathbf{d}_2 = 2\mathbf{i} + \mathbf{j} - 1\mathbf{k}$
$\Rightarrow \cos\theta = \dfrac{2+1+0}{\sqrt{2}.\sqrt{6}}$

Case 3: $\mathbf{d}_1 = \mathbf{i} + \mathbf{j} + 0\mathbf{k}$ and $\mathbf{d}_2 = 2(2\mathbf{i} + \mathbf{j} - 1\mathbf{k})$
$\Rightarrow \cos\theta = \dfrac{4+2+0}{\sqrt{2}.\sqrt{24}}$

Case 4: their ft $\pm\overrightarrow{AB} = \pm(3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k})$ and $\mathbf{d}_2 = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$
$\Rightarrow \cos\theta = \pm\left(\dfrac{6+4-5}{\sqrt{50}.\sqrt{6}}\right)$

Case 5: their ft $\overrightarrow{OA} = 2\mathbf{i} + 1\mathbf{j} - 1\mathbf{k}$ and their ft $\overrightarrow{OB} = 5\mathbf{i} + 5\mathbf{j} + 4\mathbf{k}$
$\Rightarrow \cos\theta = \pm\left(\dfrac{10+5-4}{\sqrt{6}.\sqrt{66}}\right)$

Note: If candidate use cases 2, 3, 4 and 5 they cannot gain the final three marks for this part.
Note: Candidate can only gain some/all of the final three marks if they use case 1.

Examples of awarding of marks M1M1A1 in 5.(b)

Example	Marks
$\sqrt{50}.\sqrt{2}\cos\theta = \pm(3 + 4 + 0)$	M1M1A1 (Case 1)
$\sqrt{2}.\sqrt{6}\cos\theta = 3$	M1M0A0 (Case 2)
$\sqrt{2}.\sqrt{24}\cos\theta = 4 + 2$	M1M0A0 (Case 3)

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Question Number	Scheme		Marks
6. (a)	$x = \tan^2 t\,, \qquad y = \sin t$		
	$\frac{dx}{dt} = 2(\tan t)\sec^2 t\,, \quad \frac{dy}{dt} = \cos t$	Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$	B1
	$\therefore \frac{dy}{dx} = \frac{\cos t}{2\tan t \sec^2 t} \quad \left(= \frac{\cos^4 t}{2\sin t}\right)$	$\frac{\pm\cos t}{\text{their } \frac{dx}{dt}}$	M1
		$\frac{+\cos t}{\text{their } \frac{dx}{dt}}$	A1$\sqrt{}$
			[3]
(b)	When $t = \frac{\pi}{4}$, $\quad x = 1\,,\ y = \frac{1}{\sqrt{2}}$ (need values)	The point $\underline{\left(1, \frac{1}{\sqrt{2}}\right)}$ or $\underline{(1, \text{awrt } 0.71)}$ These coordinates can be implied. ($y = \sin\left(\frac{\pi}{4}\right)$ is not sufficient for B1)	B1, B1
	When $t = \frac{\pi}{4}$, m(T) $= \frac{dy}{dx} = \frac{\cos\frac{\pi}{4}}{2\tan\frac{\pi}{4}\sec^2\frac{\pi}{4}}$		
	$= \underline{\frac{\frac{1}{\sqrt{2}}}{2.(1)\left(\frac{1}{\frac{1}{\sqrt{2}}}\right)^2}} = \underline{\frac{\frac{1}{\sqrt{2}}}{2.(1)\left(\frac{1}{\frac{1}{2}}\right)}} = \underline{\frac{\frac{1}{\sqrt{2}}}{2.(1)(2)}} = \underline{\frac{1}{4\sqrt{2}}} = \underline{\frac{\sqrt{2}}{8}}$	any of the five underlined expressions or awrt 0.18	B1 aef
	T: $y - \frac{1}{\sqrt{2}} = \frac{1}{4\sqrt{2}}(x - 1)$	Finding an equation of a tangent with *their point* and *their tangent gradient* or finds *c* by using $y = \underline{(\text{their gradient})}x + "\underline{c}"$.	M1$\sqrt{}$ aef
	T: $\underline{y = \frac{1}{4\sqrt{2}}x + \frac{3}{4\sqrt{2}}}$ or $\underline{y = \frac{\sqrt{2}}{8}x + \frac{3\sqrt{2}}{8}}$	Correct simplified EXACT equation of <u>tangent</u>	<u>A1</u> aef cso
	or $\frac{1}{\sqrt{2}} = \frac{1}{4\sqrt{2}}(1) + c \Rightarrow c = \frac{1}{\sqrt{2}} - \frac{1}{4\sqrt{2}} = \frac{3}{4\sqrt{2}}$		
	Hence T: $\underline{y = \frac{1}{4\sqrt{2}}x + \frac{3}{4\sqrt{2}}}$ or $\underline{y = \frac{\sqrt{2}}{8}x + \frac{3\sqrt{2}}{8}}$		
			[5]

Note: The x and y coordinates must be the right way round.

A candidate who incorrectly differentiates $\tan^2 t$ to give $\frac{dx}{dt} = 2\sec^2 t$ or $\frac{dx}{dt} = \sec^4 t$ is then able to fluke the correct answer in part (b). Such candidates can potentially get: (a) B0M1A1$\sqrt{}$ (b) B1B1B1M1A0 **cso**.
Note: cso means “correct solution only”.
Note: part (a) not fully correct implies candidate can achieve a maximum of 4 out of 5 marks in part (b).

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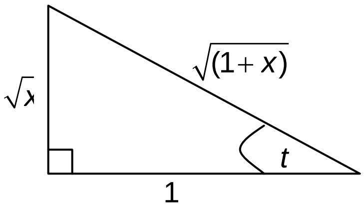
Question Number	Scheme		Marks
6. (c) Way 1	$x = \tan^2 t = \dfrac{\sin^2 t}{\cos^2 t}$ $y = \sin t$		
	$x = \dfrac{\sin^2 t}{1-\sin^2 t}$	Uses $\cos^2 t = 1-\sin^2 t$	M1
	$x = \dfrac{y^2}{1-y^2}$	Eliminates 't' *to* write an equation involving x and y.	M1
	$x(1-y^2) = y^2 \Rightarrow x - xy^2 = y^2$		
	$x = y^2 + xy^2 \Rightarrow x = y^2(1+x)$	Rearranging and factorising with an attempt to make y^2 the subject.	ddM1
	$y^2 = \dfrac{x}{1+x}$	$\dfrac{x}{1+x}$	A1
			[4]
Aliter 6. (c) Way 2	$1+\cot^2 t = cosec^2 t$	Uses $1+\cot^2 t = cosec^2 t$	M1
	$= \dfrac{1}{\sin^2 t}$	Uses $cosec^2 t = \dfrac{1}{\sin^2 t}$	M1 implied
	Hence, $1+\dfrac{1}{x} = \dfrac{1}{y^2}$	Eliminates 't' *to* write an equation involving x and y.	ddM1
	Hence, $y^2 = 1-\dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	$1-\dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	A1
			[4]

$\dfrac{1}{1+\frac{1}{x}}$ is an acceptable response for the final accuracy A1 mark.

Question Number	Scheme		Marks
Aliter 6. (c) Way 3	$x = \tan^2 t \qquad y = \sin t$		
	$1 + \tan^2 t = \sec^2 t$	Uses $1 + \tan^2 t = \sec^2 t$	M1
	$= \dfrac{1}{\cos^2 t}$	Uses $\sec^2 t = \dfrac{1}{\cos^2 t}$	M1
	$= \dfrac{1}{1 - \sin^2 t}$		
	Hence, $1 + x = \dfrac{1}{1 - y^2}$	Eliminates '*t*' *to* write an equation involving *x* and *y*.	ddM1
	Hence, $y^2 = 1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	$1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	A1
			[4]
Aliter 6. (c) Way 4	$y^2 = \sin^2 t = 1 - \cos^2 t$	Uses $\sin^2 t = 1 - \cos^2 t$	M1
	$= 1 - \dfrac{1}{\sec^2 t}$	Uses $\cos^2 t = \dfrac{1}{\sec^2 t}$	M1
	$= 1 - \dfrac{1}{(1 + \tan^2 t)}$	then uses $\sec^2 t = 1 + \tan^2 t$	ddM1
	Hence, $y^2 = 1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	$1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	A1
			[4]

$\dfrac{1}{1+\frac{1}{x}}$ is an acceptable response for the final accuracy A1 mark.

Question Number	Scheme		Marks
Aliter 6. (c) Way 5	$x = \tan^2 t \qquad y = \sin t$		
	$x = \tan^2 t \;\Rightarrow\; \tan t = \sqrt{x}$		
	 	Draws a right-angled triangle and places both $\sqrt{x}$ and 1 on the triangle	M1
		Uses Pythagoras to deduce the hypotenuse	M1
	Hence, $y = \sin t = \dfrac{\sqrt{x}}{\sqrt{1+x}}$	Eliminates '*t*' *to* write an equation involving *x* and *y*.	ddM1
	Hence, $y^2 = \dfrac{x}{1+x}$	$\dfrac{x}{1+x}$	A1 [4]
			12 marks

$\dfrac{1}{1+\frac{1}{x}}$ is an acceptable response for the final accuracy A1 mark.

There are so many ways that a candidate can proceed with part (c). If a candidate produces a correct solution then please award all four marks. If they use a method commensurate with the five ways as detailed on the mark scheme then award the marks appropriately. If you are unsure of how to apply the scheme please escalate your response up to your team leader.

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Question Number	Scheme		Marks
7. (a)	x: 0, $\frac{\pi}{16}$, $\frac{\pi}{8}$, $\frac{3\pi}{16}$, $\frac{\pi}{4}$ y: 0, $0.445995927\ldots$, $0.643594252\ldots$, $0.817421946\ldots$, 1		
	Enter marks into ePEN in the correct order.	0.446 or awrt 0.44600 awrt 0.64359 awrt 0.81742	B1 B1 B1 [3]
(b) Way 1	0 can be implied $\text{Area} \approx \frac{1}{2} \times \frac{\pi}{16}\ ; \times \underline{\left\{0 + 2(0.44600 + 0.64359 + 0.81742) + 1\right\}}$	Outside brackets $\frac{1}{2} \times \frac{\pi}{16}$ or $\frac{\pi}{32}$ For structure of trapezium rule $\{\ldots\ldots\ldots\ldots\}$; Correct expression inside brackets which all must be multiplied by $\frac{h}{2}$.	B1 M1 $\sqrt{}$ A1 $\sqrt{}$
	$= \frac{\pi}{32} \times 4.81402\ldots = 0.472615308\ldots = \underline{0.4726}$ (4dp)	for seeing $\underline{0.4726}$	A1 cao [4]
Aliter (b) Way 2	$\text{Area} \approx \frac{\pi}{16} \times \left\{\frac{0+0.44600}{2} + \frac{0.44600+0.64359}{2} + \frac{0.64359+0.81742}{2} + \frac{0.81742+1}{2}\right\}$ which is equivalent to: $\text{Area} \approx \frac{1}{2} \times \frac{\pi}{16}\ ; \times \underline{\left\{0 + 2(0.44600 + 0.64359 + 0.81742) + 1\right\}}$	$\frac{\pi}{16}$ and a divisor of 2 on all terms inside brackets. One of first and last ordinates, two of the middle ordinates inside brackets ignoring the 2. Correct expression inside brackets if $\frac{1}{2}$ was to be factorised out.	B1 M1 $\sqrt{}$ A1 $\sqrt{}$
	$= \frac{\pi}{16} \times 2.40701\ldots = 0.472615308\ldots = \underline{0.4726}$	$\underline{0.4726}$	A1 cao [4]

$\mathit{Area} = \frac{1}{2} \times \frac{\pi}{20} \times \left\{0 + 2(0.44600 + 0.64359 + 0.81742) + 1\right\} = 0.3781$, gains B0M1A1A0

In (a) for $x = \frac{\pi}{16}$ writing 0.4459959… then 0.45600 gains B1 for awrt 0.44600 even though 0.45600 is incorrect.

In (b) you can follow though a candidate's values from part (a) to award M1 ft, A1 ft

Question Number	Scheme	Marks

7. (c)	Volume $= (\pi)\underline{\int_0^{\frac{\pi}{4}} \left(\sqrt{\tan x}\right)^2 dx} = (\pi)\underline{\int_0^{\frac{\pi}{4}} \tan x \, dx}$	$\underline{\int \left(\sqrt{\tan x}\right)^2 dx}$ or $\underline{\int \tan x \, dx}$ Can be implied. Ignore limits and (π)	M1
	$= (\pi)\left[\underline{\ln \sec x}\right]_0^{\frac{\pi}{4}}$ or $= (\pi)\left[\underline{-\ln \cos x}\right]_0^{\frac{\pi}{4}}$	$\tan x \to \underline{\ln \sec x}$ or $\tan x \to \underline{-\ln \cos x}$	$\underline{\text{A1}}$
	$= (\pi)\left[\left(\ln \sec \frac{\pi}{4}\right) - \left(\ln \sec 0\right)\right]$ or $= (\pi)\left[\left(-\ln \cos \frac{\pi}{4}\right) - \left(\ln \cos 0\right)\right]$	The correct use of limits on a function other than tan x; ie $x = \frac{\pi}{4}$ 'minus' $x = 0$. $\ln(\sec 0) = 0$ may be implied. Ignore (π)	dM1
	$= \pi\left[\ln\left(\frac{1}{\frac{1}{\sqrt{2}}}\right) - \ln\left(\frac{1}{1}\right)\right] = \pi\left[\ln\sqrt{2} - \ln 1\right]$ or $= \pi\left[-\ln\left(\frac{1}{\sqrt{2}}\right) - \ln(1)\right]$		
	$= \underline{\pi \ln \sqrt{2}}$ or $\underline{\pi \ln \frac{2}{\sqrt{2}}}$ or $\underline{\frac{1}{2}\pi \ln 2}$ or $\underline{-\pi \ln\left(\frac{1}{\sqrt{2}}\right)}$ or $\underline{\frac{\pi}{2}\ln\left(\frac{1}{2}\right)}$	$\underline{\pi \ln \sqrt{2}}$ or $\underline{\pi \ln \frac{2}{\sqrt{2}}}$ or $\underline{\frac{1}{2}\pi \ln 2}$ or $\underline{-\pi \ln\left(\frac{1}{\sqrt{2}}\right)}$ or $\underline{\frac{\pi}{2}\ln\left(\frac{1}{2}\right)}$ must be exact.	A1 aef [4]
			11 marks

If a candidate gives the correct exact answer and then writes 1.088779…, then such a candidate can be awarded A1 (aef). The subsequent working would then be ignored. (isw)

Beware: In part (c) the factor of π is not needed for the first three marks.

Beware: In part (b) a candidate can also add up individual trapezia in this way:

$$\text{Area} \approx \frac{1}{2}\cdot\frac{\pi}{16}\left(\underline{0 + 0.44600}\right) + \frac{1}{2}\cdot\frac{\pi}{16}\left(\underline{0.44600 + 0.64359}\right) + \frac{1}{2}\cdot\frac{\pi}{16}\left(\underline{0.64359 + 0.81742}\right) + \frac{1}{2}\cdot\frac{\pi}{16}\left(\underline{0.81742 + 1}\right)$$

Question Number	Scheme		Marks
8. (a)	$\frac{dP}{dt} = kP$ and $t = 0,\ P = P_0$ (1)		
	$\int \frac{dP}{P} = \int k\ dt$	Separates the variables with $\int \frac{dP}{P}$ and $\int k\,dt$ on either side with integral signs not necessary.	M1
	$\ln P = kt;(+c)$	Must see $\ln P$ and kt; Correct equation with/without + c.	A1
	When $t = 0,\ P = P_0 \Rightarrow \ln P_0 = c$ $\left(\text{or } P = Ae^{kt} \Rightarrow P_0 = A\right)$	Use of boundary condition (1) to attempt to find the constant of integration.	M1
	$\ln P = kt + \ln P_0 \Rightarrow e^{\ln P} = e^{kt + \ln P_0} = e^{kt}.e^{\ln P_0}$		
	Hence, $\underline{P = P_0 e^{kt}}$	$\underline{P = P_0 e^{kt}}$	A1 [4]
(b)	$P = 2P_0\ \&\ k = 2.5 \Rightarrow \underline{2P_0 = P_0 e^{2.5t}}$	Substitutes $P = 2P_0$ into an expression involving P	M1
	$e^{2.5t} = 2 \Rightarrow \underline{\ln e^{2.5t} = \ln 2}$ or $\underline{2.5t = \ln 2}$ ...or $e^{kt} = 2 \Rightarrow \underline{\ln e^{kt} = \ln 2}$ or $\underline{kt = \ln 2}$	Eliminates P_0 and takes ln of both sides	M1
	$\Rightarrow t = \frac{1}{2.5}\ln 2 = 0.277258872...$ days		
	$t = 0.277258872... \times 24 \times 60 = 399.252776...$ minutes		
	$t = \underline{399\text{min}}$ or $t = \underline{6\text{ hr } 39\text{ mins}}$ (to nearest minute)	awrt $t = \underline{399}$ or $\underline{6\text{ hr } 39\text{ mins}}$	A1 [3]

$\underline{P = P_0 e^{kt}}$ written down without the first M1 mark given scores all four marks in part (a).

Question Number	Scheme		Marks
8. (c)	$\frac{dP}{dt} = \lambda P \cos \lambda t$ and $t = 0,\ P = P_0$ (1)		
	$\int \frac{dP}{P} = \int \lambda \cos \lambda t \ dt$	Separates the variables with $\int \frac{dP}{P}$ and $\int \lambda \cos \lambda t \, dt$ on either side with integral signs not necessary.	M1
	$\ln P = \sin \lambda t; (+c)$	Must see $\ln P$ and $\sin \lambda t$; Correct equation with/without + c.	A1
	When $t = 0,\ P = P_0 \Rightarrow \ln P_0 = c$ $\left(\text{or } P = Ae^{\sin \lambda t} \Rightarrow P_0 = A\right)$	Use of boundary condition (1) to attempt to find the constant of integration.	M1
	$\ln P = \sin \lambda t + \ln P_0 \Rightarrow e^{\ln P} = e^{\sin \lambda t + \ln P_0} = e^{\sin \lambda t}.e^{\ln P_0}$		
	Hence, $\underline{P = P_0 e^{\sin \lambda t}}$	$\underline{P = P_0 e^{\sin \lambda t}}$	A1
			[4]
(d)	$P = 2P_0$ & $\lambda = 2.5 \Rightarrow 2P_0 = P_0 e^{\sin 2.5t}$		
	$e^{\sin 2.5t} = 2 \Rightarrow \underline{\sin 2.5t = \ln 2}$ ...or ... $e^{\lambda t} = 2 \Rightarrow \underline{\sin \lambda t = \ln 2}$	Eliminates P_0 and makes $\sin \lambda t$ or $\sin 2.5t$ the subject by taking ln's	M1
	$\underline{t = \frac{1}{2.5} \sin^{-1}(\ln 2)}$	Then rearranges to make *t* the subject. (must use $\sin^{-1}$)	dM1
	$t = 0.306338477...$		
	$t = 0.306338477... \times 24 \times 60 = 441.1274082...$ minutes		
	$t = \underline{441\text{min}}$ or $t = \underline{7 \text{ hr } 21 \text{ mins}}$ (to nearest minute)	awrt $t = \underline{441}$ or $\underline{7 \text{ hr } 21 \text{ mins}}$	A1
			[3]
			14 marks

$\underline{P = P_0 e^{\sin \lambda t}}$ written down without the first M1 mark given scores all four marks in part (c).

Question Number	Scheme		Marks
	$\frac{\mathrm{d}P}{\mathrm{d}t} = kP$ and $t = 0,\ P = P_0$ (1)		
Aliter 8. (a) Way 2	$\int \frac{\mathrm{d}P}{kP} = \int 1\,\mathrm{d}t$	Separates the variables with $\int \frac{\mathrm{d}P}{kP}$ and $\int \mathrm{d}t$ on either side with integral signs not necessary.	M1
	$\frac{1}{k}\ln P = t;(+c)$	Must see $\frac{1}{k}\ln P$ and t; Correct equation with/without + c.	A1
	When $t = 0,\ P = P_0 \Rightarrow \frac{1}{k}\ln P_0 = c$ $\left(\text{or } P = Ae^{kt} \Rightarrow P_0 = A\right)$	Use of boundary condition (1) to attempt to find the constant of integration.	M1
	$\frac{1}{k}\ln P = t + \frac{1}{k}\ln P_0 \Rightarrow \ln P = kt + \ln P_0$ $\Rightarrow e^{\ln P} = e^{kt + \ln P_0} = e^{kt}.e^{\ln P_0}$		
	Hence, $\underline{P = P_0 e^{kt}}$	$\underline{P = P_0 e^{kt}}$	A1 [4]
Aliter 8. (a) Way 3	$\int \frac{\mathrm{d}P}{kP} = \int 1\,\mathrm{d}t$	Separates the variables with $\int \frac{\mathrm{d}P}{kP}$ and $\int \mathrm{d}t$ on either side with integral signs not necessary.	M1
	$\frac{1}{k}\ln(kP) = t;(+c)$	Must see $\frac{1}{k}\ln(kP)$ and t; Correct equation with/without + c.	A1
	When $t = 0,\ P = P_0 \Rightarrow \frac{1}{k}\ln(kP_0) = c$ $\left(\text{or } kP = Ae^{kt} \Rightarrow kP_0 = A\right)$	Use of boundary condition (1) to attempt to find the constant of integration.	M1
	$\frac{1}{k}\ln(kP) = t + \frac{1}{k}\ln(kP_0) \Rightarrow \ln(kP) = kt + \ln(kP_0)$ $\Rightarrow e^{\ln(kP)} = e^{kt + \ln(kP_0)} = e^{kt}.e^{\ln(kP_0)}$ $\Rightarrow kP = e^{kt}.(kP_0) \Rightarrow kP = kP_0 e^{kt}$ $\left(\text{or } kP = kP_0 e^{kt}\right)$		
	Hence, $\underline{P = P_0 e^{kt}}$	$\underline{P = P_0 e^{kt}}$	A1 [4]

Question Number	Scheme	Marks

	$\frac{dP}{dt} = \lambda P\cos\lambda t$ and $t = 0,\ P = P_0$ (1)		
Aliter 8. (c) Way 2	$\int \frac{dP}{\lambda P} = \int \cos\lambda t\ dt$	Separates the variables with $\int \frac{dP}{\lambda P}$ and $\int \cos\lambda t\ dt$ on either side with integral signs not necessary.	M1
	$\frac{1}{\lambda}\ln P = \frac{1}{\lambda}\sin\lambda t; (+c)$	Must see $\frac{1}{\lambda}\ln P$ and $\frac{1}{\lambda}\sin\lambda t$; Correct equation with/without + c.	A1
	When $t = 0,\ P = P_0 \Rightarrow \frac{1}{\lambda}\ln P_0 = c$ $\left(\text{or } P = Ae^{\sin\lambda t} \Rightarrow P_0 = A\right)$	Use of boundary condition (1) to attempt to find the constant of integration.	M1
	$\frac{1}{\lambda}\ln P = \frac{1}{\lambda}\sin\lambda t + \frac{1}{\lambda}\ln P_0 \Rightarrow \ln P = \sin\lambda t + \ln P_0$		
	$\Rightarrow e^{\ln P} = e^{\sin\lambda t + \ln P_0} = e^{\sin\lambda t}.e^{\ln P_0}$		
	Hence, $\underline{P = P_0 e^{\sin\lambda t}}$	$\underline{P = P_0 e^{\sin\lambda t}}$	A1 [4]

$\underline{P = P_0 e^{kt}}$ written down without the first M1 mark given scores all four marks in part (a).

$\underline{P = P_0 e^{\sin\lambda t}}$ written down without the first M1 mark given scores all four marks in part (c).

Question Number	Scheme		Marks
	$\frac{\mathrm{d}P}{\mathrm{d}t} = \lambda P \cos \lambda t$ and $t = 0,\ P = P_0$ (1)		
Aliter 8. (c) Way 3	$\int \frac{\mathrm{d}P}{\lambda P} = \int \cos \lambda t \ \mathrm{d}t$	Separates the variables with $\int \frac{\mathrm{d}P}{\lambda P}$ and $\int \cos \lambda t \,\mathrm{d}t$ on either side with integral signs not necessary.	M1
	$\frac{1}{\lambda}\ln(\lambda P) = \frac{1}{\lambda}\sin \lambda t ;(+ c)$	Must see $\frac{1}{\lambda}\ln(\lambda P)$ and $\frac{1}{\lambda}\sin \lambda t$; Correct equation with/without + c.	A1
	When $t = 0,\ P = P_0 \Rightarrow \frac{1}{\lambda}\ln(\lambda P_0) = c$ $\left(\text{or } \lambda P = Ae^{\sin \lambda t} \Rightarrow \lambda P_0 = A\right)$	Use of boundary condition (1) to attempt to find the constant of integration.	M1
	$\frac{1}{\lambda}\ln(\lambda P) = \frac{1}{\lambda}\sin \lambda t + \frac{1}{\lambda}\ln(\lambda P_0)$		
	$\Rightarrow \ln(\lambda P) = \sin \lambda t + \ln(\lambda P_0)$		
	$\Rightarrow e^{\ln(\lambda P)} = e^{\sin \lambda t + \ln(\lambda P_0)} = e^{\sin \lambda t}.e^{\ln(\lambda P_0)}$		
	$\Rightarrow \lambda P = e^{\sin \lambda t}.(\lambda P_0)$ $\left(\text{or } \lambda P = \lambda P_0 e^{\sin \lambda t}\right)$		
	Hence, $\underline{P = P_0 e^{\sin \lambda t}}$	$\underline{P = P_0 e^{\sin \lambda t}}$	A1
			[4]

- Note: dM1 denotes a method mark which is dependent upon the award of the previous method mark.
  ddM1 denotes a method mark which is dependent upon the award of the previous two method marks.
  depM1$*$ denotes a method mark which is dependent upon the award of M1$*$.
  ft denotes "follow through"
  cao denotes "correct answer only"
  aef denotes "any equivalent form"

Mark Scheme (Results)

January 2008

GCE

GCE Mathematics (6666/01)

edexcel

January 2008
6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme		Marks
1. (a)	x: 0, $\frac{\pi}{4}$, $\frac{\pi}{2}$, $\frac{3\pi}{4}$, π y: 0, $1.844321332\ldots$, $4.810477381\ldots$, 8.87207, 0		
		awrt 1.84432	B1
		awrt 4.81048 or 4.81047	B1
			[2]
(b) **Way 1**	0 can be implied Area $\approx \frac{1}{2} \times \frac{\pi}{4}; \times \underline{\left\{0 + 2\left(1.84432 + 4.81048 + 8.87207\right) + 0\right\}}$	Outside brackets awrt 0.39 or $\frac{1}{2} \times$ awrt 0.79 $\frac{1}{2} \times \frac{\pi}{4}$ or $\frac{\pi}{8}$	B1
		<u>For structure of trapezium</u> <u>rule</u> $\underline{\{.............\}}$;	<u>M1</u> $\sqrt{\ }$
		Correct expression <u>inside brackets</u> which all must be multiplied by their "outside constant".	<u>A1</u> $\sqrt{\ }$
	$= \frac{\pi}{8} \times 31.05374... = 12.19477518... = \underline{12.1948}$ (4dp)	<u>12.1948</u>	A1 **cao**
			[4]
Aliter (b) **Way 2**	Area $\approx \frac{\pi}{4} \times \left\{\frac{0+1.84432}{2} + \frac{1.84432+4.81048}{2} + \frac{4.81048+8.87207}{2} + \frac{8.87207+0}{2}\right\}$	$\frac{\pi}{4}$ (or awrt 0.79) and a divisor of 2 on all terms inside brackets.	B1
	which is equivalent to:	One of first and last ordinates, two of the middle ordinates inside brackets ignoring the 2.	<u>M1</u> $\sqrt{\ }$
	Area $\approx \frac{1}{2} \times \frac{\pi}{4}; \times \underline{\left\{0 + 2\left(1.84432 + 4.81048 + 8.87207\right) + 0\right\}}$	Correct expression inside brackets if $\frac{1}{2}$ was to be factorised out.	<u>A1</u> $\sqrt{\ }$
	$= \frac{\pi}{4} \times 15.52687... = 12.19477518... = \underline{12.1948}$ (4dp)	<u>12.1948</u>	A1 **cao**
			[4]
			6 marks

Note an expression like Area $\approx \frac{1}{2} \times \frac{\pi}{4} + 2\left(1.84432 + 4.81048 + 8.87207\right)$ would score B1M1A0A0

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Question Number	Scheme		Marks
	** represents a constant (which must be consistent for first accuracy mark)		
2. (a)	$(8-3x)^{\frac{1}{3}} = \underline{(8)^{\frac{1}{3}}}\left(1-\frac{3x}{8}\right)^{\frac{1}{3}} = \underline{2}\left(1-\frac{3x}{8}\right)^{\frac{1}{3}}$	Takes 8 outside the bracket to give any of $\underline{(8)^{\frac{1}{3}}}$ or $\underline{2}$.	<u>B1</u>
	$= 2\left\{\underline{1+(\tfrac{1}{3})(**x);+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(**x)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(**x)^3+...}\right\}$ **with** $** \neq 1$	Expands $(1+**x)^{\frac{1}{3}}$ to give a simplified or an un-simplified $1+(\frac{1}{3})(**x)$; A correct simplified or an un-simplified $\{\underline{.........}\}$ expansion with candidate's followed through $(**x)$	M1; A1 $\sqrt{}$
	$= 2\left\{\underline{1+(\tfrac{1}{3})(-\tfrac{3x}{8})+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(-\tfrac{3x}{8})^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(-\tfrac{3x}{8})^3+...}\right\}$	Award SC M1 if you see $\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(**x)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(**x)^3$	
	$= 2\{1-\tfrac{1}{8}x;-\tfrac{1}{64}x^2-\tfrac{5}{1536}x^3-...\}$ $= 2-\frac{1}{4}x;-\frac{1}{32}x^2-\frac{5}{768}x^3-...$	Either $2\{1-\frac{1}{8}x\ ....\}$ or anything that cancels to $2-\frac{1}{4}x$; Simplified $-\frac{1}{32}x^2-\frac{5}{768}x^3$	A1; A1 **[5]**
(b)	$(7.7)^{\frac{1}{3}} \approx 2-\frac{1}{4}(0.1)-\frac{1}{32}(0.1)^2-\frac{5}{768}(0.1)^3-...$	***Attempt to substitute*** $x=0.1$ into a candidate's binomial expansion.	M1
	$= 2-0.025-0.0003125-0.0000065104166...$		
	$= 1.97468099...$	awrt 1.9746810	A1 **[2]**
			7 marks

You would award B1M1A0 for
$$=2\left\{\underline{1+(\tfrac{1}{3})(-\tfrac{3x}{8})+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(-\tfrac{3x}{8})^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(-3x)^3+...}\right\}$$
because ** is not consistent.

If you see the constant term "2" in a candidate's final binomial expansion, then you can award B1.

Be wary of calculator value of $(7.7)^{\frac{1}{3}} = 1.974680822...$

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<table>
<tr><th>Question Number</th><th>Scheme</th><th></th><th>Marks</th></tr>
<tr><td>Aliter
2. (a)
Way 2</td><td>$(8-3x)^{\frac{1}{3}}$</td><td></td><td></td></tr>
<tr><td></td><td></td><td>2 or $(8)^{\frac{1}{3}}$ (See note $\downarrow$)</td><td>B1</td></tr>
<tr><td></td><td>$= \left\{ (8)^{\frac{1}{3}} + (\tfrac{1}{3})(8)^{-\frac{2}{3}}(**x); + \frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(8)^{-\frac{5}{3}}(**x)^2 + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(8)^{-\frac{8}{3}}(**x)^3 + ... \right\}$</td><td>Expands $(8-3x)^{\frac{1}{3}}$ to give an un-simplified or simplified $(8)^{\frac{1}{3}} + (\frac{1}{3})(8)^{-\frac{2}{3}}(**x)$;</td><td>M1;</td></tr>
<tr><td></td><td>with $** \neq 1$</td><td>A correct un-simplified or simplified $\{\underline{.........}\}$ expansion with candidate's followed through $(**x)$</td><td>A1 $\sqrt{}$</td></tr>
<tr><td></td><td>$= \left\{ (8)^{\frac{1}{3}} + (\tfrac{1}{3})(8)^{-\frac{2}{3}}(-3x); + \frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(8)^{-\frac{5}{3}}(-3x)^2 + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(8)^{-\frac{8}{3}}(-3x)^3 + ... \right\}$</td><td>Award SC M1 if you see $\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(8)^{-\frac{5}{3}}(**x)^2 + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(8)^{-\frac{8}{3}}(**x)^3$</td><td></td></tr>
<tr><td></td><td>$= \left\{ 2 + (\tfrac{1}{3})(\tfrac{1}{4})(-3x) + (-\tfrac{1}{9})(\tfrac{1}{32})(9x^2) + (\tfrac{5}{81})(\tfrac{1}{256})(-27x^3) + ... \right\}$</td><td></td><td></td></tr>
<tr><td></td><td>$= 2 - \frac{1}{4}x; - \frac{1}{32}x^2 - \frac{5}{768}x^3 - ...$</td><td>Anything that cancels to $2 - \frac{1}{4}x$; or $2\{1 - \frac{1}{8}x \text{ ....}\}$</td><td>A1;</td></tr>
<tr><td></td><td></td><td>Simplified $-\frac{1}{32}x^2 - \frac{5}{768}x^3$</td><td>A1</td></tr>
<tr><td></td><td></td><td></td><td>[5]</td></tr>
</table>

Attempts using Maclaurin expansion should be escalated up to your team leader.

Be wary of calculator value of $(7.7)^{\frac{1}{3}} = 1.974680822...$

If you see the constant term "2" in a candidate's final binomial expansion, then you can award B1.

Question Number	Scheme		Marks
3.	Volume $= \pi \int_a^b \left(\dfrac{1}{2x+1}\right)^2 \mathrm{d}x = \pi \int_a^b \dfrac{1}{(2x+1)^2}\mathrm{d}x$	Use of $V = \pi \int y^2 \; \mathrm{d}x$. Can be implied. Ignore limits.	B1
	$= \pi \int_a^b (2x+1)^{-2} \mathrm{d}x$		
	$= (\pi)\left[\dfrac{(2x+1)^{-1}}{(-1)(2)}\right]_a^b$		
	$= (\pi)\left[-\tfrac{1}{2}(2x+1)^{-1}\right]_a^b$	Integrating to give $\pm p(2x+1)^{-1}$ $-\tfrac{1}{2}(2x+1)^{-1}$	M1 A1
	$= (\pi)\left[\left(\dfrac{-1}{2(2b+1)}\right) - \left(\dfrac{-1}{2(2a+1)}\right)\right]$	Substitutes limits of b and a and subtracts the correct way round.	dM1
	$= \dfrac{\pi}{2}\left[\dfrac{-2a-1+2b+1}{(2a+1)(2b+1)}\right]$		
	$= \dfrac{\pi}{2}\left[\dfrac{2(b-a)}{(2a+1)(2b+1)}\right]$		
	$= \dfrac{\pi(b-a)}{(2a+1)(2b+1)}$	$\dfrac{\pi(b-a)}{(2a+1)(2b+1)}$	A1 aef
			[5]
			5 marks

Allow other equivalent forms such as

$$\frac{\pi b - \pi a}{(2a+1)(2b+1)} \text{ or } \frac{-\pi(a-b)}{(2a+1)(2b+1)} \text{ or } \frac{\pi(b-a)}{4ab+2a+2b+1} \text{ or } \frac{\pi b - \pi a}{4ab+2a+2b+1}.$$

Note that π is not required for the middle three marks of this question.

Question Number	Scheme		Marks
Aliter **3. Way 2**	Volume $= \underline{\pi \int_a^b \left(\frac{1}{2x+1}\right)^2} \,\mathrm{d}x = \pi \int_a^b \frac{1}{(2x+1)^2}\,\mathrm{d}x$	Use of $V = \underline{\pi \int y^2}\,\mathrm{d}x$. Can be implied. Ignore limits.	B1
	$= \pi \int_a^b (2x+1)^{-2}\,\mathrm{d}x$		
	Applying substitution $u = 2x+1 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 2$ and changing limits $x \to u$ so that $a \to 2a+1$ and $b \to 2b+1$, gives		
	$= (\pi)\int_{2a+1}^{2b+1} \frac{u^{-2}}{2}\,\mathrm{d}u$		
	$= (\pi)\left[\frac{u^{-1}}{(-1)(2)}\right]_{2a+1}^{2b+1}$		
	$= (\pi)\left[\underline{-\tfrac{1}{2}u^{-1}}\right]_{2a+1}^{2b+1}$	Integrating to give $\underline{\pm p u^{-1}}$ $\underline{-\tfrac{1}{2}u^{-1}}$	M1 A1
	$= (\pi)\left[\left(\frac{-1}{2(2b+1)}\right) - \left(\frac{-1}{2(2a+1)}\right)\right]$	Substitutes limits of $2b+1$ and $2a+1$ and subtracts the correct way round.	dM1
	$= \frac{\pi}{2}\left[\frac{-2a-1+2b+1}{(2a+1)(2b+1)}\right]$		
	$= \frac{\pi}{2}\left[\frac{2(b-a)}{(2a+1)(2b+1)}\right]$		
	$= \frac{\pi(b-a)}{(2a+1)(2b+1)}$	$\underline{\frac{\pi(b-a)}{(2a+1)(2b+1)}}$	A1 aef
			[5]
			5 marks

Note that π is not required for the middle three marks of this question.

Allow other equivalent forms such as

$$\frac{\pi b - \pi a}{(2a+1)(2b+1)} \text{ or } \frac{-\pi(a-b)}{(2a+1)(2b+1)} \text{ or } \frac{\pi(b-a)}{4ab+2a+2b+1} \text{ or } \frac{\pi b - \pi a}{4ab+2a+2b+1}.$$

Question Number	Scheme		Marks
4. (i)	$\int \ln\left(\frac{x}{2}\right) \mathrm{d}x = \int 1.\ln\left(\frac{x}{2}\right) \mathrm{d}x \Rightarrow \left\{\begin{array}{ll} u = \ln\left(\frac{x}{2}\right) & \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{\frac{1}{2}}{\frac{x}{2}} = \frac{1}{x} \\ \frac{\mathrm{d}v}{\mathrm{d}x} = 1 & \Rightarrow v = x \end{array}\right\}$		
	$\int \ln\left(\frac{x}{2}\right) \mathrm{d}x = x\ln\left(\frac{x}{2}\right) - \int x.\frac{1}{x} \mathrm{d}x$	Use of 'integration by parts' formula in the correct direction.	M1
		Correct expression.	A1
	$= x\ln\left(\frac{x}{2}\right) - \int \underline{1} \mathrm{d}x$	An attempt to multiply x by a candidate's $\frac{a}{x}$ or $\frac{1}{bx}$ or $\frac{1}{x}$.	<u>dM1</u>
	$= x\ln\left(\frac{x}{2}\right) - x + c$	Correct integration with + c	A1 aef
			[4]
(ii)	$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \mathrm{d}x$		
	$\left[\text{NB: } \underline{\cos 2x = \pm 1 \pm 2\sin^2 x} \text{ or } \underline{\sin^2 x = \tfrac{1}{2}(\pm 1 \pm \cos 2x)}\right]$	Consideration of double angle formula for $\cos 2x$	M1
	$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1-\cos 2x}{2} \mathrm{d}x = \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\underline{1 - \cos 2x}) \mathrm{d}x$		
	$= \frac{1}{2}\left[\underline{x - \tfrac{1}{2}\sin 2x}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$	<u>Integrating to give</u> <u>$\pm ax \pm b\sin 2x$</u>; $a, b \neq 0$	dM1
		Correct result of anything equivalent to $\frac{1}{2}x - \frac{1}{4}\sin 2x$	A1
	$= \frac{1}{2}\left[\left(\frac{\pi}{2} - \frac{\sin(\pi)}{2}\right) - \left(\frac{\pi}{4} - \frac{\sin\left(\frac{\pi}{2}\right)}{2}\right)\right]$	Substitutes limits of $\frac{\pi}{2}$ and $\frac{\pi}{4}$ and subtracts the correct way round.	ddM1
	$= \frac{1}{2}\left[\left(\frac{\pi}{2} - 0\right) - \left(\frac{\pi}{4} - \frac{1}{2}\right)\right]$		
	$= \frac{1}{2}\left(\frac{\pi}{4} + \frac{1}{2}\right) = \frac{\pi}{8} + \frac{1}{4}$	$\underline{\frac{1}{2}\left(\frac{\pi}{4} + \frac{1}{2}\right)}$ or $\underline{\frac{\pi}{8} + \frac{1}{4}}$ or $\underline{\frac{\pi}{8} + \frac{2}{8}}$ Candidate must collect their π term and constant term together for A1 No fluked answers, hence **cso**.	A1 **aef**, **cso** **[5]**
			9 marks

Note: $\int \ln\left(\frac{x}{2}\right) \mathrm{d}x = (\text{their } v)\ln\left(\frac{x}{2}\right) - \int (\text{their } v).\left(\text{their } \frac{\mathrm{d}u}{\mathrm{d}x}\right) \mathrm{d}x$ for M1 in part (i).

Note $\frac{\pi}{8} + \frac{1}{4} = 0.64269...$

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Question Number	Scheme		Marks
Aliter **4.** (i) **Way 2**	$\int \ln\left(\frac{x}{2}\right)\,dx = \int \left(\ln x - \ln 2\right)\,dx = \int \ln x\,dx - \int \ln 2\,dx$		
	$\int \ln x\,dx = \int 1.\ln x\,dx \Rightarrow \left\{\begin{array}{lll} u = \ln x & \Rightarrow & \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = 1 & \Rightarrow & v = x \end{array}\right\}$		
	$\int \ln x\,dx = x\ln x - \int x.\frac{1}{x}\,dx$	Use of 'integration by parts' formula in the correct direction.	M1
	$= x\ln x - x + c$	Correct integration of $\ln x$ with or without $+ c$	A1
	$\int \ln 2\,dx = x\ln 2 + c$	Correct integration of $\ln 2$ with or without $+ c$	M1
	Hence, $\int \ln\left(\frac{x}{2}\right)\,dx = x\ln x - x - x\ln 2 + c$	Correct integration with $+ c$	A1 aef
			[4]

Note: $\int \ln x\,dx = (\text{their } v)\ln x - \int (\text{their } v).(\text{their } \frac{du}{dx})\,dx$ for M1 in part (i).

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Question Number	Scheme		Marks
Aliter **4.** (i) **Way 3**	$\int \ln\left(\frac{x}{2}\right)\,dx$		
	$u = \frac{x}{2} \Rightarrow \frac{du}{dx} = \frac{1}{2}$		
	$\int \ln\left(\frac{x}{2}\right)\,dx = 2\int \ln u\,du$	Applying substitution correctly to give $\int \ln\left(\frac{x}{2}\right)\,dx = 2\int \ln u\,du$ ***Decide to award 2nd M1 here!***	
	$\int \ln u\,dx = \int 1.\ln u\,du$		
	$\int \ln u\,dx = u\ln u - \int u.\frac{1}{u}\,du$	Use of 'integration by parts' formula in the correct direction.	M1
	$= u\ln u - u + c$	Correct integration of $\ln u$ with or without + c	A1
		Decide to award 2nd M1 here!	M1
	$\int \ln\left(\frac{x}{2}\right)dx = 2\left(u\ln u - u\right) + c$		
	Hence, $\int \ln\left(\frac{x}{2}\right)\,dx = x\ln\left(\frac{x}{2}\right) - x + c$	Correct integration with + c	A1 aef **[4]**

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Question Number	Scheme		Marks
Aliter **4.** (ii) **Way 2**	$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \, dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin x . \sin x \, dx$ and $I = \int \sin^2 x \, dx$		
	$\left\{ \begin{array}{ll} u = \sin x & \Rightarrow \frac{du}{dx} = \cos x \\ \frac{dv}{dx} = \sin x & \Rightarrow v = -\cos x \end{array} \right\}$		
	$\therefore I = \underline{\left\{ -\sin x \cos x + \int \cos^2 x \, dx \right\}}$	An attempt to use the correct by parts formula.	M1
	$\therefore I = \left\{ -\sin x \cos x + \int \left(1 - \sin^2 x\right) dx \right\}$		
	$\int \sin^2 x \, dx = \left\{ -\sin x \cos x + \int 1 \, dx - \int \sin^2 x \, dx \right\}$		
	$2 \int \sin^2 x \, dx = \left\{ -\sin x \cos x + \int 1 \, dx \right\}$	For the LHS becoming $2I$	dM1
	$2 \int \sin^2 x \, dx = \left\{ -\sin x \cos x + x \right\}$		
	$\int \sin^2 x \, dx = \left\{ \underline{-\tfrac{1}{2} \sin x \cos x + \tfrac{x}{2}} \right\}$	<u>Correct integration</u>	A1
	$\therefore \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \, dx = \left[\left(-\tfrac{1}{2} \sin(\tfrac{\pi}{2}) \cos(\tfrac{\pi}{2}) + \tfrac{(\frac{\pi}{2})}{2} \right) - \left(-\tfrac{1}{2} \sin(\tfrac{\pi}{4}) \cos(\tfrac{\pi}{4}) + \tfrac{(\frac{\pi}{4})}{2} \right) \right]$ $= \left[(0 + \tfrac{\pi}{4}) - (-\tfrac{1}{4} + \tfrac{\pi}{8}) \right]$	Substitutes limits of $\frac{\pi}{2}$ and $\frac{\pi}{4}$ and subtracts the correct way round.	ddM1
	$= \tfrac{\pi}{8} + \tfrac{1}{4}$	$\underline{\tfrac{1}{2}\left(\tfrac{\pi}{4} + \tfrac{1}{2}\right)}$ or $\underline{\tfrac{\pi}{8} + \tfrac{1}{4}}$ or $\underline{\tfrac{\pi}{8} + \tfrac{2}{8}}$ Candidate must collect their π term and constant term together for A1 No fluked answers, hence **cso**.	A1 **aef cso** **[5]**

Note $\frac{\pi}{8} + \frac{1}{4} = 0.64269...$

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Question Number	Scheme		Marks
5. (a)	$x^3 - 4y^2 = 12xy$ (eqn $*$)		
	$x = -8 \Rightarrow -512 - 4y^2 = 12(-8)y$ $-512 - 4y^2 = -96y$	Substitutes $x = -8$ (at least once) into * to obtain a three term quadratic in y. Condone the loss of $= 0$.	M1
	$4y^2 - 96y + 512 = 0$ $y^2 - 24y + 128 = 0$		
	$(y-16)(y-8) = 0$ $y = \dfrac{24 \pm \sqrt{576 - 4(128)}}{2}$	An attempt to solve the quadratic in y by either factorising or by the formula or by ***completing the square***.	dM1
	$y = 16$ or $y = 8$.	Both $\underline{y = 16}$ and $\underline{y = 8}$. or $\underline{(-8, 8)}$ and $\underline{(-8, 16)}$.	A1
			[3]
(b)	$\left\{ \dfrac{\cancel{dy}}{\cancel{dx}} \cancel{\times} \right\}$ $3x^2 - 8y\dfrac{dy}{dx}; = \left(\underline{12y + 12x\dfrac{dy}{dx}} \right)$	Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $12x\frac{dy}{dx}$. Ignore $\frac{dy}{dx} = \ldots$ Correct LHS equation; <u>Correct application of product rule</u>	M1 A1; (B1)
	$\left\{ \dfrac{dy}{dx} = \dfrac{3x^2 - 12y}{12x + 8y} \right\}$	*not necessarily required.*	
	@ $(-8, 8)$, $\dfrac{dy}{dx} = \dfrac{3(64) - 12(8)}{12(-8) + 8(8)} = \dfrac{96}{-32} = \underline{-3}$,	Substitutes $x = -8$ and *at least one* of their y-values to attempt to find any one of $\frac{dy}{dx}$.	dM1
	@ $(-8, 16)$, $\dfrac{dy}{dx} = \dfrac{3(64) - 12(16)}{12(-8) + 8(16)} = \dfrac{0}{32} = \underline{0}$.	One gradient found. Both gradients of $\underline{-3}$ and $\underline{0}$ ***correctly*** found.	A1 A1 **cso**
			[6]
			9 marks

Question Number	Scheme		Marks
Aliter **5.** (b) **Way 2**	$\left\{\frac{\cancel{\mathrm{d}x}}{\cancel{\mathrm{d}y}}\cancel{\times}\right\}\quad 3x^2\frac{\mathrm{d}x}{\mathrm{d}y}-8y;\ =\underline{\left(12y\frac{\mathrm{d}x}{\mathrm{d}y}+12x\right)}$	Differentiates implicitly to include either $\pm kx^2\frac{dx}{dy}$ or $12y\frac{dx}{dy}$. Ignore $\frac{dx}{dy}=\ldots$	M1
		Correct LHS equation	A1;
		<u>Correct application of product rule</u>	(B1)
	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3x^2-12y}{12x+8y}\right\}$	*not necessarily required.*	
	@ $(-8, 8)$, $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3(64)-12(8)}{12(-8)+8(8)}=\frac{96}{-32}=\underline{-3}$,	Substitutes $x=-8$ and *at least one* of their y-values to attempt to find any one of $\frac{dy}{dx}$ or $\frac{dx}{dy}$.	dM1
	@ $(-8, 16)$, $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3(64)-12(16)}{12(-8)+8(16)}=\frac{0}{32}=\underline{0}$.	One gradient found.	A1
		Both gradients of <u>-3</u> and <u>0</u> ***correctly*** found.	A1 **cso**
			[6]

Question Number	Scheme		Marks
Aliter **5.** (b) **Way 3**	$x^3 - 4y^2 = 12xy$ (eqn $*$) $4y^2 + 12xy - x^3 = 0$ $y = \dfrac{-12x \pm \sqrt{144x^2 - 4(4)(-x^3)}}{8}$ $y = \dfrac{-12x \pm \sqrt{144x^2 + 16x^3}}{8}$ $y = \dfrac{-12x \pm 4\sqrt{9x^2 + x^3}}{8}$ $y = -\frac{3}{2}x \pm \frac{1}{2}\left(9x^2 + x^3\right)^{\frac{1}{2}}$		
	$\dfrac{dy}{dx} = -\frac{3}{2} \pm \frac{1}{2}\left(\frac{1}{2}\right)\left(9x^2 + x^3\right)^{-\frac{1}{2}};\left(18x + 3x^2\right)$	A credible attempt to make y the subject and an attempt to differentiate either $-\frac{3}{2}x$ or $\frac{1}{2}\left(9x^2 + x^3\right)^{\frac{1}{2}}$.	M1
	$\dfrac{dy}{dx} = -\dfrac{3}{2} \pm \dfrac{18x + 3x^2}{4(9x^2 + x^3)^{\frac{1}{2}}}$	$\dfrac{dy}{dx} = -\frac{3}{2} \pm k\left(9x^2 + x^3\right)^{-\frac{1}{2}}\left(g(x)\right)$	A1
		$\dfrac{dy}{dx} = -\frac{3}{2} \pm \frac{1}{2}\left(\frac{1}{2}\right)\left(9x^2 + x^3\right)^{-\frac{1}{2}};\left(18x + 3x^2\right)$	A1
	@ $x = -8$ $\quad \dfrac{dy}{dx} = -\dfrac{3}{2} \pm \dfrac{18(-8) + 3(64)}{4(9(64) + (-512))^{\frac{1}{2}}}$	Substitutes $x = -8$ find any one of $\frac{dy}{dx}$.	dM1
	$= -\dfrac{3}{2} \pm \dfrac{48}{4\sqrt{(64)}} = -\dfrac{3}{2} \pm \dfrac{48}{32}$		
	$\therefore \dfrac{dy}{dx} = -\dfrac{3}{2} \pm \dfrac{3}{2} = \underline{-3}, \underline{0}.$	One gradient correctly found. Both gradients of <u>-3</u> and <u>0</u> correctly found.	A1 A1
			[6]

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Question Number	Scheme		Marks
6. (a)	$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix}$ & $\overrightarrow{OB} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}$		
	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} = \underline{\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}}$	Finding the difference between $\overrightarrow{OB}$ and $\overrightarrow{OA}$.	M1 ±
		Correct answer.	A1
			[2]
(b)	$l_1 : \mathbf{r} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$	An expression of the form $(\text{vector}) \pm \lambda(\text{vector})$	M1
	$l_1 : \mathbf{r} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$	$\mathbf{r} = \overrightarrow{OA} \pm \lambda(\text{their } \overrightarrow{AB})$ or $\mathbf{r} = \overrightarrow{OB} \pm \lambda(\text{their } \overrightarrow{AB})$ or $\mathbf{r} = \overrightarrow{OA} \pm \lambda(\text{their } \overrightarrow{BA})$ or $\mathbf{r} = \overrightarrow{OB} \pm \lambda(\text{their } \overrightarrow{BA})$ ($\mathbf{r}$ is needed.)	A1 $\sqrt{}$ aef
			[2]
(c)	$l_2 : \mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \Rightarrow \mathbf{r} = \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$		
	$\overrightarrow{AB} = \mathbf{d}_1 = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$, $\mathbf{d}_2 = \mathbf{i} + 0\mathbf{j} + \mathbf{k}$ & θ is angle		
	$\cos\theta = \dfrac{\overrightarrow{AB} \bullet \mathbf{d}_2}{\left(\lvert\overrightarrow{AB}\rvert . \lvert\mathbf{d}_2\rvert\right)} = \dfrac{\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}}{\left(\sqrt{(1)^2 + (-2)^2 + (2)^2} . \sqrt{(1)^2 + (0)^2 + (1)^2}\right)}$	Considers dot product between $\mathbf{d}_2$ and their $\overrightarrow{AB}$.	M1 $\sqrt{}$
	$\cos\theta = \dfrac{1 + 0 + 2}{\sqrt{(1)^2 + (-2)^2 + (2)^2} . \sqrt{(1)^2 + (0)^2 + (1)^2}}$	Correct followed through expression or **equation**.	A1 $\sqrt{}$
	$\cos\theta = \dfrac{3}{3.\sqrt{2}} \Rightarrow \underline{\theta = 45^\circ \text{ or } \tfrac{\pi}{4} \text{ or awrt } 0.79}.$	$\underline{\theta = 45^\circ \text{ or } \tfrac{\pi}{4} \text{ or awrt } 0.79}$	A1 **cao**
			[3]

This means that $\cos\theta$ does not necessarily have to be the subject of the equation. It could be of the form $3\sqrt{2}\cos\theta = 3$.

Question Number	Scheme		Marks
6. (d)	If l_1 and l_2 intersect then: $\begin{pmatrix}2\\6\\-1\end{pmatrix}+\lambda\begin{pmatrix}1\\-2\\2\end{pmatrix}=\mu\begin{pmatrix}1\\0\\1\end{pmatrix}$		
	$\mathbf{i}: \quad 2+\lambda=\mu \quad (1)$ $\mathbf{j}: \quad 6-2\lambda=0 \quad (2)$ $\mathbf{k}: -1+2\lambda=\mu \quad (3)$	**Either** seeing equation (2) written down correctly with or without any other equation **or** seeing equations (1) and (3) written down correctly.	M1 $\sqrt{}$
	(2) yields $\lambda=3$ Any two yields $\lambda=3, \quad \mu=5$	Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find …	dM1
		either one of λ or μ correct.	A1
	$l_1: \mathbf{r}=\begin{pmatrix}2\\6\\-1\end{pmatrix}+3\begin{pmatrix}1\\-2\\2\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ *or* $\mathbf{r}=5\begin{pmatrix}1\\0\\1\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$	$\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ or $5\mathbf{i}+5\mathbf{k}$ Fully correct solution & no incorrect values of λ or μ seen earlier.	A1 **cso**
			[4]
Aliter 6. (d) **Way 2**	If l_1 and l_2 intersect then: $\begin{pmatrix}3\\4\\1\end{pmatrix}+\lambda\begin{pmatrix}1\\-2\\2\end{pmatrix}=\mu\begin{pmatrix}1\\0\\1\end{pmatrix}$		
	$\mathbf{i}: \quad 3+\lambda=\mu \quad (1)$ $\mathbf{j}: \quad 4-2\lambda=0 \quad (2)$ $\mathbf{k}: \quad 1+2\lambda=\mu \quad (3)$	**Either** seeing equation (2) written down correctly with or without any other equation **or** seeing equations (1) and (3) written down correctly.	M1 $\sqrt{}$
	(2) yields $\lambda=2$ Any two yields $\lambda=2, \quad \mu=5$	Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find …	dM1
		either one of λ or μ correct.	A1
	$l_1: \mathbf{r}=\begin{pmatrix}3\\4\\1\end{pmatrix}+2\begin{pmatrix}1\\-2\\2\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ *or* $\mathbf{r}=5\begin{pmatrix}1\\0\\1\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$	$\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ or $5\mathbf{i}+5\mathbf{k}$ Fully correct solution & no incorrect values of λ or μ seen earlier.	A1 **cso**
			[4]
			11 marks

Note: Be careful! λ and μ are not defined in the question, so a candidate could interchange these or use different scalar parameters.

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Question Number	Scheme		Marks
***Aliter* 6. (d) Way 3**	If l_1 and l_2 intersect then: $\begin{pmatrix}2\\6\\-1\end{pmatrix}+\lambda\begin{pmatrix}-1\\2\\-2\end{pmatrix}=\mu\begin{pmatrix}1\\0\\1\end{pmatrix}$		
	$\mathbf{i}: \quad 2-\lambda=\mu \qquad (1)$ $\mathbf{j}: \quad 6+2\lambda=0 \qquad (2)$ $\mathbf{k}: -1-2\lambda=\mu \qquad (3)$	**Either** seeing equation (2) written down correctly with or without any other equation **or** seeing equations (1) and (3) written down correctly.	M1 $\sqrt{}$
	(2) yields $\lambda=-3$ Any two yields $\lambda=-3,\ \mu=5$	Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find …	dM1
		either one of λ or μ correct.	A1
	$l_1: \mathbf{r}=\begin{pmatrix}2\\6\\-1\end{pmatrix}-3\begin{pmatrix}-1\\2\\-2\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ *or* $\mathbf{r}=5\begin{pmatrix}1\\0\\1\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$	$\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ or $5\mathbf{i}+5\mathbf{k}$ Fully correct solution & no incorrect values of λ or μ seen earlier.	A1 **cso**
			[4]
***Aliter* 6. (d) Way 4**	If l_1 and l_2 intersect then: $\begin{pmatrix}3\\4\\1\end{pmatrix}+\lambda\begin{pmatrix}-1\\2\\-2\end{pmatrix}=\mu\begin{pmatrix}1\\0\\1\end{pmatrix}$		
	$\mathbf{i}: \quad 3-\lambda=\mu \qquad (1)$ $\mathbf{j}: \quad 4+2\lambda=0 \qquad (2)$ $\mathbf{k}: \quad 1-2\lambda=\mu \qquad (3)$	**Either** seeing equation (2) written down correctly with or without any other equation **or** seeing equations (1) and (3) written down correctly.	M1 $\sqrt{}$
	(2) yields $\lambda=-2$ Any two yields $\lambda=-2,\ \mu=5$	Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find …	dM1
		either one of λ or μ correct.	A1
	$l_1: \mathbf{r}=\begin{pmatrix}3\\4\\1\end{pmatrix}-2\begin{pmatrix}-1\\2\\-2\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ *or* $\mathbf{r}=5\begin{pmatrix}1\\0\\1\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$	$\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ or $5\mathbf{i}+5\mathbf{k}$ Fully correct solution & no incorrect values of λ or μ seen earlier.	A1 **cso**
			[4]
			11 marks

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Question Number	Scheme		Marks
7. (a)	$\left[x=\ln(t+2),\ \ y=\dfrac{1}{t+1}\right],\quad \Rightarrow \dfrac{dx}{dt}=\dfrac{1}{t+2}$	Must state $\dfrac{dx}{dt}=\dfrac{1}{t+2}$	B1
	$\text{Area}(R)=\displaystyle\int_{\ln 2}^{\ln 4}\frac{1}{t+1}\,dx;=\int_0^2\left(\frac{1}{t+1}\right)\left(\frac{1}{t+2}\right)dt$	Area $=\displaystyle\int\frac{1}{t+1}\,dx$. Ignore limits.	M1;
		$\displaystyle\int\left(\frac{1}{t+1}\right)\times\left(\frac{1}{t+2}\right)dt$. Ignore limits.	A1 **AG**
	Changing limits, when: $x=\ln 2\ \Rightarrow \ln 2=\ln(t+2)\ \Rightarrow\ 2=t+2\ \Rightarrow t=0$ $x=\ln 4\ \Rightarrow \ln 4=\ln(t+2)\ \Rightarrow\ 4=t+2\ \Rightarrow t=2$	changes limits $x\to t$ so that $\ln 2\to 0$ and $\ln 4\to 2$	B1
	Hence, $\text{Area}(R)=\displaystyle\int_0^2\frac{1}{(t+1)(t+2)}\,dt$		
			[4]
(b)	$\left(\dfrac{1}{(t+1)(t+2)}\right)=\dfrac{A}{(t+1)}+\dfrac{B}{(t+2)}$	$\dfrac{A}{(t+1)}+\dfrac{B}{(t+2)}$ with A and B found	M1
	$1=A(t+2)+B(t+1)$		
	Let $t=-1,\quad 1=A(1)\quad\Rightarrow\ \underline{A=1}$ Let $t=-2,\quad 1=B(-1)\ \Rightarrow\ \underline{B=-1}$	Finds both A and B correctly. Can be implied. (See note below)	A1
	$\displaystyle\int_0^2\frac{1}{(t+1)(t+2)}\,dt=\int_0^2\frac{1}{(t+1)}-\frac{1}{(t+2)}\,dt$		
	$=\left[\ln(t+1)-\ln(t+2)\right]_0^2$	Either $\pm a\ln(t+1)$ or $\pm b\ln(t+2)$ Both ln terms correctly ft.	dM1 A1 $\sqrt{}$
	$=(\ln 3-\ln 4)-(\ln 1-\ln 2)$	Substitutes ***both*** limits of 2 and 0 and subtracts the correct way round.	ddM1
	$=\ln 3-\ln 4+\ln 2=\ln 3-\ln 2=\ln\left(\frac{3}{2}\right)$	$\underline{\ln 3-\ln 4+\ln 2}$ or $\underline{\ln\left(\frac{3}{4}\right)-\ln\left(\frac{1}{2}\right)}$ or $\underline{\ln 3-\ln 2}$ or $\underline{\ln\left(\frac{3}{2}\right)}$ (must deal with ln 1)	A1 aef isw
			[6]

Takes out brackets.

Writing down $\dfrac{1}{(t+1)(t+2)}=\dfrac{1}{(t+1)}+\dfrac{1}{(t+2)}$ means first M1A0 in (b).

Writing down $\dfrac{1}{(t+1)(t+2)}=\dfrac{1}{(t+1)}-\dfrac{1}{(t+2)}$ means first M1A1 in (b).

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Question Number	Scheme		Marks
	$x=\ln(t+2)$, $\quad y=\dfrac{1}{t+1}$		
7. (c)	$e^x = t+2 \Rightarrow t = e^x - 2$	Attempt to make $t =$… the subject giving $t = e^x - 2$	M1 A1
	$y=\dfrac{1}{e^x-2+1} \Rightarrow y=\dfrac{1}{e^x-1}$	Eliminates t by substituting in y giving $y=\dfrac{1}{e^x-1}$	dM1 A1 **[4]**
Aliter **7.** (c) **Way 2**	$t+1=\dfrac{1}{y} \Rightarrow t=\dfrac{1}{y}-1$ or $t=\dfrac{1-y}{y}$ $y(t+1)=1 \Rightarrow yt+y=1 \Rightarrow yt=1-y \Rightarrow t=\dfrac{1-y}{y}$	Attempt to make $t =$… the subject Giving either $t=\dfrac{1}{y}-1$ or $t=\dfrac{1-y}{y}$	M1 A1
	$x=\ln\left(\dfrac{1}{y}-1+2\right)$ or $x=\ln\left(\dfrac{1-y}{y}+2\right)$	Eliminates t by substituting in x	dM1
	$x=\ln\left(\dfrac{1}{y}+1\right)$ $e^x=\dfrac{1}{y}+1$ $e^x-1=\dfrac{1}{y}$		
	$y=\dfrac{1}{e^x-1}$	giving $y=\dfrac{1}{e^x-1}$	A1 **[4]**
(d)	Domain : $\underline{x>0}$	$\underline{x>0}$ or just >0	B1 **[1]**
			15 marks

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Question Number	Scheme		Marks
***Aliter* 7. (c) Way 3**	$e^x = t + 2 \;\Rightarrow\; t + 1 = e^x - 1$	Attempt to make $t + 1 = \ldots$ the subject	M1
		giving $t + 1 = e^x - 1$	A1
	$y = \dfrac{1}{t+1} \;\Rightarrow\; y = \dfrac{1}{e^x - 1}$	Eliminates t by substituting in y	dM1
		giving $y = \dfrac{1}{e^x - 1}$	A1
			[4]
***Aliter* 7. (c) Way 4**	$t + 1 = \dfrac{1}{y} \;\Rightarrow\; t + 2 = \dfrac{1}{y} + 1$ or $t + 2 = \dfrac{1+y}{y}$	Attempt to make $t + 2 = \ldots$ the subject	M1
		Either $t + 2 = \dfrac{1}{y} + 1$ or $t + 2 = \dfrac{1+y}{y}$	A1
	$x = \ln\left(\dfrac{1}{y} + 1\right)$ or $x = \ln\left(\dfrac{1+y}{y}\right)$	Eliminates t by substituting in x	dM1
	$x = \ln\left(\dfrac{1}{y} + 1\right)$		
	$e^x = \dfrac{1}{y} + 1 \;\Rightarrow\; e^x - 1 = \dfrac{1}{y}$		
	$y = \dfrac{1}{e^x - 1}$	giving $y = \dfrac{1}{e^x - 1}$	A1
			[4]

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Question Number	Scheme		Marks
8. (a)	$\frac{dV}{dt} = 1600 - c\sqrt{h}$ or $\frac{dV}{dt} = 1600 - k\sqrt{h}$,	Either of these statements	M1
	$\left(V = 4000h \Rightarrow\right) \frac{dV}{dh} = 4000$	$\frac{dV}{dh} = 4000$ or $\frac{dh}{dV} = \frac{1}{4000}$	M1
	$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} = \frac{\frac{dV}{dt}}{\frac{dV}{dh}}$		
	Either, $\frac{dh}{dt} = \frac{1600 - c\sqrt{h}}{4000} = \frac{1600}{4000} - \frac{c\sqrt{h}}{4000} = 0.4 - k\sqrt{h}$ or $\frac{dh}{dt} = \frac{1600 - k\sqrt{h}}{4000} = \frac{1600}{4000} - \frac{k\sqrt{h}}{4000} = 0.4 - k\sqrt{h}$	Convincing proof of $\frac{dh}{dt}$	A1 **AG**
			[3]
(b)	When $h = 25$ water ***leaks out such that*** $\frac{dV}{dt} = 400$		
	$400 = c\sqrt{h} \Rightarrow 400 = c\sqrt{25} \Rightarrow 400 = c(5) \Rightarrow c = 80$		
	From above; $k = \frac{c}{4000} = \frac{80}{4000} = 0.02$ as required	Proof that $k = 0.02$	B1 **AG**
			[1]
Aliter (b) **Way 2**	$400 = 4000k\sqrt{h}$		
	$\Rightarrow 400 = 4000k\sqrt{25}$		
	$\Rightarrow 400 = k(20000) \Rightarrow k = \frac{400}{20000} = 0.02$	Using 400, 4000 and $h = 25$ or $\sqrt{h} = 5$. Proof that $k = 0.02$	B1 **AG**
			[1]
(c)	$\frac{dh}{dt} = 0.4 - k\sqrt{h} \Rightarrow \int \frac{dh}{0.4 - k\sqrt{h}} = \int dt$	***Separates the variables*** with $\int \frac{dh}{0.4 - k\sqrt{h}}$ and $\int dt$ on either side with integral signs not necessary.	M1 oe
	$\therefore$ time required $= \int_0^{100} \frac{1}{0.4 - 0.02\sqrt{h}}\,dh \quad \frac{\div 0.02}{\div 0.02}$		
	time required $= \int_0^{100} \frac{50}{20 - \sqrt{h}}\,dh$	Correct proof	A1 **AG**
			[2]

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Question Number	Scheme		Marks
8. (d)	$\int_0^{100} \frac{50}{20-\sqrt{h}}\,dh$ with substitution $h=(20-x)^2$		
	$\frac{dh}{dx} = 2(20-x)(-1)$ or $\frac{dh}{dx} = -2(20-x)$	Correct $\frac{dh}{dx}$	B1 aef
	$h=(20-x)^2 \Rightarrow \sqrt{h}=20-x \Rightarrow x=20-\sqrt{h}$		
	$\int \frac{50}{20-\sqrt{h}}\,dh = \int \frac{50}{x}.-2(20-x)\,dx$	$\pm\lambda\int\frac{20-x}{x}\,dx$ or $\pm\lambda\int\frac{20-x}{20-(20-x)}\,dx$ where λ is a constant	M1
	$=100\int\frac{x-20}{x}\,dx$		
	$=100\int\left(1-\frac{20}{x}\right)dx$		
	$=100(x-20\ln x)\ (+c)$	$\pm\alpha x\pm\beta\ln x\ ;\ \alpha,\beta\neq 0$ $100x-2000\ln x$	M1 A1
	change limits: when $h=0$ then $x=20$ and when $h=100$ then $x=10$		
	$\int_0^{100}\frac{50}{20-\sqrt{h}}\,dh = [100x-2000\ln x]_{20}^{10}$		
	or $\int_0^{100}\frac{50}{20-\sqrt{h}}\,dh = \left[100\left(20-\sqrt{h}\right)-2000\ln\left(20-\sqrt{h}\right)\right]_0^{100}$		
	$= (1000-2000\ln 10)-(2000-2000\ln 20)$	Correct use of limits, ie. putting them in the correct way round Either $x=10$ and $x=20$ or $h=100$ and $h=0$	ddM1
	$= 2000\ln 20 - 2000\ln 10 - 1000$	Combining logs to give...	
	$= 2000\ln 2 - 1000$	$2000\ln 2-1000$ or $-2000\ln\left(\frac{1}{2}\right)-1000$	A1 **aef**
			[6]
(e)	Time required $= 2000\ln 2 - 1000 = 386.2943611...$ sec		
	$= 386$ seconds (nearest second)		
	$= 6$ minutes and 26 seconds (nearest second)	<u>6 minutes, 26 seconds</u>	B1
			[1]
			13 marks

Mark Scheme (Results)

Summer 2008

GCE

GCE Mathematics (6666/01)

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June 2008
6666 Core Mathematics C4
Mark Scheme

Question	Scheme		Marks
1. (a)			

x	0	0.4	0.8	1.2	1.6	2
y	e^0	$e^{0.08}$	$e^{0.32}$	$e^{0.72}$	$e^{1.28}$	e^2
or y	1	1.08329 ...	1.37713...	2.05443...	3.59664...	7.38906...

Question	Scheme		Marks
		Either $e^{0.32}$ and $e^{1.28}$ or awrt 1.38 and 3.60 (or a mixture of e's and decimals)	B1 **[1]**
(b) **Way 1**	$\text{Area} \approx \frac{1}{2}\times 0.4\,;\times\underline{\left[e^0+2\left(e^{0.08}+e^{0.32}+e^{0.72}+e^{1.28}\right)+e^2\right]}$	Outside brackets $\frac{1}{2}\times 0.4$ or 0.2	B1;
		For structure of trapezium rule $\underline{[\ \ldots\ldots\ldots\ldots\ ]}$;	M1 $\sqrt{}$
	$= 0.2\times 24.61203164\ldots = 4.922406\ldots = \underline{4.922}$ (4sf)	$\underline{4.922}$	A1 **cao** **[3]**
Aliter (b) **Way 2**	$\text{Area} \approx 0.4\times\left[\frac{e^0+e^{0.08}}{2}+\frac{e^{0.08}+e^{0.32}}{2}+\frac{e^{0.32}+e^{0.72}}{2}+\frac{e^{0.72}+e^{1.28}}{2}+\frac{e^{1.28}+e^2}{2}\right]$	0.4 and a divisor of 2 on all terms inside brackets.	B1
	which is equivalent to:	One of first and last ordinates, two of the middle ordinates inside brackets ignoring the 2.	M1 $\sqrt{}$
	$\text{Area} \approx \frac{1}{2}\times 0.4\,;\times\underline{\left[e^0+2\left(e^{0.08}+e^{0.32}+e^{0.72}+e^{1.28}\right)+e^2\right]}$		
	$= 0.2\times 24.61203164\ldots = 4.922406\ldots = \underline{4.922}$ (4sf)	$\underline{4.922}$	A1 **cao** **[3]**
			4 marks

Note an expression like $\text{Area} \approx \frac{1}{2}\times 0.4 + e^0+2\left(e^{0.08}+e^{0.32}+e^{0.72}+e^{1.28}\right)+e^2$ would score B1M1A0

Allow one term missing (slip!) in the ( ) brackets for

The M1 mark for structure is for the material found in the curly brackets ie
$\underline{\left[\text{first } y \text{ ordinate} + 2\left(\text{intermediate ft } y \text{ ordinate}\right) + \text{final } y \text{ ordinate}\right]}$

Question Number	Scheme		Marks
2. (a)	$\begin{Bmatrix} u = x & \Rightarrow & \frac{du}{dx} = 1 \\ \frac{dv}{dx} = e^x & \Rightarrow & v = e^x \end{Bmatrix}$		
	$\int x e^x \, dx = x e^x - \int e^x .1 \, dx$	Use of 'integration by parts' formula in the **correct direction**. (See note.) Correct expression. (Ignore d*x*)	M1 A1
	$= x e^x - \int e^x \, dx$		
	$= x e^x - e^x \ (+ c)$	Correct integration with/without + *c*	A1
			[3]
(b)	$\begin{Bmatrix} u = x^2 & \Rightarrow & \frac{du}{dx} = 2x \\ \frac{dv}{dx} = e^x & \Rightarrow & v = e^x \end{Bmatrix}$		
	$\int x^2 e^x \, dx = x^2 e^x - \int e^x .2x \, dx$	Use of 'integration by parts' formula in the **correct direction**. Correct expression. (Ignore d*x*)	M1 A1
	$= x^2 e^x - 2\int x e^x \, dx$		
	$= x^2 e^x - 2\left(x e^x - e^x\right) + c$	Correct expression **including + c**. **(seen at any stage! in part (b))** You can ignore subsequent working.	A1 **ISW**
			[3]
	$\begin{Bmatrix} = x^2 e^x - 2x e^x + 2e^x + c \\ = e^x\left(x^2 - 2x + 2\right) + c \end{Bmatrix}$	*Ignore subsequent working*	
			6 marks

Note integration by parts in the **correct direction** means that *u* and $\frac{dv}{dx}$ must be assigned/used as $u = x$ and $\frac{dv}{dx} = e^x$ in part (a) for example

+ *c* is not required in part (a).

Question Number	Scheme		Marks
3. (a)	From question, $\frac{dA}{dt} = 0.032$	$\frac{dA}{dt} = 0.032$ seen or implied from working.	B1
	$\left\{A = \pi x^2 \Rightarrow \frac{dA}{dx} = \right\} 2\pi x$	$2\pi x$ by itself seen or implied from working	B1
	$\frac{dx}{dt} = \frac{dA}{dt} \div \frac{dA}{dx} = (0.032)\frac{1}{2\pi x}; \left\{= \frac{0.016}{\pi x}\right\}$	$0.032 \div \text{Candidate's } \frac{dA}{dx}$;	M1;
	When $x = 2\text{cm}$, $\frac{dx}{dt} = \frac{0.016}{2\pi}$		
	Hence, $\frac{dx}{dt} = 0.002546479...$ (cm s^{-1})	awrt 0.00255	A1 **cso**
			[4]
(b)	$V = \underline{\pi x^2(5x)} = \underline{5\pi x^3}$	$V = \underline{\pi x^2(5x)}$ or $\underline{5\pi x^3}$	B1
	$\frac{dV}{dx} = 15\pi x^2$	$\frac{dV}{dx} = 15\pi x^2$ or ft from candidate's V **in one variable**	B1 $\sqrt{}$
	$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} = 15\pi x^2.\left(\frac{0.016}{\pi x}\right); \{= 0.24x\}$	Candidate's $\frac{dV}{dx} \times \frac{dx}{dt}$;	M1 $\sqrt{}$
	When $x = 2\text{cm}$, $\frac{dV}{dt} = 0.24(2) = \underline{0.48}$ $(\text{cm}^3\,\text{s}^{-1})$	$\underline{0.48}$ or $\underline{\text{awrt } 0.48}$	A1 **cso**
			[4]
			8 marks

Question Number	Scheme		Marks
4. (a)	$3x^2 - y^2 + xy = 4$ (eqn $*$)		
	$\left\{\cancel{\frac{dy}{dx}} \cancel{=}\right\}$ $\underline{6x - 2y\frac{dy}{dx}} + \underline{\underline{\left(y + x\frac{dy}{dx}\right)}} = \underline{0}$	Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $x\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$)	M1
		Correct application $\underline{\underline{(\ )}}$ of product rule	B1
		$(3x^2 - y^2) \to \underline{\left(6x - 2y\frac{dy}{dx}\right)}$ and $(4 \to \underline{0})$	<u>A1</u>
	$\left\{\frac{dy}{dx} = \frac{-6x - y}{x - 2y}\right\}$ or $\left\{\frac{dy}{dx} = \frac{6x + y}{2y - x}\right\}$	*not necessarily required.*	
	$\frac{dy}{dx} = \frac{8}{3} \Rightarrow \frac{-6x - y}{x - 2y} = \frac{8}{3}$	Substituting $\frac{dy}{dx} = \frac{8}{3}$ into their equation.	M1$*$
	giving $-18x - 3y = 8x - 16y$		
	giving $13y = 26x$	Attempt to combine either terms in x or terms in y together to give either ax or by.	dM1$*$
	Hence, $y = 2x \Rightarrow \underline{y - 2x = 0}$	simplifying to give $\underline{y - 2x = 0}$ **AG**	A1 **cso**
			[6]
(b)	At P & Q, $y = 2x$. Substituting into eqn $*$		
	gives $3x^2 - (2x)^2 + x(2x) = 4$	Attempt replacing y by $2x$ **in at least one** of the y terms in eqn$*$	M1
	Simplifying gives, $x^2 = 4 \Rightarrow \underline{x = \pm 2}$	Either $x = 2$ or $x = -2$	<u>A1</u>
	$y = 2x \Rightarrow y = \pm 4$		
	Hence coordinates are $\underline{(2,4)}$ and $\underline{(-2,-4)}$	Both $\underline{(2,4)}$ and $\underline{(-2,-4)}$	<u>A1</u>
			[3]
			9 marks

Question Number	Scheme		Marks
	** represents a constant (which must be consistent for first accuracy mark)		
5. (a)	$\frac{1}{\sqrt{(4-3x)}} = (4-3x)^{-\frac{1}{2}} = \underline{(4)^{-\frac{1}{2}}}\left(1-\frac{3x}{4}\right)^{-\frac{1}{2}} = \underline{\frac{1}{2}}\left(1-\frac{3x}{4}\right)^{-\frac{1}{2}}$	$\underline{(4)^{-\frac{1}{2}}}$ or $\underline{\frac{1}{2}}$ outside brackets	<u>B1</u>
		Expands $(1+{*}{*}x)^{-\frac{1}{2}}$ to give a simplified or an un-simplified $1+(-\frac{1}{2})({*}{*}x)$;	M1;
	$=\frac{1}{2}\left[\underline{1+(-\tfrac{1}{2})({*}{*}x);+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}({*}{*}x)^2+...}\right]$ **with** ${*}{*}\neq 1$	A correct simplified or an un-simplified $[\ \underline{.........}\ ]$ expansion with candidate's followed through $({*}{*}x)$	A1 $\sqrt{}$
	$=\frac{1}{2}\left[\underline{1+(-\tfrac{1}{2})(-\tfrac{3x}{4})+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-\tfrac{3x}{4})^2+...}\right]$	Award SC M1 if you see $(-\frac{1}{2})({*}{*}x)+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}({*}{*}x)^2$	
		$\frac{1}{2}\left[\ 1+\frac{3}{8}x;\ ...\ \right]$	A1 isw
	$=\frac{1}{2}\left[\ 1+\frac{3}{8}x;+\frac{27}{128}x^2+...\ \right]$	SC: $K\left[\ 1+\frac{3}{8}x+\frac{27}{128}x^2+...\ \right]$	
		$\frac{1}{2}\left[\ ....;\ \frac{27}{128}x^2\ \right]$	A1 isw
	$\left\{=\frac{1}{2}+\frac{3}{16}x;+\frac{27}{256}x^2+...\right\}$	*Ignore subsequent working*	
			[5]
(b)	$(x+8)\left(\frac{1}{2}+\frac{3}{16}x+\frac{27}{256}x^2+...\right)$	Writing $(x+8)$ multiplied by candidate's part (a) expansion.	M1
	$= \quad \frac{1}{2}x+\frac{3}{16}x^2+.....$ $+\ 4+\frac{3}{2}x+\frac{27}{32}x^2+.....$	Multiply out brackets to find a constant term, two x terms and two x^2 terms.	M1
	$=\ 4+2x;+\frac{33}{32}x^2+...$	Anything that cancels to $4+2x;\ \frac{33}{32}x^2$	A1; A1
			[4]
			9 marks

Question Number	Scheme		Marks
6. (a)	Lines meet where:		
	$\begin{pmatrix}-9\\0\\10\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\-1\end{pmatrix}=\begin{pmatrix}3\\1\\17\end{pmatrix}+\mu\begin{pmatrix}3\\-1\\5\end{pmatrix}$		
	Any two of $\mathbf{i}: -9+2\lambda=3+3\mu$ (1); $\mathbf{j}: \lambda=1-\mu$ (2); $\mathbf{k}: 10-\lambda=17+5\mu$ (3)	Need any two of these correct equations seen anywhere in part (a).	M1
	(1) – 2**(2)** gives: $-9=1+5\mu \Rightarrow \mu=-2$	Attempts to solve simultaneous equations to find one of either λ or μ	dM1
	(2) gives: $\lambda=1--2=3$	Both $\underline{\lambda=3}$ & $\underline{\mu=-2}$	A1
	$\mathbf{r}=\begin{pmatrix}-9\\0\\10\end{pmatrix}+3\begin{pmatrix}2\\1\\-1\end{pmatrix}$ or $\mathbf{r}=\begin{pmatrix}3\\1\\17\end{pmatrix}-2\begin{pmatrix}3\\-1\\5\end{pmatrix}$	Substitutes their value of **either** λ or μ into the line l_1 or l_2 **respectively**. This mark can be implied by any two correct components of $(-3, 3, 7)$.	ddM1
	Intersect at $\mathbf{r}=\underline{\begin{pmatrix}-3\\3\\7\end{pmatrix}}$ or $\mathbf{r}=\underline{-3\mathbf{i}+3\mathbf{j}+7\mathbf{k}}$	$\underline{\begin{pmatrix}-3\\3\\7\end{pmatrix}}$ or $\underline{-3\mathbf{i}+3\mathbf{j}+7\mathbf{k}}$ or $(-3, 3, 7)$	A1
	Either check **k:** $\lambda=3$: LHS $=10-\lambda=10-3=7$ $\mu=-2$: RHS $=17+5\mu=17-10=7$	**Either** check that $\lambda=3$, $\mu=-2$ in a third equation **or** check that $\lambda=3$, $\mu=-2$ give the same coordinates on the other line. Conclusion not needed.	B1
	(As LHS = RHS then the lines intersect.)		**[6]**
(b)	$\mathbf{d}_1=2\mathbf{i}+\mathbf{j}-\mathbf{k}$, $\mathbf{d}_2=3\mathbf{i}-\mathbf{j}+5\mathbf{k}$		
	As $\mathbf{d}_1\bullet\mathbf{d}_2=\begin{pmatrix}2\\1\\-1\end{pmatrix}\bullet\begin{pmatrix}3\\-1\\5\end{pmatrix}=\underline{(2\times3)+(1\times-1)+(-1\times5)}=0$	Dot product calculation between the ***two direction vectors***: $\underline{(2\times3)+(1\times-1)+(-1\times5)}$ or $\underline{6-1-5}$	M1
	Then l_1 is perpendicular to l_2.	Result '=0' and appropriate conclusion	A1
			[2]

Question Number	Scheme		Marks
6. (c)	Equating **i** ; $-9+2\lambda=5 \Rightarrow \lambda=7$		
	$\mathbf{r}=\begin{pmatrix}-9\\0\\10\end{pmatrix}+7\begin{pmatrix}2\\1\\-1\end{pmatrix}=\begin{pmatrix}5\\7\\3\end{pmatrix}$ $(=\overrightarrow{OA}.$ Hence the point A lies on l_1.)	Substitutes candidate's $\lambda=7$ into the line l_1 and finds $5\mathbf{i}+7\mathbf{j}+3\mathbf{k}$. The conclusion on this occasion is not needed.	B1
			[1]
(d)	Let $\overrightarrow{OX}=-3\mathbf{i}+3\mathbf{j}+7\mathbf{k}$ be point of intersection		
	$\overrightarrow{AX}=\overrightarrow{OX}-\overrightarrow{OA}=\underline{\begin{pmatrix}-3\\3\\7\end{pmatrix}-\begin{pmatrix}5\\7\\3\end{pmatrix}}=\begin{pmatrix}-8\\-4\\4\end{pmatrix}$	Finding the difference between their $\overrightarrow{OX}$ (can be implied) and $\overrightarrow{OA}$. $\overrightarrow{AX}=\pm\left(\begin{pmatrix}-3\\3\\7\end{pmatrix}-\begin{pmatrix}5\\7\\3\end{pmatrix}\right)$	M1 $\sqrt{}$ ±
	$\overrightarrow{OB}=\overrightarrow{OA}+\overrightarrow{AB}=\overrightarrow{OA}+2\overrightarrow{AX}$		
	$\overrightarrow{OB}=\begin{pmatrix}5\\7\\3\end{pmatrix}+2\begin{pmatrix}-8\\-4\\4\end{pmatrix}$	$\begin{pmatrix}5\\7\\3\end{pmatrix}+2\begin{pmatrix}\text{their } \overrightarrow{AX}\end{pmatrix}$	dM1 $\sqrt{}$
	Hence, $\overrightarrow{OB}=\underline{\begin{pmatrix}-11\\-1\\11\end{pmatrix}}$ or $\overrightarrow{OB}=\underline{-11\mathbf{i}-\mathbf{j}+11\mathbf{k}}$	$\underline{\begin{pmatrix}-11\\-1\\11\end{pmatrix}}$ or $\underline{-11\mathbf{i}-\mathbf{j}+11\mathbf{k}}$ or $\underline{(-11,-1,11)}$	A1
			[3]
			12 marks

Question Number	Scheme		Marks
7. (a)	$\dfrac{2}{4-y^2} \equiv \dfrac{2}{(2-y)(2+y)} \equiv \dfrac{A}{(2-y)} + \dfrac{B}{(2+y)}$		
	$2 \equiv A(2+y) + B(2-y)$	Forming this identity. **NB**: A & B are not assigned in this question	M1
	Let $y=-2$, $2 = B(4) \Rightarrow B = \frac{1}{2}$		
	Let $y=2$, $2 = A(4) \Rightarrow A = \frac{1}{2}$	Either one of $A=\frac{1}{2}$ or $B=\frac{1}{2}$	A1
	giving $\underline{\dfrac{\frac{1}{2}}{(2-y)} + \dfrac{\frac{1}{2}}{(2+y)}}$	$\underline{\dfrac{\frac{1}{2}}{(2-y)} + \dfrac{\frac{1}{2}}{(2+y)}}$, aef	A1 **cao**
	(If n**o working seen**, but candidate writes down ***correct partial fraction*** then award all three marks. If no working is seen but one of *A* or *B* is incorrect then M0A0A0.)		**[3]**

Question Number	Scheme		Marks
7. (b)	$\int \frac{2}{4-y^2}\,\mathrm{d}y = \int \frac{1}{\cot x}\,\mathrm{d}x$	Separates variables as shown. Can be implied. Ignore the integral signs, and the '2'.	B1
	$\int \frac{\frac{1}{2}}{(2-y)} + \frac{\frac{1}{2}}{(2+y)}\,\mathrm{d}y = \int \tan x\,\mathrm{d}x$		
		$\ln(\sec x)$ or $-\ln(\cos x)$	B1
		Either $\pm a\ln(\lambda - y)$ or $\pm b\ln(\lambda + y)$	M1;
	$\therefore\ -\frac{1}{2}\ln(2-y) + \frac{1}{2}\ln(2+y) = \ln(\sec x)\ \ + (c)$	their $\int \frac{1}{\cot x}\,\mathrm{d}x$ = LHS correct with ft for their *A* and *B* and no error with the "2" with or without $+c$	A1 $\sqrt{}$
	$y = 0,\ x = \frac{\pi}{3}\ \Rightarrow\ -\frac{1}{2}\ln 2 + \frac{1}{2}\ln 2 = \ln\left(\frac{1}{\cos\left(\frac{\pi}{3}\right)}\right) + c$	Use of $y = 0$ and $x = \frac{\pi}{3}$ in an integrated equation containing c ;	M1*
	$\{0 = \ln 2 + c\ \Rightarrow\ \underline{c = -\ln 2}\}$		
	$-\frac{1}{2}\ln(2-y) + \frac{1}{2}\ln(2+y) = \ln(\sec x) - \ln 2$		
	$\frac{1}{2}\ln\left(\frac{2+y}{2-y}\right) = \ln\left(\frac{\sec x}{2}\right)$	Using either the quotient (or product) or power laws for logarithms CORRECTLY.	M1
	$\ln\left(\frac{2+y}{2-y}\right) = 2\ln\left(\frac{\sec x}{2}\right)$		
	$\ln\left(\frac{2+y}{2-y}\right) = \ln\left(\frac{\sec x}{2}\right)^2$	Using the log laws correctly to obtain a single log term on both sides of the equation.	dM1*
	$\frac{2+y}{2-y} = \frac{\sec^2 x}{4}$		
	Hence, $\underline{\sec^2 x = \frac{8+4y}{2-y}}$	$\underline{\sec^2 x = \frac{8+4y}{2-y}}$	A1 aef
			[8]
			11 marks

Question Number	Scheme		Marks
8. (a)	At $P(4, 2\sqrt{3})$ either $\underline{4 = 8\cos t}$ or $\underline{2\sqrt{3} = 4\sin 2t}$	$\underline{4 = 8\cos t}$ or $\underline{2\sqrt{3} = 4\sin 2t}$	M1
	$\Rightarrow$ only solution is $\underline{t = \frac{\pi}{3}}$ where $0 \leqslant t \leqslant \frac{\pi}{2}$	$\underline{t = \frac{\pi}{3}}$ or $\underline{\text{awrt } 1.05}$ (radians) only stated in the range $0 \leqslant t \leqslant \frac{\pi}{2}$	A1
			[2]
(b)	$x = 8\cos t$, $y = 4\sin 2t$		
	$\frac{dx}{dt} = -8\sin t$, $\frac{dy}{dt} = 8\cos 2t$	Attempt to differentiate both x and y wrt t to give $\pm p\sin t$ and $\pm q\cos 2t$ respectively	M1
		Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$	A1
	At P, $\frac{dy}{dx} = \underline{\frac{8\cos\left(\frac{2\pi}{3}\right)}{-8\sin\left(\frac{\pi}{3}\right)}}$	Divides in correct way round and attempts to substitute their value of t (in degrees or radians) into their $\frac{dy}{dx}$ expression.	M1*
	$\left\{= \frac{8\left(-\frac{1}{2}\right)}{(-8)\left(\frac{\sqrt{3}}{2}\right)} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58\right\}$	You may need to check candidate's substitutions for M1* **Note the next two method marks are dependent on M1***	
	Hence $m(\mathbf{N}) = -\sqrt{3}$ or $\frac{-1}{\frac{1}{\sqrt{3}}}$	Uses $m(\mathbf{N}) = -\frac{1}{\text{their } m(\mathbf{T})}$.	dM1*
	N: $y - 2\sqrt{3} = -\sqrt{3}(x - 4)$	Uses $y - 2\sqrt{3} = (\text{their } m_N)(x - 4)$ or finds c using $x = 4$ and $y = 2\sqrt{3}$ and uses $y = (\text{their } m_N)x + "c"$.	dM1*
	N: $\underline{y = -\sqrt{3}x + 6\sqrt{3}}$ **AG**	$\underline{y = -\sqrt{3}x + 6\sqrt{3}}$	A1 **cso AG**
	or $2\sqrt{3} = -\sqrt{3}(4) + c \Rightarrow c = 2\sqrt{3} + 4\sqrt{3} = 6\sqrt{3}$ so **N**: $\left[\underline{y = -\sqrt{3}x + 6\sqrt{3}}\right]$		
			[6]

Question	Scheme		Marks
8. (c)	$A = \int_0^4 y\,\mathrm{d}x = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} 4\sin 2t.(-8\sin t)\,\mathrm{d}t$	attempt at $A = \int \underline{y}\,\frac{\mathrm{d}x}{\underline{\mathrm{d}t}}\,\mathrm{d}t$ correct expression (ignore limits and $\mathrm{d}t$)	M1 A1
	$A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32\sin 2t.\sin t\,\mathrm{d}t = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32(2\sin t\cos t).\sin t\,\mathrm{d}t$	Seeing $\sin 2t = 2\sin t\cos t$ anywhere in PART (c).	M1
	$A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -64.\sin^2 t\cos t\,\mathrm{d}t$ $A = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64.\sin^2 t\cos t\,\mathrm{d}t$	Correct proof. Appreciation of how the negative sign affects the limits. **Note that the answer is given in the question.**	A1 **AG**
			[4]
(d)	{Using substitution $u = \sin t \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}t} = \cos t$} {change limits: when $t = \frac{\pi}{3}$, $u = \frac{\sqrt{3}}{2}$ & when $t = \frac{\pi}{2}$, $u = 1$}		
	$A = 64\left[\frac{\sin^3 t}{3}\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$ or $A = 64\left[\frac{u^3}{3}\right]_{\frac{\sqrt{3}}{2}}^{1}$	$k\sin^3 t$ or ku^3 with $u = \sin t$ Correct integration ignoring limits.	M1 A1
	$A = 64\left[\frac{1}{3} - \left(\frac{1}{3}.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}\right)\right]$	Substitutes limits of either $\left(t = \frac{\pi}{2} \text{ and } t = \frac{\pi}{3}\right)$ or $\left(u = 1 \text{ and } u = \frac{\sqrt{3}}{2}\right)$ and subtracts the correct way round.	dM1
	$A = 64\left(\frac{1}{3} - \frac{1}{8}\sqrt{3}\right) = \underline{\frac{64}{3} - 8\sqrt{3}}$	$\underline{\frac{64}{3} - 8\sqrt{3}}$ Aef in the form $a + b\sqrt{3}$, with awrt 21.3 and anything that cancels to $a = \frac{64}{3}$ and $b = -8$.	A1 aef **isw**
			[4]
	(Note that $a = \frac{64}{3}$, $b = -8$)		
			16 marks

Mark Scheme (Final)

January 2009

GCE

GCE Core Mathematics C4 (6666/01)

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January 2009
6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme	Marks
1. (a)	C: $y^2 - 3y = x^3 + 8$	
	$\left\{\cancel{\frac{dy}{dx}}\ \cancel{=}\right\}$ $2y\frac{dy}{dx} - 3\frac{dy}{dx} = 3x^2$ Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $\pm 3\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$.)	M1
	Correct equation.	A1
	$(2y-3)\frac{dy}{dx} = 3x^2$ A correct (condoning sign error) attempt to combine or factorise their '$2y\frac{dy}{dx} - 3\frac{dy}{dx}$'. Can be implied.	M1
	$\frac{dy}{dx} = \frac{3x^2}{2y-3}$ $\underline{\frac{3x^2}{2y-3}}$	A1 oe
		[4]
(b)	$y = 3 \Rightarrow 9 - 3(3) = x^3 + 8$ Substitutes $y = 3$ into C.	M1
	$x^3 = -8 \Rightarrow \underline{x = -2}$ Only $\underline{x = -2}$	A1
	$(-2, 3) \Rightarrow \frac{dy}{dx} = \frac{3(4)}{6-3} \Rightarrow \frac{dy}{dx} = 4$ $\frac{dy}{dx} = 4$ from correct working. Also can be ft using their 'x' value and $y = 3$ in the correct part (a) of $\frac{dy}{dx} = \frac{3x^2}{2y-3}$	A1 $\sqrt{\ }$
		[3]
		7 marks

1(b) final A1 $\sqrt{\ }$ **.** Note if the candidate inserts their x value and $y = 3$ into $\frac{dy}{dx} = \frac{3x^2}{2y-3}$, then an answer of $\frac{dy}{dx} =$ their x^2, ***may*** indicate a correct follow through.

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Question Number	Scheme		Marks
2. (a)	Area(R) = $\int_0^2 \frac{3}{\sqrt{(1+4x)}}\,dx = \int_0^2 3(1+4x)^{-\frac{1}{2}}\,dx$		
	$= \left[\frac{3(1+4x)^{\frac{1}{2}}}{\frac{1}{2}.4}\right]_0^2$	***Integrating*** $3(1+4x)^{-\frac{1}{2}}$ to give $\pm k(1+4x)^{\frac{1}{2}}$.	M1
		<u>Correct integration.</u> Ignore limits.	<u>A1</u>
	$= \left[\frac{3}{2}(1+4x)^{\frac{1}{2}}\right]_0^2$		
	$= \left(\frac{3}{2}\sqrt{9}\right) - \left(\frac{3}{2}(1)\right)$	Substitutes limits of 2 and 0 into a changed function and subtracts the correct way round.	M1
	$= \frac{9}{2} - \frac{3}{2} = \underline{3}\,(\text{units})^2$	$\underline{3}$	<u>A1</u>
			[4]
	(Answer of 3 with no working scores M0A0M0A0.)		
(b)	Volume $= \underline{\pi \int_0^2 \left(\frac{3}{\sqrt{(1+4x)}}\right)^2}\,dx$	Use of $\underline{V = \pi\int y^2}\,dx$. Can be implied. Ignore limits and dx.	B1
	$= (\pi)\int_0^2 \frac{9}{1+4x}\,dx$		
	$= (\pi)\left[\frac{9}{4}\ln\lvert 1+4x\rvert\right]_0^2$	$\pm k\ln\lvert 1+4x\rvert$	M1
		$\frac{9}{4}\ln\lvert 1+4x\rvert$	A1
	$= (\pi)\left[\left(\frac{9}{4}\ln 9\right) - \left(\frac{9}{4}\ln 1\right)\right]$	Substitutes limits of 2 and 0 and subtracts the correct way round.	dM1
	So Volume $= \underline{\frac{9}{4}\pi\ln 9}$	$\underline{\frac{9}{4}\pi\ln 9}$ or $\underline{\frac{9}{2}\pi\ln 3}$ or $\underline{\frac{18}{4}\pi\ln 3}$	A1 oe isw
			[5]
			9 marks

Note the answer must be a one term exact value. Note, also you can ignore subsequent working here.

Note that $\ln 1$ can be implied as equal to 0.

Note that $= \frac{9}{4}\pi\ln 9 + c$ (oe.) would be awarded the final A0.

Question Number	Scheme		Marks
3. (a)	$27x^2+32x+16 \equiv A(3x+2)(1-x)+B(1-x)+C(3x+2)^2$	Forming this identity	M1
	$x=-\frac{2}{3},\quad 12-\frac{64}{3}+16=\left(\frac{5}{3}\right)B \Rightarrow \frac{20}{3}=\left(\frac{5}{3}\right)B \Rightarrow B=4$ $x=1,\quad 27+32+16=25C \Rightarrow 75=25C \Rightarrow C=3$	Substitutes either $x=-\frac{2}{3}$ or $x=1$ into their identity or equates 3 terms or substitutes in values to write down three simultaneous equations.	M1
		Both $B=4$ and $C=3$ **(Note the A1 is dependent on both method marks in this part.)**	A1
	Equate x^2: $27=-3A+9C \Rightarrow 27=-3A+27 \Rightarrow 0=-3A \Rightarrow A=0$ $x=0,\quad 16=2A+B+4C \Rightarrow 16=2A+4+12 \Rightarrow 0=2A \Rightarrow A=0$	Compares coefficients or substitutes in a third x-value or uses simultaneous equations to show $A=0$.	B1
			[4]
(b)	$f(x)=\dfrac{4}{(3x+2)^2}+\dfrac{3}{(1-x)}$		
	$=4(3x+2)^{-2}+3(1-x)^{-1}$	Moving powers to top on any one of the two expressions	M1
	$=4\left[2\left(1+\frac{3}{2}x\right)^{-2}\right]+3(1-x)^{-1}$		
	$=1\left(1+\frac{3}{2}x\right)^{-2}+3(1-x)^{-1}$		
	$=1\left\{\underline{1+(-2)(\frac{3x}{2});+\frac{(-2)(-3)}{2!}(\frac{3x}{2})^2+\ldots}\right\}$	Either $1\pm(-2)(\frac{3x}{2})$ or $1\pm(-1)(-x)$ from either first or second expansions respectively	dM1;
	$+3\left\{\underline{1+(-1)(-x);+\frac{(-1)(-2)}{2!}(-x)^2+\ldots}\right\}$	Ignoring 1 and 3, any one correct $\{\underline{\ldots\ldots\ldots\ldots}\}$ expansion.	A1
		Both $\{\underline{\ldots\ldots\ldots\ldots}\}$ correct.	A1
	$=\left\{1-3x+\frac{27}{4}x^2+\ldots\right\}+3\left\{1+x+x^2+\ldots\right\}$		
	$=4+0x;+\frac{39}{4}x^2$	$4+(0x)$; $\frac{39}{4}x^2$	A1; A1
			[6]

Question Number	Scheme		Marks
3. (c)	Actual $= f(0.2) = \dfrac{1.08+6.4+16}{(6.76)(0.8)}$ $= \dfrac{23.48}{5.408} = 4.341715976... = \dfrac{2935}{676}$ Or Actual $= f(0.2) = \dfrac{4}{(3(0.2)+2)^2} + \dfrac{3}{(1-0.2)}$ $= \dfrac{4}{6.76} + 3.75 = 4.341715976... = \dfrac{2935}{676}$	Attempt to find the actual value of f(0.2) or seeing awrt 4.3 and believing it is candidate's actual f(0.2). Candidates can also attempt to find the actual value by using $\dfrac{A}{(3x+2)} + \dfrac{B}{(3x+2)^2} + \dfrac{C}{(1-x)}$ with their *A*, *B* and *C*.	M1
	Estimate $= f(0.2) = 4 + \frac{39}{4}(0.2)^2$ $= 4 + 0.39 = 4.39$	Attempt to find an estimate for f(0.2) using their answer to (b)	M1 $\sqrt{}$
	%age error $= \dfrac{\lvert 4.39 - 4.341715976... \rvert}{4.341715976...} \times 100$	$\left\lvert \dfrac{\text{their estimate - actual}}{\text{actual}} \right\rvert \times 100$	M1
	$= 1.112095408... = 1.1\%\,(2\text{sf})$	1.1%	A1 **cao** **[4]**
			14 marks

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Question Number	Scheme		Marks
4. (a)	$\mathbf{d}_1 = -2\mathbf{i} + \mathbf{j} - 4\mathbf{k}$, $\mathbf{d}_2 = q\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$		
	As $\left\{\mathbf{d}_1 \bullet \mathbf{d}_2 = \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} \bullet \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix}\right\} = \underline{(-2\times q) + (1\times 2) + (-4\times 2)}$	Apply dot product calculation between two direction vectors, ie. $\underline{(-2\times q) + (1\times 2) + (-4\times 2)}$	M1
	$\mathbf{d}_1 \bullet \mathbf{d}_2 = 0 \Rightarrow -2q + 2 - 8 = 0$ $-2q = 6 \Rightarrow \underline{q = -3}$ AG	Sets $\mathbf{d}_1 \bullet \mathbf{d}_2 = 0$ and solves to find $\underline{q = -3}$	A1 **cso**
			[2]
(b)	Lines meet where:		
	$\begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} = \begin{pmatrix} -5 \\ 11 \\ p \end{pmatrix} + \mu \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix}$		
	First two of $\mathbf{i}: \ 11 - 2\lambda = -5 + q\mu \quad (1)$ $\mathbf{j}: \ 2 + \lambda = 11 + 2\mu \quad (2)$ $\mathbf{k}: \ 17 - 4\lambda = p + 2\mu \quad (3)$	Need to see equations (1) and (2). Condone one slip. (Note that $q = -3$.)	M1
	(1) + 2**(2)** gives: $15 = 17 + \mu \Rightarrow \mu = -2$	Attempts to solve (1) and (2) to find one of either λ or μ	dM1
	(2) gives: $2 + \lambda = 11 - 4 \Rightarrow \lambda = 5$	Any one of $\underline{\lambda = 5}$ or $\underline{\mu = -2}$	A1
		Both $\underline{\lambda = 5}$ and $\underline{\mu = -2}$	A1
	(3) $\Rightarrow 17 - 4(5) = p + 2(-2)$	Attempt to substitute their λ and μ into their **k** component to give an equation in p alone.	ddM1
	$\Rightarrow p = 17 - 20 + 4 \Rightarrow \underline{p = 1}$	$\underline{p = 1}$	A1 **cso**
			[6]
(c)	$\mathbf{r} = \begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + 5\begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} -5 \\ 11 \\ 1 \end{pmatrix} - 2\begin{pmatrix} -3 \\ 2 \\ 2 \end{pmatrix}$	Substitutes their value of λ or μ into the correct line l_1 or l_2 .	M1
	Intersect at $\mathbf{r} = \underline{\begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix}}$ or $\underline{(1, 7, -3)}$	$\underline{\begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix}}$ or $\underline{(1, 7, -3)}$	A1
			[2]

Question Number	Scheme		Marks
(d)	Let $\overrightarrow{OX} = \mathbf{i} + 7\mathbf{j} - 3\mathbf{k}$ be point of intersection		
	$\overrightarrow{AX} = \overrightarrow{OX} - \overrightarrow{OA} = \begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix} - \begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} = \begin{pmatrix} -8 \\ 4 \\ -16 \end{pmatrix}$	Finding vector $\overrightarrow{AX}$ by finding the difference between $\overrightarrow{OX}$ and $\overrightarrow{OA}$. Can be ft using candidate's $\overrightarrow{OX}$.	M1 $\sqrt{}$ ±
	$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OA} + 2\overrightarrow{AX}$		
	$\overrightarrow{OB} = \begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} + 2\begin{pmatrix} -8 \\ 4 \\ -16 \end{pmatrix}$	$\begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} + 2\begin{pmatrix} \text{their } \overrightarrow{AX} \end{pmatrix}$	dM1 $\sqrt{}$
	Hence, $\overrightarrow{OB} = \underline{\begin{pmatrix} -7 \\ 11 \\ -19 \end{pmatrix}}$ or $\overrightarrow{OB} = \underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$	$\underline{\begin{pmatrix} -7 \\ 11 \\ -19 \end{pmatrix}}$ or $\underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$ or $\underline{(-7, 11, -19)}$	A1
			[3]
			13 marks

Question Number	Scheme		Marks
5. (a)	Similar triangles $\Rightarrow \underline{\dfrac{r}{h}=\dfrac{16}{24}} \Rightarrow \underline{r=\dfrac{2h}{3}}$	Uses similar triangles, ratios or trigonometry to find either one of these two expressions oe.	M1
	$V=\dfrac{1}{3}\pi r^2 h = \dfrac{1}{3}\pi\left(\dfrac{2h}{3}\right)^2 h = \dfrac{4\pi h^3}{27}$ **AG**	Substitutes $r=\frac{2h}{3}$ into the formula for the volume of water V.	A1
			[2]
(b)	From the question, $\dfrac{dV}{dt}=8$	$\dfrac{dV}{dt}=8$	B1
	$\underline{\dfrac{dV}{dh}}=\underline{\dfrac{12\pi h^2}{27}}=\underline{\dfrac{4\pi h^2}{9}}$	$\underline{\dfrac{dV}{dh}}=\underline{\dfrac{12\pi h^2}{27}}$ or $\underline{\dfrac{4\pi h^2}{9}}$	B1
	$\dfrac{dh}{dt}=\dfrac{dV}{dt}\div\dfrac{dV}{dh} = \underline{8\times\dfrac{9}{4\pi h^2}} = \underline{\dfrac{18}{\pi h^2}}$	Candidate's $\dfrac{dV}{dt}\div\dfrac{dV}{dh}$;	M1;
		$\underline{8\div\left(\dfrac{12\pi h^2}{27}\right)}$ or $\underline{8\times\dfrac{9}{4\pi h^2}}$ or $\underline{\dfrac{18}{\pi h^2}}$ oe	A1
	When $h=12$, $\dfrac{dh}{dt}=\underline{\dfrac{18}{144\pi}}=\underline{\dfrac{1}{8\pi}}$	$\underline{\dfrac{18}{144\pi}}$ or $\underline{\dfrac{1}{8\pi}}$	A1 oe isw
			[5]
			7 marks

Note the answer must be a one term exact value.

Note, also you can ignore subsequent working after $\underline{\dfrac{18}{144\pi}}$.

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Question Number	Scheme		Marks
6. (a)	$\int \tan^2 x \, dx$		
	$\left[NB: \underline{\sec^2 A = 1 + \tan^2 A} \text{ gives } \underline{\tan^2 A = \sec^2 A - 1} \right]$	The correct <u>underlined identity</u>.	M1 oe
	$= \int \sec^2 x - 1 \, dx$		
	$= \underline{\tan x} - x \, (+c)$	Correct integration with/without + c	A1
			[2]
(b)	$\int \frac{1}{x^3} \ln x \, dx$		
	$\begin{cases} u = \ln x & \Rightarrow \quad \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = x^{-3} & \Rightarrow \quad v = \frac{x^{-2}}{-2} = \frac{-1}{2x^2} \end{cases}$		
	$= -\frac{1}{2x^2} \ln x - \int -\frac{1}{2x^2} \cdot \frac{1}{x} \, dx$	Use of 'integration by parts' formula in the correct direction.	M1
		Correct expression.	A1
	$= -\frac{1}{2x^2} \ln x + \frac{1}{2} \int \frac{1}{x^3} \, dx$	An attempt to multiply through $\frac{k}{x^n}, n \in \mathbb{Z}, n \geq 2$ by $\frac{1}{x}$ and an attempt to ...	
	$= \underline{-\frac{1}{2x^2} \ln x + \frac{1}{2}\left(-\frac{1}{2x^2}\right)} \, (+c)$	... "integrate"(process the result);	M1
		<u>correct solution</u> with/without + c	A1 oe
			[4]

Correct direction means that $u = \ln x$.

Question Number	Scheme		Marks
(c)	$\int \frac{e^{3x}}{1+e^x}\,dx$		
	$\left\{u = 1+e^x \Rightarrow \underline{\frac{du}{dx} = e^x},\ \underline{\frac{dx}{du} = \frac{1}{e^x}},\ \underline{\frac{dx}{du} = \frac{1}{u-1}}\right\}$	Differentiating to find any one of the <u>three underlined</u>	<u>B1</u>
	$= \int \frac{e^{2x}.e^x}{1+e^x}\,dx = \int \frac{(u-1)^2.e^x}{u}.\frac{1}{e^x}\,du$ or $= \int \frac{(u-1)^3}{u}.\frac{1}{(u-1)}\,du$	Attempt to substitute for $e^{2x} = f(u)$, their $\frac{dx}{du} = \frac{1}{e^x}$ and $u = 1+e^x$ or $e^{3x} = f(u)$, their $\frac{dx}{du} = \frac{1}{u-1}$ and $u = 1+e^x$.	M1*
	$= \int \frac{(u-1)^2}{u}\,du$	$\underline{\int \frac{(u-1)^2}{u}\,du}$	A1
	$= \int \frac{u^2-2u+1}{u}\,du$ $= \int u - 2 + \frac{1}{u}\,du$	An attempt to multiply out their numerator to give at least three terms and divide through each term by u	dM1*
	$= \frac{u^2}{2} - 2u + \ln u \ (+c)$	Correct integration with/without +c	A1
	$= \frac{(1+e^x)^2}{2} - 2(1+e^x) + \ln(1+e^x) + c$	Substitutes $u = 1+e^x$ back into their integrated expression with at least two terms.	dM1*
	$= \frac{1}{2} + e^x + \frac{1}{2}e^{2x} - 2 - 2e^x + \ln(1+e^x) + c$ $= \frac{1}{2} + e^x + \frac{1}{2}e^{2x} - 2 - 2e^x + \ln(1+e^x) + c$ $= \frac{1}{2}e^{2x} - e^x + \ln(1+e^x) - \frac{3}{2} + c$		
	$= \frac{1}{2}e^{2x} - e^x + \ln(1+e^x) + k$ **AG**	$\underline{\frac{1}{2}e^{2x} - e^x + \ln(1+e^x) + k}$ must use a $+c$ and "$-\frac{3}{2}$" combined.	A1 **cso**
			[7]
			13 marks

Question Number	Scheme		Marks
7. (a)	At A, $x = -1 + 8 = 7$ & $y = (-1)^2 = 1 \Rightarrow A(7,1)$	$A(7,1)$	B1 **[1]**
(b)	$x = t^3 - 8t$, $y = t^2$,		
	$\frac{dx}{dt} = 3t^2 - 8$, $\frac{dy}{dt} = 2t$		
	$\therefore \frac{dy}{dx} = \frac{2t}{3t^2 - 8}$	Their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$	M1
		Correct $\frac{dy}{dx}$	A1
	At A, $m(\mathbf{T}) = \underline{\frac{2(-1)}{3(-1)^2 - 8}} = \underline{\frac{-2}{3-8}} = \underline{\frac{-2}{-5}} = \underline{\frac{2}{5}}$	Substitutes for t to give any of the four underlined oe:	<u>A1</u>
	$\mathbf{T}$: $y - (\text{their } 1) = m_T(x - (\text{their } 7))$	Finding an equation of a tangent with their point and their tangent gradient	
	or $1 = \frac{2}{5}(7) + c \Rightarrow c = 1 - \frac{14}{5} = -\frac{9}{5}$	or finds c and uses $y = (\text{their gradient})x + "c"$.	dM1
	Hence $\mathbf{T}$: $y = \frac{2}{5}x - \frac{9}{5}$		
	gives $\mathbf{T}$: $\underline{2x - 5y - 9 = 0}$ **AG**	$\underline{2x - 5y - 9 = 0}$	A1 **cso** **[5]**
(c)	$2(t^3 - 8t) - 5t^2 - 9 = 0$	Substitution of both $x = t^3 - 8t$ and $y = t^2$ into $\mathbf{T}$	M1
	$2t^3 - 5t^2 - 16t - 9 = 0$		
	$(t+1)\{(2t^2 - 7t - 9) = 0\}$ $(t+1)\{(t+1)(2t - 9) = 0\}$	A realisation that $(t+1)$ is a factor.	dM1
	$\{t = -1 \text{ (at } A)\}$ $t = \frac{9}{2}$ at B	$t = \frac{9}{2}$	A1
	$x = \left(\frac{9}{2}\right)^2 - 8\left(\frac{9}{2}\right) = \frac{729}{8} - 36 = \frac{441}{8} = 55.125$ or awrt 55.1	Candidate uses their value of t to find either the x or y coordinate	ddM1
	$y = \left(\frac{9}{2}\right)^2 = \frac{81}{4} = 20.25$ or awrt 20.3	One of either x or y correct.	A1
		Both x and y correct.	A1
	Hence $B\left(\frac{441}{8}, \frac{81}{4}\right)$	awrt	**[6]**
			12 marks

- Note: dM1 denotes a method mark which is dependent upon the award of the previous method mark.
  ddM1 denotes a method mark which is dependent upon the award of the previous two method marks.
  Oe or equivalent.

January 2009
6666 Core Mathematics C4
Appendix

Question 1

Question Number	Scheme		Marks
Aliter **1.** (a) **Way 2**	$C: \quad y^2 - 3y = x^3 + 8$		
	$\left\{\cancel{\dfrac{dx}{dy}} \cancel{=}\right\} \quad 2y - 3 = 3x^2 \dfrac{dx}{dy}$	Differentiates implicitly to include either $\pm kx^2 \dfrac{dx}{dy}$. (Ignore $\left(\dfrac{dx}{dy} = \right)$.)	M1
		Correct equation.	A1
	$2y - 3 = 3x^2 \dfrac{1}{\left(\frac{dy}{dx}\right)}$	Applies $\dfrac{dx}{dy} = \dfrac{1}{\left(\frac{dy}{dx}\right)}$	dM1
	$\dfrac{dy}{dx} = \dfrac{3x^2}{2y-3}$	$\dfrac{3x^2}{2y-3}$	A1 oe
			[4]
Aliter **1.** (a) **Way 3**	$C: \quad y^2 - 3y = x^3 + 8$		
	gives $x^3 = y^2 - 3y - 8$ $\Rightarrow \quad x = \left(y^2 - 3y - 8\right)^{\frac{1}{3}}$		
	$\dfrac{dx}{dy} = \dfrac{1}{3}\left(y^2 - 3y - 8\right)^{-\frac{2}{3}}(2y - 3)$	Differentiates in the form $\frac{1}{3}(\mathrm{f}(y))^{-\frac{2}{3}}(\mathrm{f}'(y))$.	M1
		Correct differentiation.	A1
	$\dfrac{dx}{dy} = \dfrac{2y-3}{3\left(y^2 - 3y - 8\right)^{\frac{2}{3}}}$		
	$\dfrac{dy}{dx} = \dfrac{3\left(y^2 - 3y - 8\right)^{\frac{2}{3}}}{2y-3}$	Applies $\dfrac{dy}{dx} = \dfrac{1}{\left(\frac{dx}{dy}\right)}$	dM1
	$\dfrac{dy}{dx} = \dfrac{3\left(x^3\right)^{\frac{2}{3}}}{2y-3} \Rightarrow \dfrac{dy}{dx} = \dfrac{3x^2}{2y-3}$	$\dfrac{3\left(x^3\right)^{\frac{2}{3}}}{2y-3}$ or $\dfrac{3x^2}{2y-3}$	A1 oe
			[4]

Question 2

Question Number	Scheme		Marks
Aliter **2.** (a) **Way 2**	Area(R) = $\int_0^2 \frac{3}{\sqrt{(1+4x)}}\,\mathrm{d}x = \int_0^2 3(1+4x)^{-\frac{1}{2}}\,\mathrm{d}x$		
	{Using substitution $u = 1+4x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 4$ }		
	{change limits: When $x = 0$, $u = 1$ & when $x = 2$, $u = 9$ }		
	So, Area(R) = $\int_1^9 3u^{-\frac{1}{2}}\,\frac{1}{4}\mathrm{d}u$		
	$= \left[\frac{3}{4}\frac{u^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}\right]_0^2$	***Integrating*** $\pm\lambda u^{-\frac{1}{2}}$ to give $\pm k u^{\frac{1}{2}}$.	M1
		<u>Correct integration.</u> Ignore limits.	<u>A1</u>
	$= \left[\frac{3}{2}u^{\frac{1}{2}}\right]_1^9$		
	$= \left(\frac{3}{2}\sqrt{9}\right) - \left(\frac{3}{2}(1)\right)$	Substitutes limits of either $(u = 9 \text{ and } u = 1)$ or in x, $(x = 2 \text{ and } x = 0)$ into a changed function and subtracts the correct way round .	M1
	$= \frac{9}{2} - \frac{3}{2} = \underline{3}\,(\text{units})^2$	$\underline{3}$	<u>A1</u>
			[4]
Aliter **2.** (a) **Way 3**	Area(R) = $\int_0^2 \frac{3}{\sqrt{(1+4x)}}\,\mathrm{d}x = \int_0^2 3(1+4x)^{-\frac{1}{2}}\,\mathrm{d}x$		
	{Using substitution $u^2 = 1+4x \Rightarrow 2u\frac{\mathrm{d}u}{\mathrm{d}x} = 4 \Rightarrow \frac{1}{2}u\mathrm{d}u = \mathrm{d}x$ }		
	{change limits: When $x = 0$, $u = 1$ & when $x = 2$, $u = 3$ }		
	So, Area(R) = $\int_1^3 \frac{3}{u}\,\frac{1}{2}u\,\mathrm{d}u = \int_1^3 \frac{3}{2}\mathrm{d}u$		
	$= \left[\frac{3}{2}u\right]_1^3$	***Integrating*** $\pm\lambda$ to give $\pm k u$.	M1
		<u>Correct integration.</u> Ignore limits.	<u>A1</u>
	$= \left(\frac{3}{2}(3)\right) - \left(\frac{3}{2}(1)\right)$	Substitutes limits of either $(u = 3 \text{ and } u = 1)$ or in x, $(x = 2 \text{ and } x = 0)$ into a changed function and subtracts the correct way round .	M1
	$= \frac{9}{2} - \frac{3}{2} = \underline{3}\,(\text{units})^2$	$\underline{3}$	<u>A1</u>
			[4]

Question 3

Question Number	Scheme		Marks
Aliter **3.** (a) **Way 2**	$27x^2+32x+16 \equiv A(3x+2)(1-x) + B(1-x) + C(3x+2)^2$	Forming this identity	M1
	x^2 terms: $27 = -3A + 9C$ (1) x terms: $32 = A - B + 12C$ (2) constants: $16 = 2A + B + 4C$ (3)	equates 3 terms.	M1
	(2) + **(3)** gives $48 = 3A + 16C$ **(4)**		
	(1) + **(4)** gives $75 = 25C \Rightarrow C = 3$		
	(1) gives $27 = -3A + 27 \Rightarrow 0 = -3A \Rightarrow A = 0$		
	(2) gives $32 = -B + 36 \Rightarrow B = 36 - 32 = 4$	Both $B = 4$ and $C = 3$ Decide to award B1 for $A = 0$	A1 B1 **[4]**
3. (a)	If the candidate assumes $A = 0$ and writes the identity $27x^2+32x+16 \equiv B(1-x) + C(3x+2)^2$ and goes on to find $B = 4$ and $C = 3$ then the candidate is awarded M0M1A0B0.		
3. (a)	If the candidate has the incorrect identity $27x^2+32x+16 \equiv A(3x+2) + B(1-x) + C(3x+2)^2$ and goes on to find $B = 4$, $C = 3$ and $A = 0$ then the candidate is awarded M0M1A0B1.		
3. (a)	If the candidate has the incorrect identity $27x^2+32x+16 \equiv A(3x+2)^2(1-x) + B(1-x) + C(3x+2)^2$ and goes on to find $B = 4$, $C = 3$ and $A = 0$ then the candidate is awarded M0M1A0B1.		

Question Number	Scheme		Marks
Aliter **3.** (b) ***Way 2***	$\mathrm{f}(x) = \dfrac{4}{(3x+2)^2} + \dfrac{3}{(1-x)}$		
	$= 4(3x+2)^{-2} + 3(1-x)^{-1}$	Moving powers to top on any one of the two expressions	M1
	$= 4(2+3x)^{-2} + 3(1-x)^{-1}$		
	$= 4\left\{\underline{(2)^{-2} + (-2)(2)^{-3}(3x); + \frac{(-2)(-3)}{2!}(2)^{-4}(3x)^2 + }\right\}$	Either $(2)^{-2} \pm (-2)(2)^{-3}(3x)$ or $1 \pm (-1)(-x)$ from either first or second expansions respectively	dM1;
	$+ 3\left\{\underline{1 + (-1)(-x); + \frac{(-1)(-2)}{2!}(-x)^2 + ...}\right\}$	Ignoring 1 and 3, any one correct $\left\{\underline{..........}\right\}$ expansion.	A1
		Both $\left\{\underline{..........}\right\}$ correct.	A1
	$= 4\left\{\frac{1}{4} - \frac{3}{4}x + \frac{27}{16}x^2 + ...\right\} + 3\left\{1 + x + x^2 + ...\right\}$		
	$= 4 + 0x\ ; + \frac{39}{4}x^2$	$4 + (0x)\ ;\ \frac{39}{4}x^2$	A1; A1
			[6]

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Question Number	Scheme		Marks
Aliter **3.** (c) **Way 2**	Actual $= \mathrm{f}(0.2) = \dfrac{1.08+6.4+16}{(6.76)(0.8)}$ $= \dfrac{23.48}{5.408} = 4.341715976... = \dfrac{2935}{676}$	Attempt to find the actual value of f(0.2)	M1
	Estimate $= \mathrm{f}(0.2) = 4 + \frac{39}{4}(0.2)^2$ $= 4 + 0.39 = 4.39$	Attempt to find an estimate for f(0.2) using their answer to (b)	M1 $\sqrt{}$
	%age error $= \left\lvert 100 - \left(\dfrac{4.39}{4.341715976...} \times 100\right)\right\rvert$	$\left\lvert 100 - \left(\left(\dfrac{\text{their estimate}}{\text{actual}}\right)\times 100\right)\right\rvert$	M1
	$= \lvert 100 - 101.1120954\rvert$		
	$= \lvert -1.112095408...\rvert = 1.1\%(2\text{sf})$	1.1%	A1 **cao**
			[4]
3. (c)	Note that: %age error $= \dfrac{\lvert 4.39 - 4.341715976...\rvert}{4.39} \times 100$ $= 1.0998638... = 1.1\%(2\text{sf})$	Should be awarded the final marks of M0A0	
3. (c)	Also note that: %age error $= \left\lvert 100 - \left(\dfrac{4.341715976...}{4.39} \times 100\right)\right\rvert$ $= 1.0998638... = 1.1\%(2\text{sf})$	Should be awarded the final marks of M0A0	
	…so be wary of 1.0998638…		

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Question 4

Question Number	Scheme		Marks
4. (a)	$-2q+2-8$ is sufficient for M1.		
Aliter **4.** (b) **Way 2**	Only apply **Way 2** if candidate does not find both λ and μ.		
	Lines meet where:		
	$\begin{pmatrix}11\\2\\17\end{pmatrix}+\lambda\begin{pmatrix}-2\\1\\-4\end{pmatrix}=\begin{pmatrix}-5\\11\\p\end{pmatrix}+\mu\begin{pmatrix}q\\2\\2\end{pmatrix}$		
	First two of $\mathbf{i}: \ 11-2\lambda=-5+q\mu$ (1) $\mathbf{j}: \ 2+\lambda=11+2\mu$ (2) $\mathbf{k}: \ 17-4\lambda=p+2\mu$ (3)	Need to see equations (2) and (2). Condone one slip. (Note that $q=-3$.)	M1
	(2) gives $\lambda=9+2\mu$		
	(1) gives $11-2(9+2\mu)=-5-3\mu$	Attempts to solve (1) and (2) to find one of either λ or μ	dM1
	$11-18-4\mu=-5-3\mu$		
	gives: $11-18+5=\mu \ \Rightarrow \ \mu=-2$	Any one of $\underline{\lambda=5}$ or $\underline{\mu=-2}$	A1
	(3) gives $17-4(9+2\mu)=p+2\mu$	Candidate writes down a correct equation containing p and one of either λ or μ which has already been found.	A1
	(3) $\Rightarrow \ 17-4(9+2(-2))=p+2(-2)$	Attempt to substitute their value for $\lambda\ (=9+2\mu)$ and μ into their **k** component to give an equation in p alone.	ddM1
	$\Rightarrow 17-20=p-4 \Rightarrow \underline{p=1}$	$\underline{p=1}$	A1 **cso** **[6]**
4. (c)	If no working is shown then any two out of the three coordinates can imply the first M1 mark.		M1
	Intersect at $\mathbf{r}=\underline{\begin{pmatrix}1\\7\\-3\end{pmatrix}}$ or $\underline{(1,7,-3)}$	$\underline{\begin{pmatrix}1\\7\\-3\end{pmatrix}}$ or $\underline{(1,7,-3)}$	A1 **[2]**

Question Number	Scheme		Marks
Aliter **4.** (d) **Way 2**	Let $\overrightarrow{OX} = \mathbf{i} + 7\mathbf{j} - 3\mathbf{k}$ be point of intersection		
	$\overrightarrow{AX} = \overrightarrow{OX} - \overrightarrow{OA} = \begin{pmatrix}1\\7\\-3\end{pmatrix} - \begin{pmatrix}9\\3\\13\end{pmatrix} = \begin{pmatrix}-8\\4\\-16\end{pmatrix}$	Finding the difference between their $\overrightarrow{OX}$ (can be implied) and $\overrightarrow{OA}$. $\overrightarrow{AX} = \pm\left(\begin{pmatrix}1\\7\\-3\end{pmatrix} - \begin{pmatrix}9\\3\\13\end{pmatrix}\right)$	M1 $\sqrt{\ }$ $\pm$
	$\overrightarrow{OB} = \overrightarrow{OX} + \overrightarrow{XB} = \overrightarrow{OX} + \overrightarrow{AX}$		
	$\overrightarrow{OB} = \begin{pmatrix}1\\7\\-3\end{pmatrix} + \begin{pmatrix}-8\\4\\-16\end{pmatrix}$	$\left(\text{their } \overrightarrow{OX}\right) + \left(\text{their } \overrightarrow{AX}\right)$	dM1 $\sqrt{\ }$
	Hence, $\overrightarrow{OB} = \underline{\begin{pmatrix}-7\\11\\-19\end{pmatrix}}$ or $\overrightarrow{OB} = \underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$	$\underline{\begin{pmatrix}-7\\11\\-19\end{pmatrix}}$ or $\underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$ or $\underline{(-7, 11, -19)}$	A1
			[3]
Aliter **4.** (d) **Way 3**	At A, $\lambda = 1$. At X, $\lambda = 5$.		
	Hence at B, $\lambda = 5 + (5-1) = 9$	$\lambda_B = (\text{their } \lambda_X) + (\text{their } \lambda_X - \text{their } \lambda_A)$ $\lambda_B = 2(\text{their } \lambda_X) - (\text{their } \lambda_A)$	M1 $\sqrt{\ }$
	$\overrightarrow{OB} = \begin{pmatrix}11\\2\\17\end{pmatrix} + 9\begin{pmatrix}-2\\1\\-4\end{pmatrix}$	Substitutes their value of λ into the line l_1.	dM1 $\sqrt{\ }$
	Hence, $\overrightarrow{OB} = \underline{\begin{pmatrix}-7\\11\\-19\end{pmatrix}}$ or $\overrightarrow{OB} = \underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$	$\underline{\begin{pmatrix}-7\\11\\-19\end{pmatrix}}$ or $\underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$ or $\underline{(-7, 11, -19)}$	A1
			[3]

Question Number	Scheme	Marks
***Aliter* 4.** (d) **Way 4**	$\overrightarrow{OA} = 9\mathbf{i} + 3\mathbf{j} + 13\mathbf{k}$ and the point of intersection $\overrightarrow{OX} = \mathbf{i} + 7\mathbf{j} - 3\mathbf{k}$	
	$\begin{pmatrix}9\\3\\13\end{pmatrix} \to \begin{pmatrix}\text{Minus } 8\\\text{Plus } 4\\\text{Minus } 16\end{pmatrix} \to \begin{pmatrix}1\\7\\-3\end{pmatrix}$ Finding the difference between their $\overrightarrow{OX}$ (can be implied) and $\overrightarrow{OA}$. $\left(\overrightarrow{AX} =\right) \pm \left(\begin{pmatrix}1\\7\\-3\end{pmatrix} - \begin{pmatrix}9\\3\\13\end{pmatrix}\right)$	M1 $\sqrt{}$ ±
	$\begin{pmatrix}1\\7\\-3\end{pmatrix} \to \begin{pmatrix}\text{Minus } 8\\\text{Plus } 4\\\text{Minus } 16\end{pmatrix} \to \begin{pmatrix}-7\\11\\-19\end{pmatrix}$ $\left(\text{their } \overrightarrow{OX}\right) + \left(\text{their } \overrightarrow{AX}\right)$	dM1 $\sqrt{}$
	Hence, $\overrightarrow{OB} = \underline{\begin{pmatrix}-7\\11\\-19\end{pmatrix}}$ or $\overrightarrow{OB} = \underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$ $\underline{\begin{pmatrix}-7\\11\\-19\end{pmatrix}}$ or $\underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$ or $\underline{(-7, 11, -19)}$	A1
		[3]
***Aliter* 4.** (d) **Way 5**	$\overrightarrow{OA} = 9\mathbf{i} + 3\mathbf{j} + 13\mathbf{k}$ and $\overrightarrow{OB} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ and the point of intersection $\overrightarrow{OX} = \mathbf{i} + 7\mathbf{j} - 3\mathbf{k}$ As X is the midpoint of AB, then	
	$(1, 7, -3) = \left(\frac{9+a}{2}, \frac{3+b}{2}, \frac{13+c}{2}\right)$ Writing down any two of these "equations" correctly.	M1 $\sqrt{}$
	$a = 2(1) - 9 = -7$ $b = 2(7) - 3 = 11$ $c = 2(-3) - 13 = -19$ An attempt to find at least two of a, b or c.	dM1 $\sqrt{}$
	Hence, $\overrightarrow{OB} = \underline{\begin{pmatrix}-7\\11\\-19\end{pmatrix}}$ or $\overrightarrow{OB} = \underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$ $\underline{\begin{pmatrix}-7\\11\\-19\end{pmatrix}}$ or $\underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$ or $\underline{(-7, 11, -19)}$ or $a = -7, b = 11, c = -19$	A1
		[3]

Question Number	Scheme		Marks
Aliter **4.** (d) **Way 6**	Let $\overrightarrow{OX} = \mathbf{i} + 7\mathbf{j} - 3\mathbf{k}$ be point of intersection		
	$\overrightarrow{AX} = \overrightarrow{OX} - \overrightarrow{OA} = \begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix} - \begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} = \begin{pmatrix} -8 \\ 4 \\ -16 \end{pmatrix}$ and $\left\lvert\overrightarrow{AX}\right\rvert = \sqrt{64 + 16 + 256} = \sqrt{336} = 4\sqrt{21}$	Finding the difference between their $\overrightarrow{OX}$ (can be implied) and $\overrightarrow{OA}$. $\overrightarrow{AX} = \pm\left(\begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix} - \begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix}\right)$ Note $\left\lvert\overrightarrow{AX}\right\rvert = \sqrt{336}$ would imply M1.	M1$\sqrt{\ }$ $\pm$
	$\overrightarrow{BX} = \overrightarrow{OX} - \overrightarrow{OB} = \begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix} - \begin{pmatrix} 11 - 2\lambda \\ 2 + \lambda \\ 17 - 4\lambda \end{pmatrix} = \begin{pmatrix} -10 + 2\lambda \\ 5 - \lambda \\ -20 + 4\lambda \end{pmatrix}$		
	Hence $\left\lvert\overrightarrow{BX}\right\rvert = \left\lvert\overrightarrow{AX}\right\rvert = \sqrt{336}$ gives $(-10 + 2\lambda)^2 + (5 - \lambda)^2 + (-20 + 4\lambda)^2 = 336$	Writes distance equation of $\left\lvert\overrightarrow{BX}\right\rvert^2 = 336$ where $\overrightarrow{BX} = \overrightarrow{OX} - \overrightarrow{OB}$ and $\overrightarrow{OB} = \begin{pmatrix} 11 - 2\lambda \\ 2 + \lambda \\ 17 - 4\lambda \end{pmatrix}$	dM1$\sqrt{\ }$
	$100 - 40\lambda + 4\lambda^2 + 25 - 10\lambda + \lambda^2 + 400 - 160\lambda + 16\lambda^2 = 336$ $21\lambda^2 - 210\lambda + 525 = 336$ $21\lambda^2 - 210\lambda + 189 = 0$ $\lambda^2 - 10\lambda + 9 = 0$ $(\lambda - 1)(\lambda - 9) = 0$		
	At A, $\lambda = 1$ and at B $\lambda = 9$, so, $\overrightarrow{OB} = \begin{pmatrix} 11 - 2(9) \\ 2 + 9 \\ 17 - 4(9) \end{pmatrix}$		
	Hence, $\overrightarrow{OB} = \underline{\begin{pmatrix} -7 \\ 11 \\ -19 \end{pmatrix}}$ or $\overrightarrow{OB} = \underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$	$\underline{\begin{pmatrix} -7 \\ 11 \\ -19 \end{pmatrix}}$ or $\underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$ or $\underline{(-7, 11, -19)}$	A1 **[3]**

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Question 5

Question Number	Scheme	Marks
5. (a)	Similar shapes $\Rightarrow$ either	
	$\underline{\dfrac{\frac{1}{3}\pi(16)^2 24}{V} = \left(\dfrac{24}{h}\right)^3}$ or $\underline{\dfrac{V}{\frac{1}{3}\pi(16)^2 24} = \left(\dfrac{h}{24}\right)^3}$	
	$\underline{\dfrac{\frac{1}{3}\pi r^2(24)}{V} = \left(\dfrac{24}{h}\right)^3}$ or $\underline{\dfrac{V}{\frac{1}{3}\pi r^2(24)} = \left(\dfrac{h}{24}\right)^3}$ Uses similar shapes to find either one of these two expressions oe.	M1
	$V = 2048\pi \times \left(\dfrac{h}{24}\right)^3 = \dfrac{4\pi h^3}{27}$ **AG** Substitutes their equation to give the correct formula for the volume of water V.	A1
		[2]
5. (a)	Candidates simply writing:	
	$V = \dfrac{4}{9}\times\dfrac{1}{3}\pi h^3$ or $V = \dfrac{1}{3}\pi\left(\dfrac{16}{24}\right)^2 h^3$ would be awarded M0A0.	
(b)	From question, $\dfrac{dV}{dt} = 8 \Rightarrow V = 8t\ (+c)$ $\dfrac{dV}{dt} = 8$ or $V = 8t$	B1
	$h = \left(\dfrac{27V}{4\pi}\right)^{\frac{1}{3}} \Rightarrow h = \underline{\left(\dfrac{27(8t)}{4\pi}\right)^{\frac{1}{3}}} = \underline{\left(\dfrac{54t}{\pi}\right)^{\frac{1}{3}}} = \underline{3\left(\dfrac{2t}{\pi}\right)^{\frac{1}{3}}}$ $\underline{\left(\dfrac{27(8t)}{4\pi}\right)^{\frac{1}{3}}}$ or $\underline{\left(\dfrac{54t}{\pi}\right)^{\frac{1}{3}}}$ or $\underline{3\left(\dfrac{2t}{\pi}\right)^{\frac{1}{3}}}$	B1
	$\dfrac{dh}{dt} = 3\left(\dfrac{2}{\pi}\right)^{\frac{1}{3}}\dfrac{1}{3}t^{-\frac{2}{3}}$ $\dfrac{dh}{dt} = \pm k t^{-\frac{2}{3}}$;	M1;
	$\dfrac{dh}{dt} = 3\left(\dfrac{2}{\pi}\right)^{\frac{1}{3}}\dfrac{1}{3}t^{-\frac{2}{3}}$	A1 oe
	When $h = 12$, $t = \left(\dfrac{12}{3}\right)^3 \times \dfrac{\pi}{2} = 32\pi$	
	So when $h = 12$, $\dfrac{dh}{dt} = \left(\dfrac{2}{\pi}\right)^{\frac{1}{3}}\left(\dfrac{1}{32\pi}\right)^{\frac{2}{3}} = \left(\dfrac{2}{1024\pi^3}\right)^{\frac{1}{3}} = \underline{\dfrac{1}{8\pi}}$ $\underline{\dfrac{1}{8\pi}}$	A1 oe
		[5]

Question 7

Question Number	Scheme		Marks
7. (a)	It is acceptable for a candidate to write $x = 7,\ y = 1,$ to gain B1.	$A(7,1)$	B1
			[1]
***Aliter* (c) Way 2**	$x = t^3 - 8t = t(t^2 - 8) = t(y - 8)$		
	So, $x^2 = t^2(y-8)^2 = y(y-8)^2$		
	$2x - 5y - 9 = 0 \Rightarrow 2x = 5y + 9 \Rightarrow 4x^2 = (5y+9)^2$		
	Hence, $4y(y-8)^2 = (5y+9)^2$	Forming an equation in terms of y only.	M1
	$4y(y^2 - 16y + 64) = 25y^2 + 90y + 81$		
	$4y^3 - 64y^2 + 256y = 25y^2 + 90y + 81$		
	$4y^3 - 89y^2 + 166y - 81 = 0$		
	$(y-1)(y-1)(4y-81) = 0$	A realisation that $(y-1)$ is a factor.	dM1
		Correct factorisation	A1
	$y = \frac{81}{4} = 20.25$ (or awrt 20.3)	***Correct y-coordinate (see below!)***	
	$x^2 = \frac{81}{4}(\frac{81}{4} - 8)^2$	Candidate uses their y-coordinate to find their x-coordinate.	ddM1
		Decide to award A1 here for correct y-coordinate.	A1
	$x = \frac{441}{8} = 55.125$ (or awrt 55.1)	Correct x-coordinate	A1
	Hence $B\left(\frac{441}{8}, \frac{81}{4}\right)$		**[6]**

Question Number	Scheme		Marks
Aliter 7. (c) **Way 3**	$t = \sqrt{y}$		
	So $x = \left(\sqrt{y}\right)^3 - 8\left(\sqrt{y}\right)$		
	$2x - 5y - 9 = 0$ yields		
	$2\left(\sqrt{y}\right)^3 - 16\left(\sqrt{y}\right) - 5y - 9 = 0$	Forming an equation in terms of y only.	M1
	$\Rightarrow 2\left(\sqrt{y}\right)^3 - 5y - 16\left(\sqrt{y}\right) - 9 = 0$		
	$\left(\sqrt{y}+1\right)\left\{\left(2y - 7\sqrt{y} - 9\right) = 0\right\}$	A realisation that $\left(\sqrt{y}+1\right)$ is a factor.	dM1
	$\left(\sqrt{y}+1\right)\left\{\left(\sqrt{y}+1\right)\left(2\sqrt{y} - 9\right) = 0\right\}$	Correct factorisation.	A1
	$y = \frac{81}{4} = 20.25$ (or awrt 20.3)	***Correct y-coordinate (see below!)***	
	$x = \left(\sqrt{\frac{81}{4}}\right)^3 - 8\left(\sqrt{\frac{81}{4}}\right)$	Candidate uses their y-coordinate to find their x-coordinate.	ddM1
		Decide to award A1 here for correct y-coordinate.	A1
	$x = \frac{441}{8} = 55.125$ (or awrt 55.1)	Correct x-coordinate	A1
	Hence $B\left(\frac{441}{8}, \frac{81}{4}\right)$		**[6]**

Mark Scheme (Results)
Summer 2009

GCE

GCE Mathematics (6666/01)

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6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme	Marks
Q1	$\mathrm{f}(x)=\dfrac{1}{\sqrt{(4+x)}}=(4+x)^{-\frac{1}{2}}$	M1
	$=(4)^{-\frac{1}{2}}(1+\ \dots\ )^{\dots}$ $\qquad \dfrac{1}{2}(1+\ \dots\ )^{\dots}$ or $\dfrac{1}{2\sqrt{(1+\dots)}}$	B1
	$=\ \dots\left(1+\left(-\frac{1}{2}\right)\left(\dfrac{x}{4}\right)+\dfrac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}\left(\dfrac{x}{4}\right)^2+\dfrac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!}\left(\dfrac{x}{4}\right)^3+\ \dots\right)$	M1 A1ft
	ft their $\left(\dfrac{x}{4}\right)$	
	$=\dfrac{1}{2}-\dfrac{1}{16}x,+\dfrac{3}{256}x^2-\dfrac{5}{2048}x^3+\ \dots$	A1, A1 (6)
		[6]
	Alternative	
	$\mathrm{f}(x)=\dfrac{1}{\sqrt{(4+x)}}=(4+x)^{-\frac{1}{2}}$	M1
	$=\underline{4^{-\frac{1}{2}}}+\left(-\frac{1}{2}\right)4^{-\frac{3}{2}}x+\dfrac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1.2}4^{-\frac{5}{2}}x^2+\dfrac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{1.2.3}4^{-\frac{7}{2}}x^3+\ \dots$	<u>B1</u> M1 A1
	$=\dfrac{1}{2}-\dfrac{1}{16}x,+\dfrac{3}{256}x^2-\dfrac{5}{2048}x^3+\ \dots$	A1, A1 (6)

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Question Number	Scheme		Marks	
Q2 (a)	1.14805	awrt 1.14805	B1	(1)
(b)	$A \approx \frac{1}{2} \times \frac{3\pi}{8}(\ \ldots\ )$		B1	
	$= \ \ldots\ \left(3+2\left(2.77164+2.12132+1.14805\right)+0\right)$	0 can be implied	M1	
	$= \ \frac{3\pi}{16}\left(3+2\left(2.77164+2.12132+1.14805\right)\right)$	ft their (a)	A1ft	
	$= \frac{3\pi}{16} \times 15.08202\ \ldots\ = 8.884$	cao	A1	(4)
(c)	$\int 3\cos\left(\frac{x}{3}\right)\mathrm{d}x = \frac{3\sin\left(\frac{x}{3}\right)}{\frac{1}{3}}$		M1 A1	
	$= 9\sin\left(\frac{x}{3}\right)$			
	$A = \left[9\sin\left(\frac{x}{3}\right)\right]_0^{\frac{3\pi}{2}} = 9-0=9$	cao	A1	(3)
				[8]

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Question Number	Scheme	Marks
Q3 (a)	$\mathrm{f}(x)=\dfrac{4-2x}{(2x+1)(x+1)(x+3)}=\dfrac{A}{2x+1}+\dfrac{B}{x+1}+\dfrac{C}{x+3}$	
	$4-2x=A(x+1)(x+3)+B(2x+1)(x+3)+C(2x+1)(x+1)$	M1
	A method for evaluating one constant	M1
	$x\to-\frac{1}{2},\quad 5=A\left(\frac{1}{2}\right)\left(\frac{5}{2}\right)\ \Rightarrow\ A=4$ any one correct constant	A1
	$x\to-1,\quad 6=B(-1)(2)\ \Rightarrow\ B=-3$	
	$x\to-3,\quad 10=C(-5)(-2)\ \Rightarrow\ C=1$ all three constants correct	A1 (4)
(b)	(i) $\displaystyle\int\left(\frac{4}{2x+1}-\frac{3}{x+1}+\frac{1}{x+3}\right)\mathrm{d}x$	
	$=\frac{4}{2}\ln(2x+1)-3\ln(x+1)+\ln(x+3)+C$ A1 two ln terms correct	M1 A1ft
	All three ln terms correct and "+*C*" ; ft constants	A1ft (3)
	(ii) $\left[2\ln(2x+1)-3\ln(x+1)+\ln(x+3)\right]_0^2$	
	$=(2\ln5-3\ln3+\ln5)-(2\ln1-3\ln1+\ln3)$	M1
	$=3\ln5-4\ln3$	
	$=\ln\left(\frac{5^3}{3^4}\right)$	M1
	$=\ln\left(\frac{125}{81}\right)$	A1 (3)
		[10]

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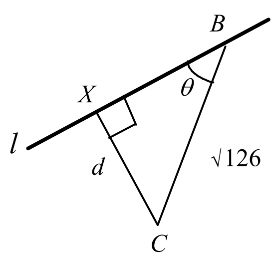
Question Number	Scheme	Marks
Q4 (a)	$e^{-2x}\dfrac{dy}{dx} - 2ye^{-2x} = 2 + 2y\dfrac{dy}{dx}$ A1 correct RHS	M1 A1
	$\dfrac{d}{dx}\left(ye^{-2x}\right) = e^{-2x}\dfrac{dy}{dx} - 2ye^{-2x}$	B1
	$\left(e^{-2x} - 2y\right)\dfrac{dy}{dx} = 2 + 2ye^{-2x}$	M1
	$\dfrac{dy}{dx} = \dfrac{2+2ye^{-2x}}{e^{-2x}-2y}$	A1 (5)
(b)	At P , $\dfrac{dy}{dx} = \dfrac{2+2e^0}{e^0-2} = -4$	M1
	Using $mm' = -1$	
	$m' = \dfrac{1}{4}$	M1
	$y - 1 = \dfrac{1}{4}(x - 0)$	M1
	$x - 4y + 4 = 0$ or any integer multiple	A1 (4)
		[9]
	Alternative for (a) *differentiating implicitly with respect to y.*	
	$e^{-2x} - 2ye^{-2x}\dfrac{dx}{dy} = 2\dfrac{dx}{dy} + 2y$ A1 correct RHS	M1 A1
	$\dfrac{d}{dy}\left(ye^{-2x}\right) = e^{-2x} - 2ye^{-2x}\dfrac{dx}{dy}$	B1
	$\left(2 + 2ye^{-2x}\right)\dfrac{dx}{dy} = e^{-2x} - 2y$	M1
	$\dfrac{dx}{dy} = \dfrac{e^{-2x}-2y}{2+2ye^{-2x}}$	
	$\dfrac{dy}{dx} = \dfrac{2+2ye^{-2x}}{e^{-2x}-2y}$	A1 (5)

Question Number	Scheme	Marks
Q5 (a)	$\frac{dx}{dt} = -4\sin 2t, \quad \frac{dy}{dt} = 6\cos t$	B1, B1
	$\frac{dy}{dx} = -\frac{6\cos t}{4\sin 2t} \quad \left(= -\frac{3}{4\sin t}\right)$	M1
	At $t = \frac{\pi}{3}$, $\quad m = -\frac{3}{4\times\frac{\sqrt{3}}{2}} = -\frac{\sqrt{3}}{2}$ accept equivalents, awrt -0.87	A1 (4)
(b)	Use of $\cos 2t = 1 - 2\sin^2 t$	M1
	$\cos 2t = \frac{x}{2}, \ \sin t = \frac{y}{6}$	
	$\frac{x}{2} = 1 - 2\left(\frac{y}{6}\right)^2$	M1
	Leading to $y = \sqrt{(18-9x)} \quad \left(= 3\sqrt{(2-x)}\right)$ cao	A1
	$-2 \le x \le 2$ $\quad k = 2$	B1 (4)
(c)	$0 \le f(x) \le 6$ either $0 \le f(x)$ or $f(x) \le 6$	B1
	Fully correct. Accept $0 \le y \le 6$, $[0, 6]$	B1 (2)
		[10]
	Alternatives to (a) *where the parameter is eliminated*	
	① $y = (18-9x)^{\frac{1}{2}}$	
	$\frac{dy}{dx} = \frac{1}{2}(18-9x)^{-\frac{1}{2}} \times (-9)$	B1
	At $t = \frac{\pi}{3}$, $x = \cos\frac{2\pi}{3} = -1$	B1
	$\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{\sqrt{(27)}} \times -9 = -\frac{\sqrt{3}}{2}$	M1 A1 (4)
	② $y^2 = 18 - 9x$	
	$2y\frac{dy}{dx} = -9$	B1
	At $t = \frac{\pi}{3}$, $y = 6\sin\frac{\pi}{3} = 3\sqrt{3}$	B1
	$\frac{dy}{dx} = -\frac{9}{2\times 3\sqrt{3}} = -\frac{\sqrt{3}}{2}$	M1 A1 (4)

Question Number	Scheme	Marks
Q6 (a)	$\int \sqrt{(5-x)}\,\mathrm{d}x = \int (5-x)^{\frac{1}{2}}\,\mathrm{d}x = \dfrac{(5-x)^{\frac{3}{2}}}{-\frac{3}{2}} \quad (+C)$	M1 A1 (2)
	$\left(= -\dfrac{2}{3}(5-x)^{\frac{3}{2}} + C\right)$	
(b)	(i) $\int (x-1)\sqrt{(5-x)}\,\mathrm{d}x = -\dfrac{2}{3}(x-1)(5-x)^{\frac{3}{2}} + \dfrac{2}{3}\int (5-x)^{\frac{3}{2}}\,\mathrm{d}x$	M1 A1ft
	$= \quad \ldots \quad + \dfrac{2}{3} \times \dfrac{(5-x)^{\frac{5}{2}}}{-\frac{5}{2}} \quad (+C)$	M1
	$= -\dfrac{2}{3}(x-1)(5-x)^{\frac{3}{2}} - \dfrac{4}{15}(5-x)^{\frac{5}{2}} \quad (+C)$	A1 (4)
	(ii) $\left[-\dfrac{2}{3}(x-1)(5-x)^{\frac{3}{2}} - \dfrac{4}{15}(5-x)^{\frac{5}{2}}\right]_1^5 = (0-0) - \left(0 - \dfrac{4}{15} \times 4^{\frac{5}{2}}\right)$	
	$= \dfrac{128}{15}\left(= 8\dfrac{8}{15} \approx 8.53\right)$ awrt 8.53	M1 A1 (2)
		[8]
	Alternatives for (b) *and* (c)	
	(b) $u^2 = 5 - x \Rightarrow 2u\dfrac{\mathrm{d}u}{\mathrm{d}x} = -1 \quad \left(\Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}u} = -2u\right)$	
	$\int (x-1)\sqrt{(5-x)}\,\mathrm{d}x = \int (4-u^2)u\dfrac{\mathrm{d}x}{\mathrm{d}u}\,\mathrm{d}u = \int (4-u^2)u(-2u)\,\mathrm{d}u$	M1 A1
	$= \int (2u^4 - 8u^2)\,\mathrm{d}u = \dfrac{2}{5}u^5 - \dfrac{8}{3}u^3 \quad (+C)$	M1
	$= \dfrac{2}{5}(5-x)^{\frac{5}{2}} - \dfrac{8}{3}(5-x)^{\frac{3}{2}} \quad (+C)$	A1
	(c) $x = 1 \Rightarrow u = 2, \quad x = 5 \Rightarrow u = 0$	
	$\left[\dfrac{2}{5}u^5 - \dfrac{8}{3}u^3\right]_2^0 = (0-0) - \left(\dfrac{64}{5} - \dfrac{64}{3}\right)$	M1
	$= \dfrac{128}{15}\left(= 8\dfrac{8}{15} \approx 8.53\right)$ awrt 8.53	A1 (2)

Question Number	Scheme	Marks
Q7 (a)	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 10 \\ 14 \\ -4 \end{pmatrix} - \begin{pmatrix} 8 \\ 13 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ or $\overrightarrow{BA} = \begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix}$	M1
	$\mathbf{r} = \begin{pmatrix} 8 \\ 13 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ *or* $\mathbf{r} = \begin{pmatrix} 10 \\ 14 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ accept equivalents	M1 A1ft (3)
(b)	$\overrightarrow{CB} = \overrightarrow{OB} - \overrightarrow{OC} = \begin{pmatrix} 10 \\ 14 \\ -4 \end{pmatrix} - \begin{pmatrix} 9 \\ 9 \\ 6 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ -10 \end{pmatrix}$ or $\overrightarrow{BC} = \begin{pmatrix} -1 \\ -5 \\ 10 \end{pmatrix}$	
	$CB = \sqrt{\left(1^2 + 5^2 + (-10)^2\right)} = \sqrt{(126)} \quad (= 3\sqrt{14} \approx 11.2)$ awrt 11.2	M1 A1 (2)
(c)	$\overrightarrow{CB}.\overrightarrow{AB} = \lvert\overrightarrow{CB}\rvert\lvert\overrightarrow{AB}\rvert \cos\theta$	
	$(\pm)(2+5+20) = \sqrt{126}\sqrt{9}\cos\theta$	M1 A1
	$\cos\theta = \dfrac{3}{\sqrt{14}} \Rightarrow \theta \approx 36.7°$ awrt 36.7°	A1 (3)
(d)		
	$\dfrac{d}{\sqrt{126}} = \sin\theta$	M1 A1ft
	$d = 3\sqrt{5} (\approx 6.7)$ awrt 6.7	A1 (3)
(e)	$BX^2 = BC^2 - d^2 = 126 - 45 = 81$	M1
	! $CBX = \dfrac{1}{2} \times BX \times d = \dfrac{1}{2} \times 9 \times 3\sqrt{5} = \dfrac{27\sqrt{5}}{2} (\approx 30.2)$ awrt 30.1 or 30.2	M1 A1 (3)
		[14]
	Alternative for (e)	
	! $CBX = \dfrac{1}{2} \times d \times BC \sin\angle XCB$	M1
	$= \dfrac{1}{2} \times 3\sqrt{5} \times \sqrt{126} \sin(90 - 36.7)°$ sine of correct angle	M1
	≈ 30.2 $\dfrac{27\sqrt{5}}{2}$, awrt 30.1 or 30.2	A1 (3)

Question Number	Scheme	Marks
Q8 (a)	$\int \sin^2\theta \, d\theta = \frac{1}{2}\int (1-\cos 2\theta)\, d\theta = \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta \quad (+C)$	M1 A1 (2)
(b)	$x = \tan\theta \Rightarrow \frac{dx}{d\theta} = \sec^2\theta$	
	$\pi\int y^2\, dx = \pi\int y^2 \frac{dx}{d\theta}\, d\theta = \pi\int (2\sin 2\theta)^2 \sec^2\theta\, d\theta$	M1 A1
	$= \pi\int \frac{(2\times 2\sin\theta\cos\theta)^2}{\cos^2\theta}\, d\theta$	M1
	$= 16\pi\int \sin^2\theta\, d\theta \qquad k = 16\pi$	A1
	$x = 0 \Rightarrow \tan\theta = 0 \Rightarrow \theta = 0, \quad x = \frac{1}{\sqrt{3}} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{\pi}{6}$	B1 (5)
	$\left(V = 16\pi\int_0^{\frac{\pi}{6}} \sin^2\theta\, d\theta\right)$	
(c)	$V = 16\pi\left[\frac{1}{2}\theta - \frac{\sin 2\theta}{4}\right]_0^{\frac{\pi}{6}}$	M1
	$= 16\pi\left[\left(\frac{\pi}{12} - \frac{1}{4}\sin\frac{\pi}{3}\right) - (0-0)\right]$ Use of correct limits	M1
	$= 16\pi\left(\frac{\pi}{12} - \frac{\sqrt{3}}{8}\right) = \frac{4}{3}\pi^2 - 2\pi\sqrt{3} \qquad p = \frac{4}{3},\ q = -2$	A1 (3)
		[10]

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Mark Scheme (Results)

January 2010

GCE

GCE Core Mathematics C4 (6666/01)

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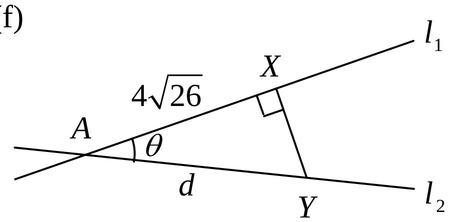
January 2010
6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme		Marks	
Q1	(a) $(1-8x)^{\frac{1}{2}} = 1 + \left(\tfrac{1}{2}\right)(-8x) + \dfrac{\left(\tfrac{1}{2}\right)\left(-\tfrac{1}{2}\right)}{2}(-8x)^2 + \dfrac{\left(\tfrac{1}{2}\right)\left(-\tfrac{1}{2}\right)\left(-\tfrac{3}{2}\right)}{3!}(-8x)^3 + \ldots$		M1 A1	
	$= 1 - 4x - 8x^2; -32x^3 - \ldots$		A1; A1	(4)
	(b) $\sqrt{(1-8x)} = \sqrt{\left(1-\dfrac{8}{100}\right)}$		M1	
	$= \sqrt{\dfrac{92}{100}} = \sqrt{\dfrac{23}{25}} = \dfrac{\sqrt{23}}{5}$ *	cso	A1	(2)
	(c) $1 - 4x - 8x^2 - 32x^3 = 1 - 4(0.01) - 8(0.01)^2 - 32(0.01)^3$			
	$= 1 - 0.04 - 0.0008 - 0.000\,032 = 0.959\,168$		M1	
	$\sqrt{23} = 5 \times 0.959\,168$		M1	
	$= 4.795\,84$	cao	A1	(3)
				[9]

Question Number	Scheme	Marks
Q2	(a) 1.386, 2.291 awrt 1.386, 2.291	B1 B1 (2)
	(b) $A \approx \frac{1}{2} \times 0.5(\ ... \ )$	B1
	$= \ ... \ \left(0+2\left(0.608+1.386+2.291+3.296+4.385\right)+5.545\right)$	M1
	$= \ 0.25\left(0+2\left(0.608+1.386+2.291+3.296+4.385\right)+5.545\right)$ ft their (a)	A1ft
	$= 0.25 \times 29.477 \ ... \ \approx 7.37$ cao	A1 (4)
	(c)(i) $\int x \ln x \, dx = \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \times \frac{1}{x} dx$	M1 A1
	$= \frac{x^2}{2} \ln x - \int \frac{x}{2} dx$	
	$= \frac{x^2}{2} \ln x - \frac{x^2}{4} \ (+C)$	M1 A1
	(ii) $\left[\frac{x^2}{2} \ln x - \frac{x^2}{4}\right]_1^4 = \left(8 \ln 4 - 4\right) - \left(-\frac{1}{4}\right)$	M1
	$= 8 \ln 4 - \frac{15}{4}$	
	$= 8\left(2 \ln 2\right) - \frac{15}{4}$ $\ln 4 = 2 \ln 2$ seen or implied	M1
	$= \frac{1}{4}\left(64 \ln 2 - 15\right)$ $a = 64, b = -15$	A1 (7)
		[13]

Question Number	Scheme	Marks	
Q3	(a) $-2\sin 2x - 3\sin 3y\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	M1 A1	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2\sin 2x}{3\sin 3y}$ Accept $\frac{2\sin 2x}{-3\sin 3y}$, $\frac{-2\sin 2x}{3\sin 3y}$	A1	(3)
	(b) At $x = \frac{\pi}{6}$, $\cos\left(\frac{2\pi}{6}\right) + \cos 3y = 1$	M1	
	$\cos 3y = \frac{1}{2}$	A1	
	$3y = \frac{\pi}{3} \Rightarrow y = \frac{\pi}{9}$ awrt 0.349	A1	(3)
	(c) At $\left(\frac{\pi}{6}, \frac{\pi}{9}\right)$, $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2\sin 2\left(\frac{\pi}{6}\right)}{3\sin 3\left(\frac{\pi}{9}\right)} = -\frac{2\sin\frac{\pi}{3}}{3\sin\frac{\pi}{3}} = -\frac{2}{3}$	M1	
	$y - \frac{\pi}{9} = -\frac{2}{3}\left(x - \frac{\pi}{6}\right)$	M1	
	Leading to $6x + 9y - 2\pi = 0$	A1	(3) **[9]**

Question Number	Scheme		Marks	
Q4	(a) A: $(-6, 4, -1)$	Accept vector forms	B1	(1)
	(b) $\begin{pmatrix}4\\-1\\3\end{pmatrix}.\begin{pmatrix}3\\-4\\1\end{pmatrix} = 12+4+3 = \sqrt{4^2+(-1)^2+3^2}\sqrt{3^2+(-4)^2+1^2}\cos\theta$		M1 A1	
	$\cos\theta = \frac{19}{26}$	awrt 0.73	A1	(3)
	(c) X: $(10, 0, 11)$	Accept vector forms	B1	(1)
	(d) $\overrightarrow{AX} = \begin{pmatrix}10\\0\\11\end{pmatrix} - \begin{pmatrix}-6\\4\\-1\end{pmatrix}$	Either order	M1	
	$= \begin{pmatrix}16\\-4\\12\end{pmatrix}$	cao	A1	(2)
	(e) $\lvert\overrightarrow{AX}\rvert = \sqrt{16^2+(-4)^2+12^2}$		M1	
	$= \sqrt{416} = \sqrt{16\times 26} = 4\sqrt{26}$ ✱	Do not penalise if consistent incorrect signs in (d)	A1	(2)
	(f) 	Use of correct right angled triangle	M1	
	$\frac{\lvert\overrightarrow{AX}\rvert}{d} = \cos\theta$		M1	
	$d = \frac{4\sqrt{26}}{\frac{19}{26}} \approx 27.9$	awrt 27.9	A1	(3)
				[12]

Question Number	Scheme		Marks	
Q5	(a) $\int \frac{9x+6}{x}\,\mathrm{d}x = \int \left(9+\frac{6}{x}\right)\mathrm{d}x$		M1	
	$= 9x + 6\ln x \;\; (+C)$		A1	(2)
	(b) $\int \frac{1}{y^{\frac{1}{3}}}\,\mathrm{d}y = \int \frac{9x+6}{x}\,\mathrm{d}x$	Integral signs not necessary	B1	
	$\int y^{-\frac{1}{3}}\,\mathrm{d}y = \int \frac{9x+6}{x}\,\mathrm{d}x$			
	$\frac{y^{\frac{2}{3}}}{\frac{2}{3}} = 9x + 6\ln x \;\; (+C)$	$\pm k y^{\frac{2}{3}} =$ their (a)	M1	
	$\frac{3}{2}y^{\frac{2}{3}} = 9x + 6\ln x \;\; (+C)$	ft their (a)	A1ft	
	$y = 8,\; x = 1$			
	$\frac{3}{2}8^{\frac{2}{3}} = 9 + 6\ln 1 + C$		M1	
	$C = -3$		A1	
	$y^{\frac{2}{3}} = \frac{2}{3}(9x + 6\ln x - 3)$			
	$y^2 = (6x + 4\ln x - 2)^3 \quad \left(= 8(3x + 2\ln x - 1)^3\right)$		A1	(6)
				[8]

Question Number	Scheme	Marks
Q6	$\frac{dA}{dt} = 1.5$	B1
	$A = \pi r^2 \Rightarrow \frac{dA}{dr} = 2\pi r$	B1
	When $A = 2$	
	$2 = \pi r^2 \Rightarrow r = \sqrt{\frac{2}{\pi}} \quad (= 0.797\,884\,...)$	M1
	$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$	
	$1.5 = 2\pi r \frac{dr}{dt}$	M1
	$\frac{dr}{dt} = \frac{1.5}{2\pi\sqrt{\frac{2}{\pi}}} \approx 0.299$ awrt 0.299	A1
		[5]

Question Number	Scheme		Marks	
Q7	(a) $y=0 \Rightarrow t(9-t^2)=t(3-t)(3+t)=0$			
	$t=0,3,-3$	Any one correct value	B1	
	At $t=0$, $x=5(0)^2-4=-4$	Method for finding one value of x	M1	
	At $t=3$, $x=5(3)^2-4=41$			
	$\left(\text{At } t=-3,\ x=5(-3)^2-4=41\right)$			
	At A, $x=-4$; at B, $x=41$	Both	A1	(3)
	(b) $\dfrac{\mathrm{d}x}{\mathrm{d}t}=10t$	Seen or implied	B1	
	$\int y\,\mathrm{d}x=\int y\dfrac{\mathrm{d}x}{\mathrm{d}t}\,\mathrm{d}t=\int t(9-t^2)10t\,\mathrm{d}t$		M1 A1	
	$=\int(90t^2-10t^4)\,\mathrm{d}t$			
	$=\dfrac{90t^3}{3}-\dfrac{10t^5}{5}\ (+C)\quad \left(=30t^3-2t^5\ (+C)\right)$		A1	
	$\left[\dfrac{90t^3}{3}-\dfrac{10t^5}{5}\right]_0^3=30\times3^3-2\times3^5\quad(=324)$		M1	
	$A=2\int y\,\mathrm{d}x=648\quad(\text{units}^2)$		A1	(6)
				[9]

Question Number	Scheme	Marks
Q8	(a) $\frac{\mathrm{d}x}{\mathrm{d}u} = -2\sin u$	B1
	$\int \frac{1}{x^2\sqrt{4-x^2}}\,\mathrm{d}x = \int \frac{1}{(2\cos u)^2\sqrt{4-(2\cos u)^2}} \times -2\sin u\,\mathrm{d}u$	M1
	$= \int \frac{-2\sin u}{4\cos^2 u\sqrt{4\sin^2 u}}\,\mathrm{d}u$ Use of $1-\cos^2 u = \sin^2 u$	M1
	$= -\frac{1}{4}\int \frac{1}{\cos^2 u}\,\mathrm{d}u$ $\pm k\int \frac{1}{\cos^2 u}\,\mathrm{d}u$	M1
	$= -\frac{1}{4}\tan u \ (+C)$ $\pm k \tan u$	M1
	$x = \sqrt{2} \Rightarrow \sqrt{2} = 2\cos u \Rightarrow u = \frac{\pi}{4}$	
	$x = 1 \Rightarrow 1 = 2\cos u \Rightarrow u = \frac{\pi}{3}$	M1
	$\left[-\frac{1}{4}\tan u\right]_{\frac{\pi}{3}}^{\frac{\pi}{4}} = -\frac{1}{4}\left(\tan\frac{\pi}{4} - \tan\frac{\pi}{3}\right)$	
	$= -\frac{1}{4}\left(1-\sqrt{3}\right) \quad \left(= \frac{\sqrt{3}-1}{4}\right)$	A1 (7)
	(b) $V = \pi\int_1^{\sqrt{2}} \left(\frac{4}{x\left(4-x^2\right)^{\frac{1}{4}}}\right)^2 \mathrm{d}x$	M1
	$= 16\pi\int_1^{\sqrt{2}} \frac{1}{x^2\sqrt{4-x^2}}\,\mathrm{d}x$ $16\pi \times$ integral in (a)	M1
	$= 16\pi\left(\frac{\sqrt{3}-1}{4}\right)$ $16\pi \times$ their answer to part (a)	A1ft (3)
		[10]

Mark Scheme (Results)
Summer 2010

GCE

Core Mathematics C4 (6666)

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June 2010
6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme	Marks
1.	(a) $y\left(\frac{\pi}{6}\right) \approx 1.2247,\ \ y\left(\frac{\pi}{4}\right) = 1.1180$ accept awrt 4 d.p.	B1 B1 **(2)**
	(b)(i) $I \approx \left(\frac{\pi}{12}\right)(1.3229 + 2 \times 1.2247 + 1)$ B1 for $\frac{\pi}{12}$	B1 M1
	≈ 1.249 cao	A1
	(ii) $I \approx \left(\frac{\pi}{24}\right)(1.3229 + 2 \times (1.2973 + 1.2247 + 1.1180) + 1)$ B1 for $\frac{\pi}{24}$	B1 M1
	≈ 1.257 cao	A1 **(6)** **[8]**

Question Number	Scheme		Marks	
2.	$\frac{\mathrm{d}u}{\mathrm{d}x} = -\sin x$		B1	
	$\int \sin x\, \mathrm{e}^{\cos x+1} \mathrm{d}x = -\int \mathrm{e}^u \mathrm{d}u$		M1 A1	
	$= -\mathrm{e}^u$	ft sign error	A1ft	
	$= -\mathrm{e}^{\cos x+1}$			
	$\left[-\mathrm{e}^{\cos x+1}\right]_0^{\frac{\pi}{2}} = -\mathrm{e}^1 - \left(-\mathrm{e}^2\right)$	or equivalent with u	M1	
	$= \mathrm{e}\left(\mathrm{e}-1\right)$ ∗	cso	A1	**(6)**
				[6]

Question Number	Scheme	Marks
3.	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(2^x\right) = \ln 2.2^x$	B1
	$\ln 2.2^x + 2y\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2y + 2x\dfrac{\mathrm{d}y}{\mathrm{d}x}$	M1 A1= A1
	Substituting $(3, 2)$	
	$8\ln 2 + 4\dfrac{\mathrm{d}y}{\mathrm{d}x} = 4 + 6\dfrac{\mathrm{d}y}{\mathrm{d}x}$	M1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 4\ln 2 - 2$ Accept exact equivalents	M1 A1 **(7)**
		[7]

Question Number	Scheme		Marks
4.	(a) $\frac{dx}{dt} = 2\sin t\cos t,\ \frac{dy}{dt} = 2\sec^2 t$		B1 B1
	$\frac{dy}{dx} = \frac{\sec^2 t}{\sin t\cos t}\left(= \frac{1}{\sin t\cos^3 t}\right)$	or equivalent	M1 A1 **(4)**
	(b) At $t = \frac{\pi}{3},\ x = \frac{3}{4},\ y = 2\sqrt{3}$		B1
	$\frac{dy}{dx} = \frac{\sec^2 \frac{\pi}{3}}{\sin\frac{\pi}{3}\cos\frac{\pi}{3}} = \frac{16}{\sqrt{3}}$		M1 A1
	$y - 2\sqrt{3} = \frac{16}{\sqrt{3}}\left(x - \frac{3}{4}\right)$		M1
	$y = 0 \quad \Rightarrow \quad x = \frac{3}{8}$		M1 A1 **(6)**
			[10]

Question Number	Scheme		Marks
5.	(a) $A = 2$		B1
	$2x^2+5x-10 = A(x-1)(x+2)+B(x+2)+C(x-1)$		
	$x \to 1$ $\quad -3 = 3B \Rightarrow B = -1$		M1 A1
	$x \to -2$ $\quad -12 = -3C \Rightarrow C = 4$		A1 **(4)**
	(b) $\dfrac{2x^2+5x-10}{(x-1)(x+2)} = 2+(1-x)^{-1}+2\left(1+\dfrac{x}{2}\right)^{-1}$		M1
	$(1-x)^{-1} = 1+x+x^2+\ ...$		B1
	$\left(1+\dfrac{x}{2}\right)^{-1} = 1-\dfrac{x}{2}+\dfrac{x^2}{4}+\ ...$		B1
	$\dfrac{2x^2+5x-10}{(x-1)(x+2)} = (2+1+2)+(1-1)x+\left(1+\dfrac{1}{2}\right)x^2+\ ...$		M1
	$= 5+\ ...$	ft their $A-B+\frac{1}{2}C$	A1 ft
	$= \ ...\ +\dfrac{3}{2}x^2+\ ...$	$0x$ stated or implied	A1 A1 **(7)**
			[11]

Question Number	Scheme		Marks	
6.	(a) $\mathrm{f}(\theta) = 4\cos^2\theta - 3\sin^2\theta$			
	$= 4\left(\frac{1}{2}+\frac{1}{2}\cos 2\theta\right) - 3\left(\frac{1}{2}-\frac{1}{2}\cos 2\theta\right)$		M1 M1	
	$= \frac{1}{2}+\frac{7}{2}\cos 2\theta$ $\ast$	cso	A1	**(3)**
	(b) $\int \theta\cos 2\theta \,\mathrm{d}\theta = \frac{1}{2}\theta\sin 2\theta - \frac{1}{2}\int \sin 2\theta \,\mathrm{d}\theta$		M1 A1	
	$= \frac{1}{2}\theta\sin 2\theta + \frac{1}{4}\cos 2\theta$		A1	
	$\int \theta \mathrm{f}(\theta)\,\mathrm{d}\theta = \frac{1}{4}\theta^2 + \frac{7}{4}\theta\sin 2\theta + \frac{7}{8}\cos 2\theta$		M1 A1	
	$\left[\ \dots\ \right]_0^{\frac{\pi}{2}} = \left[\frac{\pi^2}{16}+0-\frac{7}{8}\right]-\left[0+0+\frac{7}{8}\right]$		M1	
	$= \frac{\pi^2}{16}-\frac{7}{4}$		A1	**(7)**
				[10]

Question Number	Scheme	Marks
7.	(a) **j** components $3+2\lambda=9 \quad \Rightarrow \lambda=3$ $\quad (\mu=1)$	M1 A1
	Leading to $C:(5,9,-1)$ accept vector forms	A1 **(3)**
	(b) Choosing correct directions or finding $\overrightarrow{AC}$ and $\overrightarrow{BC}$	M1
	$\begin{pmatrix}1\\2\\1\end{pmatrix}.\begin{pmatrix}5\\0\\2\end{pmatrix}=5+2=\sqrt{6}\sqrt{29}\cos\angle ACB$ use of scalar product	M1 A1
	$\angle ACB=57.95°$ awrt $57.95°$	A1 **(4)**
	(c) $A:(2,3,-4) \quad B:(-5,9,-5)$	
	$\overrightarrow{AC}=\begin{pmatrix}3\\6\\3\end{pmatrix}, \quad \overrightarrow{BC}=\begin{pmatrix}10\\0\\4\end{pmatrix}$	
	$AC^2=3^2+6^2+3^2 \quad \Rightarrow \quad AC=3\sqrt{6}$	M1 A1
	$BC^2=10^2+4^2 \quad \Rightarrow \quad BC=2\sqrt{29}$	A1
	$\triangle ABC=\frac{1}{2}AC\times BC\sin\angle ACB$	
	$=\frac{1}{2}3\sqrt{6}\times 2\sqrt{29}\sin\angle ACB\approx 33.5$ $\quad 15\sqrt{5}$, awrt 34	M1 A1 **(5)**
		[12]
	Alternative method for (b) *and* (c)	
	(b) $A:(2,3,-4) \quad B:(-5,9,-5) \quad C:(5,9,-1)$	
	$AB^2=7^2+6^2+1^2=86$	
	$AC^2=3^2+6^2+3^2=54$	
	$BC^2=10^2+0^2+4^2=116$ Finding all three sides	M1
	$\cos\angle ACB=\frac{116+54-86}{2\sqrt{116}\sqrt{54}} \quad (=0.53066\,...)$	M1 A1
	$\angle ACB=57.95°$ awrt $57.95°$	A1 **(4)**
	If this method is used some of the working may gain credit in part (c) and appropriate marks may be awarded if there is an attempt at part (c).	

Question Number	Scheme		Marks
8.	(a) $\frac{\mathrm{d}V}{\mathrm{d}t} = 0.48\pi - 0.6\pi h$		M1 A1
	$V = 9\pi h \quad \Rightarrow \frac{\mathrm{d}V}{\mathrm{d}t} = 9\pi \frac{\mathrm{d}h}{\mathrm{d}t}$		B1
	$9\pi \frac{\mathrm{d}h}{\mathrm{d}t} = 0.48\pi - 0.6\pi h$		M1
	Leading to $75\frac{\mathrm{d}h}{\mathrm{d}t} = 4 - 5h$ *	cso	A1 **(5)**
	(b) $\int \frac{75}{4-5h}\,\mathrm{d}h = \int 1\,\mathrm{d}t$	separating variables	M1
	$-15\ln(4-5h) = t \ (+C)$		M1 A1
	$-15\ln(4-5h) = t + C$		
	When $t = 0$, $h = 0.2$		
	$-15\ln 3 = C$		M1
	$t = 15\ln 3 - 15\ln(4-5h)$		
	When $h = 0.5$		
	$t = 15\ln 3 - 15\ln 1.5 = 15\ln\left(\frac{3}{1.5}\right) = 15\ln 2$	awrt 10.4	M1 A1
	Alternative for last 3 marks		
	$t = \left[-15\ln(4-5h)\right]_{0.2}^{0.5}$		
	$= -15\ln 1.5 + 15\ln 3$		M1 M1
	$= 15\ln\left(\frac{3}{1.5}\right) = 15\ln 2$	awrt 10.4	A1 **(6)**

Mark Scheme (Results)

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GCE

GCE Core Mathematics C4 (6666) Paper 1

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January 2011
Core Mathematics C4 6666
Mark Scheme

Question Number	Scheme	Marks
1.	$\int x\sin 2x\,\mathrm{d}x = -\frac{x\cos 2x}{2} + \int \frac{\cos 2x}{2}\,\mathrm{d}x$	M1 A1 A1
	$= \quad \ldots \quad + \frac{\sin 2x}{4}$	M1
	$\left[\ \ldots\ \right]_0^{\frac{\pi}{2}} = \frac{\pi}{4}$	M1 A1
		[6]
2.	$\frac{\mathrm{d}I}{\mathrm{d}t} = -16\ln(0.5)0.5^t$	M1 A1
	At $t=3$ $\quad \frac{\mathrm{d}I}{\mathrm{d}t} = -16\ln(0.5)0.5^3$	M1
	$= -2\ln 0.5 = \ln 4$	M1 A1
		[5]

Question Number	Scheme	Marks
3.		
(a)	$\dfrac{5}{(x-1)(3x+2)} = \dfrac{A}{x-1} + \dfrac{B}{3x+2}$	
	$5 = A(3x+2) + B(x-1)$	
	$x \to 1 \qquad 5 = 5A \Rightarrow A = 1$	M1 A1
	$x \to -\dfrac{2}{3} \qquad 5 = -\dfrac{5}{3}B \Rightarrow B = -3$	A1 (3)
(b)	$\displaystyle\int \frac{5}{(x-1)(3x+2)}\,dx = \int \left(\frac{1}{x-1} - \frac{3}{3x+2}\right) dx$	
	$= \ln(x-1) - \ln(3x+2) \quad (+C)$ ft constants	M1 A1ft A1ft
		(3)
(c)	$\displaystyle\int \frac{5}{(x-1)(3x+2)}\,dx = \int \left(\frac{1}{y}\right) dy$	M1
	$\ln(x-1) - \ln(3x+2) = \ln y \quad (+C)$	M1 A1
	$y = \dfrac{K(x-1)}{3x+2}$ depends on first two Ms in (c)	M1 dep
	Using $(2, 8)$ $\quad 8 = \dfrac{K}{8}$ depends on first two Ms in (c)	M1 dep
	$y = \dfrac{64(x-1)}{3x+2}$	A1 (6)
		[12]

Question Number	Scheme	Marks
4. (a)	$\overrightarrow{AB} = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k} - (\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}) = -3\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$	M1 A1 (2)
(b)	$\mathbf{r} = \mathbf{i} - 3\mathbf{j} + 2\mathbf{k} + \lambda(-3\mathbf{i} + 5\mathbf{j} - 3\mathbf{k})$ or $\mathbf{r} = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k} + \lambda(-3\mathbf{i} + 5\mathbf{j} - 3\mathbf{k})$	M1 A1ft (2)
(c)	$\overrightarrow{AC} = 2\mathbf{i} + p\mathbf{j} - 4\mathbf{k} - (\mathbf{i} - 3\mathbf{j} + 2\mathbf{k})$ $= \mathbf{i} + (p+3)\mathbf{j} - 6\mathbf{k}$ or $\overrightarrow{CA}$ $\overrightarrow{AC}.\overrightarrow{AB} = \begin{pmatrix} 1 \\ p+3 \\ -6 \end{pmatrix} . \begin{pmatrix} -3 \\ 5 \\ -3 \end{pmatrix} = 0$ $-3 + 5p + 15 + 18 = 0$ Leading to $p = -6$	 B1 M1 M1 A1 (4)
(d)	$AC^2 = (2-1)^2 + (-6+3)^2 + (-4-2)^2 \quad (= 46)$ $AC = \sqrt{46}$ accept awrt 6.8	M1 A1 (2) **[10]**

Question Number	Scheme	Marks
5.		
(a)	$(2-3x)^{-2} = 2^{-2}\left(1-\frac{3}{2}x\right)^{-2}$	B1
	$\left(1-\frac{3}{2}x\right)^{-2} = 1+(-2)\left(-\frac{3}{2}x\right)+\frac{-2.-3}{1.2}\left(-\frac{3}{2}x\right)^2+\frac{-2.-3.-4}{1.2.3}\left(-\frac{3}{2}x\right)^3+\ ...$	M1 A1
	$= 1+3x+\frac{27}{4}x^2+\frac{27}{2}x^3+\ ...$	
	$(2-3x)^{-2} = \frac{1}{4}+\frac{3}{4}x+\frac{27}{16}x^2+\frac{27}{8}x^3+\ ...$	M1 A1 (5)
(b)	$\text{f}(x) = (a+bx)\left(\frac{1}{4}+\frac{3}{4}x+\frac{27}{16}x^2+\frac{27}{8}x^3+\ ...\right)$	
	Coefficient of x; $\frac{3a}{4}+\frac{b}{4}=0$ $(3a+b=0)$	M1
	Coefficient of x^2; $\frac{27a}{16}+\frac{3b}{4}=\frac{9}{16}$ $(9a+4b=3)$ A1 either correct	M1 A1
	Leading to $a=-1,\ b=3$	M1 A1 (5)
(c)	Coefficient of x^3 is $\frac{27a}{8}+\frac{27b}{16}=\frac{27}{8}\times(-1)+\frac{27}{16}\times 3$	M1 A1ft
	$=\frac{27}{16}$ cao	A1 (3)
		[13]

Question Number	Scheme	Marks
6.		
(a)	$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{t}, \quad \frac{\mathrm{d}y}{\mathrm{d}t} = 2t$	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 2t^2$	M1 A1
	Using $mm' = -1$, at $t = 3$	
	$m' = -\frac{1}{18}$	M1 A1
	$y - 7 = -\frac{1}{18}(x - \ln 3)$	M1 A1 (6)
(b)	$x = \ln t \Rightarrow t = \mathrm{e}^x$	B1
	$y = \mathrm{e}^{2x} - 2$	M1 A1 (3)
(c)	$V = \pi \int \left(\mathrm{e}^{2x} - 2\right)^2 \mathrm{d}x$	M1
	$\int \left(\mathrm{e}^{2x} - 2\right)^2 \mathrm{d}x = \int \left(\mathrm{e}^{4x} - 4\mathrm{e}^{2x} + 4\right) \mathrm{d}x$	M1
	$= \frac{\mathrm{e}^{4x}}{4} - \frac{4\mathrm{e}^{2x}}{2} + 4x$	M1 A1
	$\pi\left[\frac{\mathrm{e}^{4x}}{4} - \frac{4\mathrm{e}^{2x}}{2} + 4x\right]_{\ln 2}^{\ln 4} = \pi\left[(64 - 32 + 4\ln 4) - (4 - 8 + 4\ln 2)\right]$	M1
	$= \pi(36 + 4\ln 2)$	A1
		(6) **[15]**
	Alternative to (c) *using parameters*	
	$V = \pi \int \left(t^2 - 2\right)^2 \frac{\mathrm{d}x}{\mathrm{d}t} \mathrm{d}t$	M1
	$\int \left(\left(t^2 - 2\right)^2 \times \frac{1}{t}\right) \mathrm{d}t = \int \left(t^3 - 4t + \frac{4}{t}\right) \mathrm{d}t$	M1
	$= \frac{t^4}{4} - 2t^2 + 4\ln t$	M1 A1
	The limits are $t = 2$ and $t = 4$	
	$\pi\left[\frac{t^4}{4} - 2t^2 + 4\ln t\right]_2^4 = \pi\left[(64 - 32 + 4\ln 4) - (4 - 8 + 4\ln 2)\right]$	M1
	$= \pi(36 + 4\ln 2)$	A1
		(6)

Question Number	Scheme		Marks
7. **(a)**	$x = 3 \Rightarrow y = 0.1847$	awrt	B1
	$x = 5 \Rightarrow y = 0.1667$	awrt or $\frac{1}{6}$	B1 (2)
(b)	$I \approx \frac{1}{\underline{\underline{2}}}\left[0.2 + 0.1667 + 2\left(0.1847 + 0.1745\right)\right]$		$\underline{\underline{\text{B1}}}$ M1 A1ft
	≈ 0.543	0.542 or 0.543	A1 (4)
(c)	$\frac{\mathrm{d}x}{\mathrm{d}u} = 2(u-4)$		B1
	$\int \frac{1}{4+\sqrt{(x-1)}}\,\mathrm{d}x = \int \frac{1}{u} \times 2(u-4)\,\mathrm{d}u$		M1
	$= \int \left(2 - \frac{8}{u}\right)\mathrm{d}u$		A1
	$= 2u - 8\ln u$		M1 A1
	$x = 2 \Rightarrow u = 5,\quad x = 5 \Rightarrow u = 6$		B1
	$\left[2u - 8\ln u\right]_5^6 = (12 - 8\ln 6) - (10 - 8\ln 5)$		M1
	$= 2 + 8\ln\left(\frac{5}{6}\right)$		A1
			(8) **[14]**

Mark Scheme (Results)

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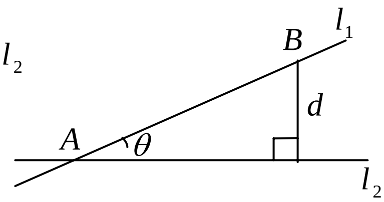
Question Number	Scheme		Marks	
1.	$9x^2 = A(x-1)(2x+1) + B(2x+1) + C(x-1)^2$		B1	
	$x \to 1 \qquad 9 = 3B \Rightarrow B = 3$		M1	
	$x \to -\frac{1}{2} \qquad \frac{9}{4} = \left(-\frac{3}{2}\right)^2 C \Rightarrow C = 1$	Any two of *A*, *B*, *C*	A1	
	x^2 terms $\qquad 9 = 2A + C \Rightarrow A = 4$	All three correct	A1	**(4)** **[4]**
	Alternatives for finding A. x terms $\qquad 0 = -A + 2B - 2C \Rightarrow A = 4$ Constant terms $\quad 0 = -A + B + C \Rightarrow A = 4$			

Question Number	Scheme		Marks	
2.	$\mathrm{f}(x) = (\ \dots\ +\ \dots)^{-\frac{1}{2}}$		M1	
	$= 9^{-\frac{1}{2}}(\ \dots\ +\ \dots)^{\dots}$	3^{-1}, $\frac{1}{3}$ or $\frac{1}{9^{\frac{1}{2}}}$	B1	
	$(1+kx^2)^n = 1 + nkx^2 + \dots$	n not a natural number, $k \neq 1$	M1	
	$(1+kx^2)^{-\frac{1}{2}} = \dots + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2}(kx^2)^2$	ft their $k \neq 1$	A1 ft	
	$\left(1+\frac{4}{9}x^2\right)^{-\frac{1}{2}} = 1 - \frac{2}{9}x^2 + \frac{2}{27}x^4$		A1	
	$\mathrm{f}(x) = \frac{1}{3} - \frac{2}{27}x^2 + \frac{2}{81}x^4$		A1	**(6)**
				[6]

Question Number	Scheme		Marks	
3.	(a) $\dfrac{dV}{dh} = \dfrac{1}{2}\pi h - \pi h^2$	or equivalent	M1 A1	
	At $h = 0.1$, $\dfrac{dV}{dh} = \dfrac{1}{2}\pi(0.1) - \pi(0.1)^2 = 0.04\pi$	$\dfrac{\pi}{25}$	M1 A1	**(4)**
	(b) $\dfrac{dh}{dt} = \dfrac{dV}{dt} \div \dfrac{dV}{dh} = \dfrac{\pi}{800} \times \dfrac{1}{\frac{1}{2}\pi h - \pi h^2}$	or $\dfrac{\pi}{800} \div$ their (a)	M1	
	At $h = 0.1$, $\dfrac{dh}{dt} = \dfrac{\pi}{800} \times \dfrac{25}{\pi} = \dfrac{1}{32}$	awrt 0.031	A1	**(2)**
				[6]

Question Number	Scheme	Marks
4.	(a) 0.0333, 1.3596 awrt 0.0333, 1.3596	B1 B1 **(2)**
	(b) Area $(R) \approx \frac{1}{2} \times \frac{\sqrt{2}}{4}[\ ...]$	B1
	$\approx\ ...\ \left[0+2\left(0.0333+0.3240+1.3596\right)+3.9210\right]$	M1
	≈ 1.30 Accept 1.3	A1 **(3)**
	(c) $u = x^2+2 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 2x$	B1
	Area $(R) = \int_0^{\sqrt{2}} x^3 \ln\left(x^2+2\right)\mathrm{d}x$	B1
	$\int x^3 \ln\left(x^2+2\right)\mathrm{d}x = \int x^2 \ln\left(x^2+2\right) x\,\mathrm{d}x = \int (u-2)(\ln u)\tfrac{1}{2}\mathrm{d}u$	M1
	Hence Area $(R) = \frac{1}{2}\int_2^4 (u-2)\ln u\,\mathrm{d}u$ ✱ cso	A1 **(4)**
	(d) $\int (u-2)\ln u\,\mathrm{d}u = \left(\frac{u^2}{2}-2u\right)\ln u - \int \left(\frac{u^2}{2}-2u\right)\frac{1}{u}\mathrm{d}u$	M1 A1
	$= \left(\frac{u^2}{2}-2u\right)\ln u - \int \left(\frac{u}{2}-2\right)\mathrm{d}u$	
	$= \left(\frac{u^2}{2}-2u\right)\ln u - \left(\frac{u^2}{4}-2u\right) \quad (+C)$	M1 A1
	Area $(R) = \frac{1}{2}\left[\left(\frac{u^2}{2}-2u\right)\ln u - \left(\frac{u^2}{4}-2u\right)\right]_2^4$	
	$= \tfrac{1}{2}\left[(8-8)\ln 4 - 4 + 8 - \left((2-4)\ln 2 - 1 + 4\right)\right]$	M1
	$= \tfrac{1}{2}(2\ln 2 + 1)$ $\ln 2 + \tfrac{1}{2}$	A1 **(6)** **[15]**

Question Number	Scheme		Marks	
5.	$\frac{1}{y}\frac{dy}{dx} = \ \dots$		B1	
	$\dots \quad = 2\ln x + 2x\left(\frac{1}{x}\right)$		M1 A1	
	At $x = 2$, $\quad \ln y = 2(2)\ln 2$		M1	
	leading to $\quad y = 16$	Accept $y = e^{4\ln 2}$	A1	
	At $(2, 16)$ $\quad \frac{1}{16}\frac{dy}{dx} = 2\ln 2 + 2$		M1	
	$\frac{dy}{dx} = 16(2 + 2\ln 2)$		A1	**(7)**
				[7]
	Alternative			
	$y = e^{2x\ln x}$		B1	
	$\frac{d}{dx}(2x\ln x) = 2\ln x + 2x\left(\frac{1}{x}\right)$		M1 A1	
	$\frac{dy}{dx} = \left(2\ln x + 2x\left(\frac{1}{x}\right)\right)e^{2x\ln x}$		M1 A1	
	At $x = 2$, $\quad \frac{dy}{dx} = (2\ln 2 + 2)e^{4\ln 2}$		M1	
	$= 16(2 + 2\ln 2)$		A1	**(7)**

Question Number	Scheme		Marks	
6.	(a) **i**: $6-\lambda=-5+2\mu$			
	j: $-3+2\lambda=15-3\mu$	Any two equations	M1	
	leading to $\lambda=3,\ \mu=4$		M1 A1	
	$\mathbf{r}=\begin{pmatrix}6\\-3\\-2\end{pmatrix}+3\begin{pmatrix}-1\\2\\3\end{pmatrix}=\begin{pmatrix}3\\3\\7\end{pmatrix}$ *or* $\mathbf{r}=\begin{pmatrix}-5\\15\\3\end{pmatrix}+4\begin{pmatrix}2\\-3\\1\end{pmatrix}=\begin{pmatrix}3\\3\\7\end{pmatrix}$		M1 A1	
	k: LHS $=-2+3(3)=7$**,** RHS $=3+4(1)=7$		B1	**(6)**
	(As LHS = RHS, lines intersect)			
	Alternatively for B1, showing that $\lambda=3$ and $\mu=4$ both give $\begin{pmatrix}3\\3\\7\end{pmatrix}$			
	(b) $\begin{pmatrix}-1\\2\\3\end{pmatrix}.\begin{pmatrix}2\\-3\\1\end{pmatrix}=-2-6+3=\sqrt{14}\sqrt{14}\cos\theta$ $(\theta\approx110.92°)$		M1 A1	
	Acute angle is $69.1°$	awrt 69.1	A1	**(3)**
	(c) $\mathbf{r}=\begin{pmatrix}6\\-3\\-2\end{pmatrix}+1\begin{pmatrix}-1\\2\\3\end{pmatrix}=\begin{pmatrix}5\\-1\\1\end{pmatrix}$ $(\Rightarrow B$ lies on $l_1)$		B1	**(1)**
	(d) Let d be shortest distance from B to l_2 			
	$\overrightarrow{AB}=\begin{pmatrix}5\\-1\\1\end{pmatrix}-\begin{pmatrix}3\\3\\7\end{pmatrix}=\begin{pmatrix}2\\-4\\-6\end{pmatrix}$		M1	
	$\lvert\overrightarrow{AB}\rvert=\sqrt{\left(2^2+(-4)^2+(-6)^2\right)}=\sqrt{56}$	awrt 7.5	A1	
	$\frac{d}{\sqrt{56}}=\sin\theta$		M1	
	$d=\sqrt{56}\sin 69.1°\approx6.99$	awrt 6.99	A1	**(4)**
				[14]

Question Number	Scheme		Marks	
7.	(a) $\tan\theta = \sqrt{3}$ *or* $\sin\theta = \frac{\sqrt{3}}{2}$		M1	
	$\theta = \frac{\pi}{3}$	awrt 1.05	A1	**(2)**
	(b) $\frac{dx}{d\theta} = \sec^2\theta,\ \frac{dy}{d\theta} = \cos\theta$			
	$\frac{dy}{dx} = \frac{\cos\theta}{\sec^2\theta}\quad \left(=\cos^3\theta\right)$		M1 A1	
	At P, $m = \cos^3\left(\frac{\pi}{3}\right) = \frac{1}{8}$	Can be implied	A1	
	Using $mm' = -1$, $m' = -8$		M1	
	For normal $y - \frac{1}{2}\sqrt{3} = -8\left(x - \sqrt{3}\right)$		M1	
	At Q, $y = 0$ $-\frac{1}{2}\sqrt{3} = -8\left(x - \sqrt{3}\right)$			
	leading to $x = \frac{17}{16}\sqrt{3}$ $\left(k = \frac{17}{16}\right)$	1.0625	A1	**(6)**
	(c) $\int y^2\,dx = \int y^2 \frac{dx}{d\theta}\,d\theta = \int \sin^2\theta \sec^2\theta\,d\theta$		M1 A1	
	$= \int \tan^2\theta\,d\theta$		A1	
	$= \int \left(\sec^2\theta - 1\right)d\theta$		M1	
	$= \tan\theta - \theta\quad (+C)$		A1	
	$V = \pi\int_0^{\frac{\pi}{3}} y^2\,dx = \left[\tan\theta - \theta\right]_0^{\frac{\pi}{3}} = \pi\left[\left(\sqrt{3} - \frac{\pi}{3}\right) - (0-0)\right]$		M1	
	$= \sqrt{3}\pi - \frac{1}{3}\pi^2\qquad \left(p = 1, q = -\frac{1}{3}\right)$		A1	**(7)**
				[15]

Question Number	Scheme	Marks
8.	(a) $\int (4y+3)^{-\frac{1}{2}}\,dx = \frac{(4y+3)^{\frac{1}{2}}}{(4)(\frac{1}{2})} \quad (+C)$	M1 A1 **(2)**
	$\left(= \frac{1}{2}(4y+3)^{\frac{1}{2}} + C\right)$	
	(b) $\int \frac{1}{\sqrt{(4y+3)}}\,dy = \int \frac{1}{x^2}\,dx$	B1
	$\int (4y+3)^{-\frac{1}{2}}\,dy = \int x^{-2}\,dx$	
	$\frac{1}{2}(4y+3)^{\frac{1}{2}} = -\frac{1}{x} \quad (+C)$	M1
	Using $(-2, 1.5)$ $\quad \frac{1}{2}(4\times1.5+3)^{\frac{1}{2}} = -\frac{1}{-2} + C$	M1
	leading to $\quad C = 1$	A1
	$\frac{1}{2}(4y+3)^{\frac{1}{2}} = -\frac{1}{x} + 1$	
	$(4y+3)^{\frac{1}{2}} = 2 - \frac{2}{x}$	M1
	$y = \frac{1}{4}\left(2 - \frac{2}{x}\right)^2 - \frac{3}{4}$ $\qquad$ or equivalent	A1 **(6)**
		[8]

Mark Scheme (Results)

January 2012

GCE Core Mathematics C4 (6666) Paper 1

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6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme	Marks
1. (a)	$\left\{\cancel{\frac{dy}{dx}} = \right\}$ $\quad \underline{2 + 6y\frac{dy}{dx}} + \left(\underline{\underline{6xy + 3x^2\frac{dy}{dx}}}\right) = \underline{8x}$	M1 A1 B1
	$\left\{\frac{dy}{dx} = \frac{8x - 2 - 6xy}{6y + 3x^2}\right\}$ *not necessarily required.*	
	At $P(-1, 1)$, $\quad \mathrm{m}(\mathbf{T}) = \frac{dy}{dx} = \frac{8(-1) - 2 - 6(-1)(1)}{6(1) + 3(-1)^2} = -\frac{4}{9}$	dM1 A1 **cso**
		[5]
(b)	So, $\mathrm{m}(\mathbf{N}) = \frac{-1}{-\frac{4}{9}} \left\{= \frac{9}{4}\right\}$	M1
	N: $y - 1 = \frac{9}{4}(x + 1)$	M1
	N: $9x - 4y + 13 = 0$	A1
		[3]
		8
(a)	**M1**: Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $3x^2\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$).	
	A1: $\left(2x + 3y^2\right) \rightarrow \left(\underline{2 + 6y\frac{dy}{dx}}\right)$ and $\left(4x^2 \rightarrow \underline{8x}\right)$. **Note:** If an extra "sixth" term appears then award A0.	
	B1: $6xy + 3x^2\frac{dy}{dx}$.	
	dM1: Substituting $x = -1$ and $y = 1$ into an equation involving $\frac{dy}{dx}$. Allow this mark if either the numerator or denominator of $\frac{dy}{dx} = \frac{8x - 2 - 6xy}{6y + 3x^2}$ is substituted into or evaluated correctly. If it is clear, however, that the candidate is intending to substitute $x = 1$ and $y = -1$, then award M0. Candidates who substitute $x = 1$ and $y = -1$, will usually achieve $\mathrm{m}(\mathbf{T}) = -4$ Note that this mark is dependent on the previous method mark being awarded.	
	A1: For $-\frac{4}{9}$ or $-\frac{8}{18}$ or $-0.\dot{4}$ or awrt -0.44 If the candidate's solution is not completely correct, then do not give this mark.	
(b)	**M1:** Applies $\mathrm{m}(\mathbf{N}) = -\frac{1}{\text{their } \mathrm{m}(\mathbf{T})}$.	
	M1: Uses $y - 1 = (m_N)(x - -1)$ or finds c using $x = -1$ and $y = 1$ and uses $y = (m_N)x + "c"$, Where $m_N = -\frac{1}{\text{their } \mathrm{m}(\mathbf{T})}$ or $m_N = \frac{1}{\text{their } \mathrm{m}(\mathbf{T})}$ or $m_N = -\text{their } \mathrm{m}(\mathbf{T})$.	
	A1: $9x - 4y + 13 = 0$ or $-9x + 4y - 13 = 0$ or $4y - 9x - 13 = 0$ or $18x - 8y + 26 = 0$ etc. Must be "$= 0$". So do not allow $9x + 13 = 4y$ etc.	
	Note: $m_N = -\left(\frac{6y + 3x^2}{8x - 2 - 6xy}\right)$ is M0M0 unless a numerical value is then found for m_N.	

<u>Alternative method for part (a): Differentiating with respect to y</u>

$$\left\{\cancel{\frac{\mathrm{d}x}{\mathrm{d}y}}\cancel{=}\right\}\quad \underline{2\frac{\mathrm{d}x}{\mathrm{d}y}+6y}+\left(\underline{\underline{6xy\frac{\mathrm{d}x}{\mathrm{d}y}+3x^2}}\right)=\underline{8x\frac{\mathrm{d}x}{\mathrm{d}y}}$$

M1: Differentiates implicitly to include either $2\frac{\mathrm{d}x}{\mathrm{d}y}$ or $6xy\frac{\mathrm{d}x}{\mathrm{d}y}$ or $\pm kx\frac{\mathrm{d}x}{\mathrm{d}y}$. (Ignore $\left(\frac{\mathrm{d}x}{\mathrm{d}y}=\right)$).

A1: $\left(2x+3y^2\right)\rightarrow\left(\underline{2\frac{\mathrm{d}x}{\mathrm{d}y}+6y}\right)$ and $\left(4x^2\rightarrow\underline{8x\frac{\mathrm{d}x}{\mathrm{d}y}}\right)$. **Note:** If an extra "sixth" term appears then award A0.

B1: $6xy+3x^2\frac{\mathrm{d}y}{\mathrm{d}x}$.

dM1: Substituting $x=-1$ and $y=1$ into an equation involving $\frac{\mathrm{d}x}{\mathrm{d}y}$ or $\frac{\mathrm{d}y}{\mathrm{d}x}$. Allow this mark if either the numerator or denominator of $\frac{\mathrm{d}x}{\mathrm{d}y}=\frac{6y+3x^2}{8x-2-6xy}$ is substituted into or evaluated correctly.

If it is clear, however, that the candidate is intending to substitute $x=1$ and $y=-1$, then award M0.

Candidates who substitute $x=1$ and $y=-1$, will usually achieve $\mathrm{m}(\mathbf{T})=-4$

Note that this mark is dependent on the previous method mark being awarded.

A1: For $-\frac{4}{9}$ or $-\frac{8}{18}$ or $-0.\dot{4}$ or awrt -0.44

If the candidate's solution is not completely correct, then do not give this mark.

Question Number	Scheme	Marks
2. (a)	$\int x\sin 3x \,\mathrm{d}x = -\frac{1}{3}x\cos 3x - \int -\frac{1}{3}\cos 3x \,\{\mathrm{d}x\}$	M1 A1
	$= -\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x \,\{+c\}$	A1
		[3]
(b)	$\int x^2\cos 3x \,\mathrm{d}x = \frac{1}{3}x^2\sin 3x - \int \frac{2}{3}x\sin 3x \,\{\mathrm{d}x\}$	M1 A1
	$= \frac{1}{3}x^2\sin 3x - \frac{2}{3}\left(-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x\right) \;\{+c\}$	A1 **isw**
	$\left\{= \frac{1}{3}x^2\sin 3x + \frac{2}{9}x\cos 3x - \frac{2}{27}\sin 3x \;\{+c\}\right\}$ *Ignore subsequent working*	**[3]**
		6
(a)	**M1:** Use of 'integration by parts' formula $uv - \int vu'$ (whether stated or not stated) in the **correct direction**, where $u = x \to u' = 1$ and $v' = \sin 3x \to v = k\cos 3x$ (seen or implied), where k is a positive or negative constant. (Allow $k = 1$). This means that the candidate must achieve $x(k\cos 3x) - \int (k\cos 3x)$, where k is a consistent constant. If x^2 appears after the integral, this would imply that the candidate is applying integration by parts in the wrong direction, so M0. **A1:** $-\frac{1}{3}x\cos 3x - \int -\frac{1}{3}\cos 3x \,\{\mathrm{d}x\}$. Can be un-simplified. Ignore the $\{\mathrm{d}x\}$. **A1:** $-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x$ with/without + c. Can be un-simplified.	
(b)	**M1:** Use of 'integration by parts' formula $uv - \int vu'$ (whether stated or not stated) in the **correct direction**, where $u = x^2 \to u' = 2x$ or x and $v' = \cos 3x \to v = \lambda\sin 3x$ (seen or implied), where λ is a positive or negative constant. (Allow $\lambda = 1$). This means that the candidate must achieve $x^2(\lambda\sin 3x) - \int 2x(\lambda\sin 3x)$, where $u' = 2x$ or $x^2(\lambda\sin 3x) - \int x(\lambda\sin 3x)$, where $u' = x$. If x^3 appears after the integral, this would imply that the candidate is applying integration by parts in the wrong direction, so M0. **A1:** $\frac{1}{3}x^2\sin 3x - \int \frac{2}{3}x\sin 3x \,\{\mathrm{d}x\}$. Can be un-simplified. Ignore the $\{\mathrm{d}x\}$. **A1:** $\frac{1}{3}x^2\sin 3x - \frac{2}{3}\left(-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x\right)$ with/without + c, can be un-simplified. You can ignore subsequent working here. **Special Case**: If the candidate scores the first two marks of M1A1 in part (b), then you can award the final A1 as a follow through for $\frac{1}{3}x^2\sin 3x - \frac{2}{3}(\text{their follow through part(a) answer})$.	

Question Number	Scheme	Marks
3. (a)	$\dfrac{1}{(2-5x)^2} = (2-5x)^{-2} = \underline{(2)^{-2}}\left(1-\dfrac{5x}{2}\right)^{-2} = \underline{\dfrac{1}{4}}\left(1-\dfrac{5x}{2}\right)^{-2}$ $\underline{(2)^{-2}}$ or $\underline{\dfrac{1}{4}}$	<u>B1</u>
	$= \left\{\dfrac{1}{4}\right\}\left[1+(-2)(**x)+\dfrac{(-2)(-3)}{2!}(**x)^2+\ldots\right]$ see notes	M1 A1ft
	$= \left\{\dfrac{1}{4}\right\}\left[\underline{1+(-2)\left(-\dfrac{5x}{2}\right)+\dfrac{(-2)(-3)}{2!}\left(-\dfrac{5x}{2}\right)^2+\ldots}\right]$	
	$= \dfrac{1}{4}\left[1+5x;+\dfrac{75}{4}x^2+\ldots\right]$ See notes below!	
	$= \dfrac{1}{4}+\dfrac{5}{4}x;+\dfrac{75}{16}x^2+\ldots$	A1; A1
		[5]
(b)	$\left\{\dfrac{2+kx}{(2-5x)^2}\right\} = (2+kx)\left(\dfrac{1}{4}+\dfrac{5}{4}x+\left\{\dfrac{75}{16}x^2+\ldots\right\}\right)$ *Can be implied by later work even in part (c).*	M1
	x terms: $\dfrac{2(5x)}{4}+\dfrac{kx}{4}=\dfrac{7x}{4}$	
	giving, $10+k=7 \Rightarrow \underline{k=-3}$ $\underline{k=-3}$	A1
		[2]
(c)	x^2 terms: $\dfrac{150x^2}{16}+\dfrac{5kx^2}{4}$	M1
	So, $A = \dfrac{75}{8}+\dfrac{5(-3)}{4} = \dfrac{75}{8}-\dfrac{15}{4} = \underline{\dfrac{45}{8}}$ $\underline{\dfrac{45}{8}}$ or $\underline{5\dfrac{5}{8}}$ or $\underline{5.625}$	A1
		[2] **9**
(a)	<u>**B1**</u>: $\underline{(2)^{-2}}$ or $\underline{\dfrac{1}{4}}$ outside brackets or $\underline{\dfrac{1}{4}}$ as candidate's constant term in their binomial expansion. **M1:** Expands to give a simplified or an un-simplified, $1+(-2)(**x)$ or $(-2)(**x)+\dfrac{(-2)(-3)}{2!}(**x)^2$ or $1+\ldots\ldots+\dfrac{(-2)(-3)}{2!}(**x)^2$, **where** $**\neq 1$. **A1:** A correct simplified or an un-simplified $1+(-2)(**x)+\dfrac{(-2)(-3)}{2!}(**x)^2$ expansion with candidate's follow through $(**x)$. Note that $(**x)$ must be consistent. You would award B1M1A0 for $=\dfrac{1}{4}\left[\underline{1+(-2)\left(-\dfrac{5x}{2}\right)+\dfrac{(-2)(-3)}{2!}(-5x)^2+\ldots}\right]$ because ** is not consistent. ***Invisible brackets*** $\left\{\dfrac{1}{4}\right\}\left[\underline{1+(-2)\left(-\dfrac{5x}{2}\right)+\dfrac{(-2)(-3)}{2!}\left(-\dfrac{5x^2}{2}\right)+\ldots}\right]$ is M1A0 unless recovered. **A1:** For $\dfrac{1}{4}+\dfrac{5}{4}x$ **(simplified fractions)** or Also allow $0.25+1.25x$ or $\dfrac{1}{4}+1\dfrac{1}{4}x$. **Allow Special Case A1 for either SC:** $\dfrac{1}{4}[1+5x;\ldots]$ or **SC**: $K\left[1+5x+\dfrac{75}{4}x^2+\ldots\right]$. **A1:** Accept only $\dfrac{75}{16}x^2$ or $4\dfrac{11}{16}x^2$ or $4.6875x^2$ <u>**Alternative method:**</u> Candidates can apply an alternative form of the binomial expansion. (See next page).	

3. (b)	**M1:** Candidate writes down $(2+kx)\big($their part (a) answer, at least up to the term in x.$\big)$ $(2+kx)\left(\frac{1}{4}+\frac{5}{4}x+\ ...\right)$ or $(2+kx)\left(\frac{1}{4}+\frac{5}{4}x+\frac{75}{16}x^2+...\right)$ are fine. This mark can also be implied by candidate multiplying out to find two terms (or coefficients) in x. **A1:** $k=-3$
(c)	**M1:** Multiplies out their $(2+kx)\left(\frac{1}{4}+\frac{5}{4}x+\frac{75}{16}x^2+...\right)$ to give **exactly** two terms (or coefficients) in x^2 and attempts to find A using a numerical value of k. **A1:** Either $\frac{45}{8}$ or $5\frac{5}{8}$ or 5.625 **Note:** $\frac{45}{8}x^2$ is A0. *Alternative method for part (a)* $(2-5x)^{-2}\ =\ (2)^{-2}+(-2)(2)^{-3}(-5x);+\frac{(-2)(-3)}{2!}(2)^{-4}(-5x)^2$ **B1:** $\frac{1}{4}$ or $(2)^{-2}$, **M1:** Any two of three (un-simplified) terms correct. **A1:** All three (un-simplified) terms correct. **A1:** $\frac{1}{4}+\frac{5}{4}x$ **A1:** $\frac{75}{16}x^2$ **Note:** The terms in C need to be evaluated, so ${}^{-2}C_0(2)^{-2}+{}^{-2}C_1(2)^{-3}(-5x);+{}^{-2}C_2(2)^{-4}(-5x)^2$ without further working is B0M0A0. *Alternative method for parts (b) and (c)* $(2+kx)=\ (2-5x)^2\left(\frac{1}{2}+\frac{7}{4}x+Ax^2+...\right)$ $(2+kx)=\ (4-20x+25x^2)\left(\frac{1}{2}+\frac{7}{4}x+Ax^2+...\right)$ $(2+kx)=\ 2+(7x-10x)+\left(4Ax^2-35x^2+\frac{25}{2}x^2\right)$ Equate x terms: $\underline{k=-3}$ Equate x^2 terms: $0=4A-35+\frac{25}{2}\ \Rightarrow\ 4A=\frac{45}{2}\ \Rightarrow\ \underline{A=\frac{45}{8}}$
(b)	**M1:** For $(2+kx)=\ (4\pm\lambda x+25x^2)\left(\frac{1}{2}+\frac{7}{4}x+Ax^2+...\right)$, where $\lambda\neq 0$ **A1:** $k=-3$
(c)	**M1:** Multiplies out to obtain three x^2 terms/coefficients, equates to 0 and attempts to find A. **A1:** Either $\frac{45}{8}$ or $5\frac{5}{8}$ or 5.625 **Note:** $\frac{45}{8}x^2$ is A0.

Question Number	Scheme		Marks
4.	Volume $= \pi \underline{\int_0^2 \left(\sqrt{\left(\frac{2x}{3x^2+4}\right)}\right)^2 \, dx}$	Use of $V = \pi \underline{\int y^2} \, dx$.	<u>B1</u>
	$= (\pi)\left[\frac{1}{3}\ln\left(3x^2+4\right)\right]_0^2$	$\pm k\ln\left(3x^2+4\right)$	M1
		$\frac{1}{3}\ln\left(3x^2+4\right)$	A1
	$= (\pi)\left[\left(\frac{1}{3}\ln 16\right) - \left(\frac{1}{3}\ln 4\right)\right]$	Substitutes limits of 2 and 0 and subtracts the correct way round.	dM1
	So Volume $= \underline{\frac{1}{3}\pi\ln 4}$	$\underline{\frac{1}{3}\pi\ln 4}$ or $\underline{\frac{2}{3}\pi\ln 2}$	A1 oe isw
			[5] **5**

NOTE: π is required for the B1 mark and the final A1 mark. It is not required for the 3 intermediate marks.

B1: For applying $\pi\int y^2$. Ignore limits and dx. This can be implied by later working,

but the pi and $\int \frac{2x}{3x^2+4}$ must appear on one line somewhere in the candidate's working.

B1 can also be implied by a correct final answer. **Note**: $\pi\left(\int y\right)^2$ would be B0.

<u>Working in x</u>

M1: For $\pm k\ln\left(3x^2+4\right)$ or $\pm k\ln\left(x^2+\frac{4}{3}\right)$ where k is a constant and k can be 1.

Note: M0 for $\pm k\,x\ln\left(3x^2+4\right)$.

Note: M1 can also be given for $\pm k\ln\left(p\left(3x^2+4\right)\right)$, where k and p are constants and k can be 1.

A1: For $\frac{1}{3}\ln\left(3x^2+4\right)$ or $\frac{1}{3}\ln\left(\frac{1}{3}\left(3x^2+4\right)\right)$ or $\frac{1}{3}\ln\left(x^2+\frac{4}{3}\right)$ or $\frac{1}{3}\ln\left(p(3x^2+4)\right)$.

You may allow M1 A1 for $\frac{1}{3}\left(\frac{x}{x}\right)\ln\left(3x^2+4\right)$ or $\frac{1}{3}\left(\frac{2x}{6x}\right)\ln\left(3x^2+4\right)$

dM1: Substitutes limits of 2 and 0 and subtracts the correct way round. Working in decimals is fine for dM1.

A1: For either $\frac{1}{3}\pi\ln 4$, $\frac{1}{3}\ln 4^{\pi}$, $\frac{2}{3}\pi\ln 2$, $\pi\ln 4^{\frac{1}{3}}$, $\pi\ln 2^{\frac{2}{3}}$, $\frac{1}{3}\pi\ln\left(\frac{16}{4}\right)$, $2\pi\ln\left(\frac{16^{\frac{1}{6}}}{4^{\frac{1}{6}}}\right)$, etc.

Note: $\frac{1}{3}\pi(\ln 16 - \ln 4)$ would be A0.

<u>Working in u:</u> where $u = 3x^2+4$,

M1: For $\pm k\ln u$ where k is a constant and k can be 1.

Note: M1 can also be given for $\pm k\ln(pu)$, where k and p are constants and k can be 1.

A1: For $\frac{1}{3}\ln u$ or $\frac{1}{3}\ln 3u$ or $\frac{1}{3}\ln pu$.

dM1: Substitutes limits of 16 and 4 in u or limits of 2 and 0 in x and subtracts the correct way round.

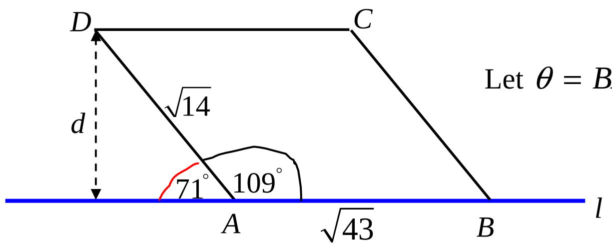
A1: As above!

Question Number	Scheme	Marks
5.	$x = 4\sin\left(t + \frac{\pi}{6}\right), \quad y = 3\cos 2t, \quad 0 \leq t < 2\pi$	
(a)	$\frac{dx}{dt} = 4\cos\left(t + \frac{\pi}{6}\right), \quad \frac{dy}{dt} = -6\sin 2t$	B1 B1
	So, $\frac{dy}{dx} = \underline{\frac{-6\sin 2t}{4\cos\left(t + \frac{\pi}{6}\right)}}$	B1$\sqrt{}$ oe
		[3]
(b)	$\left\{\frac{dy}{dx} = 0 \Rightarrow\right\} -6\sin 2t = 0$	M1 oe
	@ $t = 0$, $x = 4\sin\left(\frac{\pi}{6}\right) = 2$, $y = 3\cos 0 = 3 \rightarrow (2, 3)$	M1
	@ $t = \frac{\pi}{2}$, $x = 4\sin\left(\frac{2\pi}{3}\right) = \frac{4\sqrt{3}}{2}$, $y = 3\cos\pi = -3 \rightarrow (2\sqrt{3}, -3)$	
	@ $t = \pi$, $x = 4\sin\left(\frac{7\pi}{6}\right) = -2$, $y = 3\cos 2\pi = 3 \rightarrow (-2, 3)$	
	@ $t = \frac{3\pi}{2}$, $x = 4\sin\left(\frac{5\pi}{3}\right) = \frac{4(-\sqrt{3})}{2}$, $y = 3\cos 3\pi = -3 \rightarrow (-2\sqrt{3}, -3)$	A1A1A1
		[5] **8**
(a)	**B1:** Either one of $\frac{dx}{dt} = 4\cos\left(t + \frac{\pi}{6}\right)$ or $\frac{dy}{dt} = -6\sin 2t$. They do not have to be simplified. **B1:** Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ correct. They do not have to be simplified. Any or both of the first two marks can be implied. Don't worry too much about their notation for the first two B1 marks. **B1:** Their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ or their $\frac{dy}{dt} \times \frac{1}{\text{their}\left(\frac{dx}{dt}\right)}$. **Note:** This is a follow through mark. *<u>Alternative differentiation in part (a)</u>* $x = 2\sqrt{3}\sin t + 2\cos t \Rightarrow \frac{dx}{dt} = 2\sqrt{3}\cos t - 2\sin t$ $y = 3(2\cos^2 t - 1) \Rightarrow \frac{dy}{dt} = 3(-4\cos t\sin t)$ or $y = 3\cos^2 t - 3\sin^2 t \Rightarrow \frac{dy}{dt} = -6\cos t\sin t - 6\sin t\cos t$ or $y = 3(1 - 2\sin^2 t) \Rightarrow \frac{dy}{dt} = 3(-4\cos t\sin t)$	

5. (b)	**M1:** Candidate sets their numerator from part (a) or their $\frac{\mathrm{d}y}{\mathrm{d}t}$ equal to 0. Note that their numerator must be a trig function. Ignore $\frac{\mathrm{d}x}{\mathrm{d}t}$ equal to 0 at this stage. **M1:** Candidate substitutes a found value of t, to attempt to find either one of x or y. The first two method marks can be implied by ONE correct set of coordinates for $(x,\ y)$ or $(y,\ x)$ interchanged. A correct point coming from NO WORKING can be awarded M1M1. **A1:** At least TWO sets of coordinates. **A1:** At least THREE sets of coordinates. **A1:** ONLY FOUR correct sets of coordinates. If there are more than 4 sets of coordinates then award A0. **Note:** Candidate can use the diagram's symmetry to write down some of their coordinates. **Note:** When $x = 4\sin\left(\frac{\pi}{6}\right) = 2$, $y = 3\cos 0 = 3$ is acceptable for a pair of coordinates. Also it is fine for candidates to display their coordinates on a table of values. **Note:** The coordinates must be exact for the accuracy marks. Ie (3.46..., -3) or (-3.46..., -3) is A0. **Note:** $\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow \sin t = 0$ ONLY is fine for the first M1, and potentially the following M1A1A0A0. **Note:** $\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow \cos t = 0$ ONLY is fine for the first M1 and potentially the following M1A1A0A0. **Note:** $\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow \sin t = 0\ \&\ \cos t = 0$ has the potential to achieve all five marks. **Note:** It is possible for a candidate to gain full marks in part (b) if they make sign errors in part (a). *(b) An alternative method for finding the coordinates of the two maximum points.* Some candidates may use $y = 3\cos 2t$ to write down that the y-coordinate of a maximum point is 3. They will then deduce that $t = 0$ or π and proceed to find the x-coordinate of their maximum point. These candidates will receive no credit until they attempt to find one of the x-coordinates for the maximum point. **M1M1:** Candidate states $y = 3$ and attempts to substitute $t = 0$ or π into $x = 4\sin\left(t + \frac{\pi}{6}\right)$. M1M1 can be implied by candidate stating either $(2, 3)$ or $(2, -3)$. **Note:** these marks can only be awarded together for a candidate using this method. **A1:** For both $(2, 3)$ or $(-2, 3)$. **A0A0**: Candidate cannot achieve the final two marks by using this method. They can, however, achieve these marks by subsequently solving their numerator equal to 0.

Question Number	Scheme	Marks
6. (a)	0.73508	B1 **cao** **[1]**
(b)	Area $\approx \frac{1}{2}\times\frac{\pi}{8}\ ;\times\underline{\left[0+2\left(\text{their } 0.73508+1.17157+1.02280\right)+0\right]}$	B1 <u>M1</u>
	$=\frac{\pi}{16}\times 5.8589... = 1.150392325... = 1.1504$ (4 dp) awrt 1.1504	A1 **[3]**
(c)	$\underline{\{u=1+\cos x\} \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x}=-\sin x}$	<u>B1</u>
	$\left\{\int \frac{2\sin 2x}{(1+\cos x)}\,\mathrm{d}x=\right\}\int \frac{2(2\sin x\cos x)}{(1+\cos x)}\,\mathrm{d}x$ $\sin 2x = 2\sin x\cos x$	B1
	$=\int \frac{4(u-1)}{u}.(-1)\,\mathrm{d}u \quad \left\{= 4\int \frac{(1-u)}{u}\,\mathrm{d}u\right\}$	M1
	$=4\int\left(\frac{1}{u}-1\right)\mathrm{d}u = 4(\ln u - u)+c$	dM1
	$=4\ln(1+\cos x)-4(1+\cos x)+c \quad = 4\ln(1+\cos x)-4\cos x+k$ **AG**	A1 **cso** **[5]**
(d)	$=\left[4\ln\left(1+\cos\frac{\pi}{2}\right)-4\cos\frac{\pi}{2}\right]-\left[4\ln(1+\cos 0)-4\cos 0\right]$ Applying limits $x=\frac{\pi}{2}$ and $x=0$ either way round.	M1
	$=\left[4\ln 1-0\right]-\left[4\ln 2-4\right]$	
	$=4-4\ln 2 \ \{=1.227411278...\}$ $\pm4(1-\ln 2)$ or $\pm(4-4\ln 2)$ or awrt ±1.2, ***however*** found.	A1
	Error $=\left\|(4-4\ln 2)-1.1504...\right\|$ $=0.0770112776... = 0.077$ (2sf) awrt ±0.077 or awrt $\pm6.3(\%)$	A1 **cso** **[3]**
		12
(a)	**B1:** 0.73508 correct answer only. Look for this on the table or in the candidate's working.	
(b)	**B1**: Outside brackets $\frac{1}{2}\times\frac{\pi}{8}$ or $\frac{\pi}{16}$ or awrt 0.196 **M1:** <u>For structure of trapezium rule</u>$\left[\ \underline{............}\ \right]$; (0 can be implied). **A1:** anything that rounds to 1.1504 <u>Bracketing mistake</u>: **Unless the final answer implies that the calculation has been done correctly** Award B1M0A0 for $\frac{1}{2}\times\frac{\pi}{8}+2\left(\text{their } 0.73508+1.17157+1.02280\right)$ (nb: answer of 6.0552). Award B1M0A0 for $\frac{1}{2}\times\frac{\pi}{8}(0+0)+2\left(\text{their } 0.73508+1.17157+1.02280\right)$ (nb: answer of 5.8589). <u>*Alternative method for part (b): Adding individual trapezia*</u> Area $\approx \frac{\pi}{8}\times\left[\frac{0+0.73508}{2}+\frac{0.73508+1.17157}{2}+\frac{1.17157+1.02280}{2}+\frac{1.02280+0}{2}\right] = 1.150392325...$ B1: $\frac{\pi}{8}$ and a divisor of 2 on all terms inside brackets. M1: One of first and last ordinates, two of the middle ordinates inside brackets ignoring the 2. A1: anything that rounds to 1.1504	

6. (c)	**B1:** $\frac{du}{dx} = -\sin x$ or $du = -\sin x\,dx$ or $\frac{dx}{du} = \frac{1}{-\sin x}$ oe. **B1:** For seeing, applying or implying $\sin 2x = 2\sin x\cos x$. **M1:** After applying substitution candidate achieves $\pm k\int \frac{(u-1)}{u}(du)$ or $\pm k\int \frac{(1-u)}{u}(du)$. Allow M1 for "invisible" brackets here, eg: $\pm\int \frac{(\lambda u-1)}{u}(du)$ or $\pm\int \frac{(-\lambda+u)}{u}(du)$, where λ is a positive constant. **dM1:** An attempt to divide through each term by u and $\pm k\int\left(\frac{1}{u}-1\right)du \to \pm k(\ln u - u)$ with/without $+c$. Note that this mark is dependent on the previous M1 mark being awarded. **<u>Alternative method:</u>** Candidate can also gain this mark for applying integration by parts followed by a correct method for integrating $\ln u$. (See below). **A1:** Correctly combines their $+c$ and "-4" together to give $\underline{4\ln(1+\cos x) - 4\cos x + k}$ As a minimum candidate must write either $4\ln(1+\cos x) - 4(1+\cos x) + c \to 4\ln(1+\cos x) - 4\cos x + k$ or $4\ln(1+\cos x) - 4(1+\cos x) + k \to 4\ln(1+\cos x) - 4\cos x + k$ **Note**: that this mark is also for a correct solution only. **Note**: those candidates who attempt to find the value of k will usually achieve A0.
(d)	**M1:** Substitutes limits of $x = \frac{\pi}{2}$ and $x = 0$ into $\{4\ln(1+\cos x) - 4\cos x\}$ or their answer from part (c) and subtracts the either way round. **Note** that: $\left[4\ln\left(1+\cos\frac{\pi}{2}\right) - 4\cos\frac{\pi}{2}\right] - [0]$ is M0. **A1:** $4(1-\ln 2)$ or $4 - 4\ln 2$ or awrt 1.2, however found. This mark can be implied by the final answer of either awrt ± 0.077 or awrt ± 6.3 **A1:** For either awrt ± 0.077 or awrt ± 6.3 (for percentage error). Note this mark is for a **correct solution only**. Therefore if there if a candidate substitutes limits the incorrect way round and final achieves (usually fudges) the final correct answer then this mark can be withheld. Note that awrt 6.7 (for percentage error) is A0. *<u>Alternative method for dM1 in part (c)</u>* $\int \frac{(1-u)}{u}du = \left((1-u)\ln u - \int -\ln u\,du\right) = \left((1-u)\ln u + u\ln u - \int \frac{u}{u}du\right) = ((1-u)\ln u + u\ln u - u)$ or $\int \frac{(u-1)}{u}du = \left((u-1)\ln u - \int \ln u\,du\right) = \left((u-1)\ln u - \left(u\ln u - \int \frac{u}{u}du\right)\right) = ((u-1)\ln u - u\ln u + u)$ So **dM1** is for $\int \frac{(1-u)}{u}du$ going to $((1-u)\ln u + u\ln u - u)$ or $((u-1)\ln u - u\ln u + u)$ oe. *<u>Alternative method for part (d)</u>* **M1A1** for $\left\{4\int_2^1\left(\frac{1}{u}-1\right)du = \right\}\ 4[\ln u - u]_2^1 = 4[(\ln 1 - 1) - (\ln 2 - 2)] = 4(1-\ln 2)$ *<u>Alternative method for part (d): Using an extra constant λ from their integration.</u>* $\left[4\ln\left(1+\cos\frac{\pi}{2}\right) - 4\cos\frac{\pi}{2} + \lambda\right] - [4\ln(1+\cos 0) - 4\cos 0 + \lambda]$ λ is usually -4, but can be a value of k that the candidate has found in part (d). **Note:** The extra constant λ should cancel out and so the candidate can gain all three marks using this method, even the final A1 cso.

Question Number	Scheme		Marks
7.	$\overrightarrow{OA} = 2\mathbf{i} - \mathbf{j} + 5\mathbf{k}$, $\overrightarrow{OB} = 5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}$, $\left\{\overrightarrow{OC} = 2\mathbf{i} + 4\mathbf{j} + 9\mathbf{k}\right\}$ & $\overrightarrow{OD} = -\mathbf{i} + \mathbf{j} + 4\mathbf{k}$		
(a)	$\overrightarrow{AB} = = \pm\left((5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}) - (2\mathbf{i} - \mathbf{j} + 5\mathbf{k})\right); = 3\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$		M1; A1 **[2]**
(b)	$l: \mathbf{r} = \begin{pmatrix}2\\-1\\5\end{pmatrix} + \lambda\begin{pmatrix}3\\3\\5\end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix}5\\2\\10\end{pmatrix} + \lambda\begin{pmatrix}3\\3\\5\end{pmatrix}$	See notes	M1 A1ft **[2]**
	 Let $\theta = B\hat{A}D$	Let d be the shortest distance from C to l.	
(c)	$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA} = \begin{pmatrix}-1\\1\\4\end{pmatrix} - \begin{pmatrix}2\\-1\\5\end{pmatrix} = \begin{pmatrix}-3\\2\\-1\end{pmatrix}$ or $\overrightarrow{DA} = \begin{pmatrix}3\\-2\\1\end{pmatrix}$		M1
	$\cos\theta = \dfrac{\overrightarrow{AB} \bullet \overrightarrow{AD}}{\left\lvert\overrightarrow{AB}\right\rvert . \left\lvert\overrightarrow{AD}\right\rvert} = \dfrac{\begin{pmatrix}3\\3\\5\end{pmatrix} \bullet \begin{pmatrix}-3\\2\\-1\end{pmatrix}}{\sqrt{(3)^2 + (3)^2 + (5)^2} . \sqrt{(-3)^2 + (2)^2 + (-1)^2}}$	Applies dot product formula between their $\left(\overrightarrow{AB}\text{ or }\overrightarrow{BA}\right)$ and their $\left(\overrightarrow{AD}\text{ or }\overrightarrow{DA}\right)$.	M1
	$\cos\theta = \pm\left(\dfrac{-9 + 6 - 5}{\sqrt{(3)^2 + (3)^2 + (5)^2} . \sqrt{(-3)^2 + (2)^2 + (-1)^2}}\right)$	Correct followed through expression or **equation**.	A1 $\sqrt{}$
	$\cos\theta = \dfrac{-8}{\sqrt{43}.\sqrt{14}} \Rightarrow \theta = 109.029544... = 109 \text{ (nearest }^\circ)$	awrt 109	A1 **cso AG** **[4]**
(d)	$\overrightarrow{OC} = \overrightarrow{OD} + \overrightarrow{DC} = \overrightarrow{OD} + \overrightarrow{AB} = (-\mathbf{i} + \mathbf{j} + 4\mathbf{k}) + (3\mathbf{i} + 3\mathbf{j} + 5\mathbf{k})$ $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} = \overrightarrow{OB} + \overrightarrow{AD} = (5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}) + (-3\mathbf{i} + 2\mathbf{j} - \mathbf{k})$		M1
	So, $\overrightarrow{OC} = 2\mathbf{i} + 4\mathbf{j} + 9\mathbf{k}$		A1 **[2]**
(e)	Area $ABCD = \left(\frac{1}{2}(\sqrt{43})(\sqrt{14})\sin 109^\circ\right); \times 2 = 23.19894905$	awrt 23.2	M1; dM1 A1 **[3]**
(f)	$\dfrac{d}{\sqrt{14}} = \sin 71$ or $\sqrt{43}\,d = 23.19894905...$		M1
	$\therefore\ d = \sqrt{14}\sin 71^\circ = 3.537806563...$	awrt 3.54	A1 **[2]** **15**

7. (a)	**M1:** Finding the difference between $\overrightarrow{OB}$ and $\overrightarrow{OA}$. Can be implied by two out of three components correct in $3\mathbf{i}+3\mathbf{j}+5\mathbf{k}$ or $-3\mathbf{i}-3\mathbf{j}-5\mathbf{k}$ **A1:** $3\mathbf{i}+3\mathbf{j}+5\mathbf{k}$
(b)	**M1:** An expression of the form $(\text{3 component vector})\pm\lambda(\text{3 component vector})$ **A1ft:** $\mathbf{r}=\overrightarrow{OA}+\lambda(\text{their }\pm\overrightarrow{AB})$ or $\mathbf{r}=\overrightarrow{OB}+\lambda(\text{their }\pm\overrightarrow{AB})$. **Note:** Candidate must begin writing their line as $\mathbf{r}=$ or $l=\dots$ or $\begin{pmatrix}x\\y\\z\end{pmatrix}=\dots$ So, Line = ... would be A0.
(c)	**M1:** An attempt to find either the vector $\overrightarrow{AD}$ or $\overrightarrow{DA}$. Can be implied by two out of three components correct in $-3\mathbf{i}+2\mathbf{j}-\mathbf{k}$ or $3\mathbf{i}-2\mathbf{j}+\mathbf{k}$, respectively. **M1:** Applies dot product formula between their $(\overrightarrow{AB}\text{ or }\overrightarrow{BA})$ and their $(\overrightarrow{AD}\text{ or }\overrightarrow{DA})$. **A1ft:** Correct followed through expression or **equation**. The dot product must be correctly followed through correctly and the square roots although they can be un-simplified must be followed through correctly. **A1:** Obtains an angle of awrt 109 ***by correct solution only***. Award the final A1 mark if candidate achieves awrt 109 by either taking the dot product between: (i) $\begin{pmatrix}3\\3\\5\end{pmatrix}$ and $\begin{pmatrix}-3\\2\\-1\end{pmatrix}$ or (ii) $\begin{pmatrix}-3\\-3\\-5\end{pmatrix}$ and $\begin{pmatrix}3\\-2\\1\end{pmatrix}$. Ignore if any of these vectors are labelled incorrectly. Award A0, cso for those candidates who take the dot product between: (iii) $\begin{pmatrix}-3\\-3\\-5\end{pmatrix}$ and $\begin{pmatrix}-3\\2\\-1\end{pmatrix}$ or (iv) $\begin{pmatrix}3\\3\\5\end{pmatrix}$ and $\begin{pmatrix}3\\-2\\1\end{pmatrix}$. They will usually find awrt 71 and apply 180 – awrt 71 to give awrt 109. If these candidates give a convincing detailed explanation which must include reference to the direction of their vectors then this can be given A1 cso. If still in doubt, here, send to review.
(d)	**M1:** Applies either $\overrightarrow{OD}$ + their $\overrightarrow{AB}$ or $\overrightarrow{OB}$ + their $\overrightarrow{AD}$. This mark can be implied by two out of three correctly followed through components in their $\overrightarrow{OD}$. **A1:** For $2\mathbf{i}+4\mathbf{j}+9\mathbf{k}$.
(e)	**M1:** $\frac{1}{2}(\text{their }AB)(\text{their }CB)\sin(\text{their }109^\circ\text{ or }71^\circ\text{ from (b)})$. Awrt 11.6 will usually imply this mark. **dM1:** Multiplies this by 2 for the parallelogram. Can be implied. **Note:** $\frac{1}{2}((\text{their }AB+\text{their }AB))(\text{their }CB)\sin(\text{their }109^\circ\text{ or }71^\circ\text{ from (b)})$ **A1:** awrt 23.2
(f)	**M1:** $\dfrac{d}{\text{their }AD}=\sin(\text{their }109^\circ\text{ or }71^\circ\text{ from (b)})$ or $(\text{their }AB)\,d=(\text{their Area }ABCD)$ Award M0 for $(\text{their }AB)$ in part (f), if the area of their parallelogram in part (e) is $(\text{their }AB)(\text{their }CB)$. Award M0 for $\dfrac{d}{\text{their }\sqrt{43}}=\sin 71$ or $(\text{their }\sqrt{14})\,d=23.19894905\dots$ **A1:** awrt 3.54 **Note: Some candidates will use their answer to part (f) in order to answer part (e).**

7.	*Alternative method for part (c): Applying the cosine rule:*	
	$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA} = \begin{pmatrix} -1 \\ 1 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}$ or $\overrightarrow{DA} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$ $\overrightarrow{DB} = \overrightarrow{OD} - \overrightarrow{OA} = \begin{pmatrix} 5 \\ 2 \\ 10 \end{pmatrix} - \begin{pmatrix} -1 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \\ 6 \end{pmatrix}$ or $\overrightarrow{BD} = \begin{pmatrix} -6 \\ -1 \\ -6 \end{pmatrix}$	**M1:** as above.
	So $\left\lvert\overrightarrow{AB}\right\rvert = \sqrt{43}$, $\left\lvert\overrightarrow{AD}\right\rvert = \sqrt{14}$ and $\left\lvert\overrightarrow{DB}\right\rvert = \sqrt{73}$	
	$\cos\theta = \dfrac{\left(\sqrt{43}\right)^2 + \left(\sqrt{14}\right)^2 - \left(\sqrt{73}\right)^2}{2\sqrt{43}.\sqrt{14}}$	**M1:** Cosine rule structure of $\cos\theta = \dfrac{a^2+b^2-c^2}{2ab}$ assigned each of $\left\lvert\overrightarrow{AB}\right\rvert$, $\left\lvert\overrightarrow{AD}\right\rvert$ and $\left\lvert\overrightarrow{DB}\right\rvert$ in any order as their *a*, *b* and *c*. **A1:** Correct application of cosine rule.
	$\left\{\cos\theta = \dfrac{-16}{2\sqrt{43}.\sqrt{14}} \Rightarrow \theta = 109.029544...\right\} = 109$ (nearest °)	A1: awrt 109 (no errors seen). **AG**
	Alternative method for part (d):	
	$\overrightarrow{OE} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + \lambda\begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix}$	
	$\overrightarrow{DE} = \begin{pmatrix} 2+3\lambda \\ -1+3\lambda \\ 5+5\lambda \end{pmatrix} - \begin{pmatrix} -1 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 3+3\lambda \\ -2+3\lambda \\ 1+5\lambda \end{pmatrix}$	
	$\overrightarrow{DE} \bullet \overrightarrow{AB} = 0 \Rightarrow \begin{pmatrix} 3+3\lambda \\ -2+3\lambda \\ 1+5\lambda \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix} = 0$ $9 + 9\lambda - 6 + 9\lambda + 5 + 3\lambda = 0 \Rightarrow \lambda = -\dfrac{8}{43}$	**M1:** Takes the dot product between $\overrightarrow{DE}$ and $\overrightarrow{AB}$ and progresses to find a value of λ
	$\overrightarrow{DE} = \begin{pmatrix} 2+3\lambda \\ -1+3\lambda \\ 5+5\lambda \end{pmatrix} - \begin{pmatrix} -1 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} \frac{103}{43} \\ -\frac{110}{43} \\ \frac{3}{43} \end{pmatrix}$	**dM1:** Uses their value of λ to find $\overrightarrow{DE}$
	Length DE = 3.537806563...	**A1:** awrt 3.54

Question Number	Scheme		Marks
8. (a)	$1 = A(5-P) + BP$	Can be implied.	M1
	$A = \frac{1}{5},\ B = \frac{1}{5}$	Either one.	A1
	giving $\dfrac{\frac{1}{5}}{P} + \dfrac{\frac{1}{5}}{(5-P)}$	See notes.	A1 **cao, aef**
			[3]
(b)	$\displaystyle\int \frac{1}{P(5-P)}\,\mathrm{d}P = \int \frac{1}{15}\,\mathrm{d}t$		B1
	$\frac{1}{5}\ln P - \frac{1}{5}\ln(5-P) = \frac{1}{15}t\ \ (+c)$		M1* A1ft
	$\{t=0,\ P=1 \Rightarrow\}\ \ \frac{1}{5}\ln 1 - \frac{1}{5}\ln(4) = 0 + c\ \ \left\{\Rightarrow\ c = -\frac{1}{5}\ln 4\right\}$		dM1*
	eg: $\frac{1}{5}\ln\left(\frac{P}{5-P}\right) = \frac{1}{15}t - \frac{1}{5}\ln 4$	Using any of the subtraction (or addition) laws for logarithms CORRECTLY	dM1*
	$\ln\left(\frac{4P}{5-P}\right) = \frac{1}{3}t$		
	eg: $\frac{4P}{5-P} = \mathrm{e}^{\frac{1}{3}t}$ or eg: $\frac{5-P}{4P} = \mathrm{e}^{-\frac{1}{3}t}$	Eliminate ln's correctly.	dM1*
	gives $4P = 5\mathrm{e}^{\frac{1}{3}t} - P\mathrm{e}^{\frac{1}{3}t} \Rightarrow P(4+\mathrm{e}^{\frac{1}{3}t}) = 5\mathrm{e}^{\frac{1}{3}t}$		
	$P = \dfrac{5\mathrm{e}^{\frac{1}{3}t}}{(4+\mathrm{e}^{\frac{1}{3}t})}\ \ \left\{\dfrac{(\div\,\mathrm{e}^{\frac{1}{3}t})}{(\div\,\mathrm{e}^{\frac{1}{3}t})}\right\}$	Make P the subject.	dM1*
	$P = \dfrac{5}{(1+4\mathrm{e}^{-\frac{1}{3}t})}$ or $P = \dfrac{25}{(5+20\mathrm{e}^{-\frac{1}{3}t})}$ etc.		A1
			[8]
(c)	$1 + 4\mathrm{e}^{-\frac{1}{3}t} > 1 \Rightarrow P < 5$. So population cannot exceed 5000.		B1
			[1] **12**
(a)	**M1:** Forming a correct identity. For example, $1 = A(5-P) + BP$. **Note** A and B not referred to in question. **A1:** Either one of $A = \frac{1}{5}$ or $B = \frac{1}{5}$. **A1:** $\dfrac{\frac{1}{5}}{P} + \dfrac{\frac{1}{5}}{(5-P)}$ or any equivalent form, eg: $\dfrac{1}{5P} + \dfrac{1}{25-5P}$, etc. Ignore subsequent working. This answer must be stated in part (a) only. A1 can also be given for a candidate who finds both $A = \frac{1}{5}$ and $B = \frac{1}{5}$ and $\dfrac{A}{P} + \dfrac{B}{5-P}$ is seen in their working. Candidate can use 'cover-up' rule to write down $\dfrac{\frac{1}{5}}{P} + \dfrac{\frac{1}{5}}{(5-P)}$, as so gain all three marks. Candidate cannot gain the marks for part (a) in part (b).		

8. (b)	**B1:** Separates variables as shown. dP and dt should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs. **M1*: Both** $\pm\lambda\ln P$ and $\pm\mu\ln(\pm 5\pm P)$, where λ and μ are constants. **Or** $\pm\lambda\ln mP$ and $\pm\mu\ln(n(\pm 5\pm P))$, where λ, μ, m and n are constants. **A1ft:** Correct follow through integration of both sides from their $\int \frac{\lambda}{P}+\frac{\mu}{(5-P)}\,\mathrm{d}P = \int K\,\mathrm{d}t$ with or without $+c$ **dM1*:** Use of $t=0$ and $P=1$ in an integrated equation containing c **dM1*:** Using ANY of the subtraction (or addition) laws for logarithms CORRECTLY. **dM1*:** Apply logarithms (or take exponentials) to eliminate ln's CORRECTLY from their equation. **dM1*:** A full ACCEPTABLE method of rearranging to make P the subject. **(See below for examples!)** **A1:** $P = \frac{5}{(1+4\mathrm{e}^{-\frac{1}{3}t})}$ $\{$where $a=5, b=1, c=4\}$. Also allow any "integer" multiples of this expression. For example: $P = \frac{25}{(5+20\mathrm{e}^{-\frac{1}{3}t})}$ **Note: If the first method mark (M1*) is not awarded then the candidate cannot gain any of the six remaining marks for this part of the question.** **Note:** $\int \frac{1}{P(5-P)}\,\mathrm{d}P = \int 15\,\mathrm{d}t \Rightarrow \int \frac{\frac{1}{5}}{P}+\frac{\frac{1}{5}}{(5-P)}\,\mathrm{d}P = \int 15\,\mathrm{d}t \Rightarrow \ln P - \ln(5-P) = 15t$ is B0M1A1ft. ***<u>dM1* for making P the subject</u>*** **Note there are three type of manipulations here which are considered acceptable to make P the subject.** **(1)** M1 for $\frac{P}{5-P} = \mathrm{e}^{\frac{1}{3}t} \Rightarrow P = 5\mathrm{e}^{\frac{1}{3}t} - P\mathrm{e}^{\frac{1}{3}t} \Rightarrow P(1+\mathrm{e}^{\frac{1}{3}t}) = 5\mathrm{e}^{\frac{1}{3}t} \Rightarrow P = \frac{5}{(1+\mathrm{e}^{-\frac{1}{3}t})}$ **(2)** M1 for $\frac{P}{5-P} = \mathrm{e}^{\frac{1}{3}t} \Rightarrow \frac{5-P}{P} = \mathrm{e}^{\frac{1}{3}t} \Rightarrow \frac{5}{P} - 1 = \mathrm{e}^{\frac{1}{3}t} \Rightarrow \frac{5}{P} = \mathrm{e}^{\frac{1}{3}t} + 1 \Rightarrow P = \frac{5}{(1+\mathrm{e}^{\frac{1}{3}t})}$ **(3)** M1 for $P(5-P) = 4\mathrm{e}^{\frac{1}{3}t} \Rightarrow P^2 - 5P = -4\mathrm{e}^{\frac{1}{3}t} \Rightarrow \left(P-\frac{5}{2}\right)^2 - \frac{25}{4} = -4\mathrm{e}^{\frac{1}{3}t}$ leading to $P = \ldots$ **Note**: The incorrect manipulation of $\frac{P}{5-P} = \frac{P}{5} - 1$ or equivalent is awarded this dM0*. **Note:** $(P) - (5-P) = \mathrm{e}^{\frac{1}{3}t} \Rightarrow 2P - 5 = \frac{1}{3}t$ leading to $P = \ldots$ or equivalent is awarded this dM0*
(c)	**B1:** $1+4\mathrm{e}^{-\frac{1}{3}t} > 1$ **and** $P < 5$ **and** a conclusion relating population (or even P) or meerkats to 5000. **For** $P = \frac{25}{(5+20\mathrm{e}^{-\frac{1}{3}t})}$, B1 can be awarded for $5+20\mathrm{e}^{-\frac{1}{3}t} > 5$ **and** $P < 5$ **and** a conclusion relating population (or even P) or meerkats to 5000. B1 can only be obtained if candidates have correct values of a and b in their $P = \frac{a}{(b+c\mathrm{e}^{-\frac{1}{3}t})}$. **Award B0 for:** As $t\to\infty$, $\mathrm{e}^{-\frac{1}{3}t} \to 0$. So $P \to \frac{5}{(1+0)} = 5$, so population cannot exceed 5000, **unless** the candidate also proves that $P = \frac{5}{(1+4\mathrm{e}^{-\frac{1}{3}t})}$ oe. is an increasing function. **If unsure here, then send to review!**

8.	<u>*Alternative method for part (b)*</u>
	B1M1*A1: as before for $\frac{1}{5}\ln P - \frac{1}{5}\ln(5-P) = \frac{1}{15}t \ \ (+c)$
	Award 3rd M1for $\ln\left(\frac{P}{5-P}\right) = \frac{1}{3}t + c$
	Award 4th M1 for $\frac{P}{5-P} = Ae^{\frac{1}{3}t}$
	Award 2nd M1 for $t=0,\ P=1 \Rightarrow \frac{1}{5-1} = Ae^{0} \ \left\{\Rightarrow \ A = \frac{1}{4}\right\}$
	$\frac{P}{5-P} = \frac{1}{4}e^{\frac{1}{3}t}$
	then award the final M1A1 in the same way.

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Mark Scheme (Results)

Summer 2012

GCE Core Mathematics C4
(6666) Paper 1

PEARSON

June 2012
6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme	Marks
1.	(a) $1 = A(3x-1)^2 + Bx(3x-1) + Cx$	B1
	$x \to 0$ $\quad (1 = A)$	M1
	$x \to \frac{1}{3}$ $\quad 1 = \frac{1}{3}C \Rightarrow C = 3$ any two constants correct	A1
	Coefficients of x^2	
	$0 = 9A + 3B \Rightarrow B = -3$ all three constants correct	A1 **(4)**
	(b)(i) $\int \left(\frac{1}{x} - \frac{3}{3x-1} + \frac{3}{(3x-1)^2} \right) dx$	
	$= \ln x - \frac{3}{3}\ln(3x-1) + \frac{3}{(-1)3}(3x-1)^{-1} \quad (+C)$	M1 A1ft A1ft
	$\left(= \ln x - \ln(3x-1) - \frac{1}{3x-1} \quad (+C) \right)$	
	(ii) $\int_1^2 f(x)\,dx = \left[\ln x - \ln(3x-1) - \frac{1}{3x-1} \right]_1^2$	
	$= \left(\ln 2 - \ln 5 - \frac{1}{5} \right) - \left(\ln 1 - \ln 2 - \frac{1}{2} \right)$	M1
	$= \ln \frac{2 \times 2}{5} + \ \ldots$	M1
	$= \frac{3}{10} + \ln\left(\frac{4}{5}\right)$	A1 **(6)**
		[10]

Question Number	Scheme		Marks	
2.	(a) $V = x^3 \Rightarrow \frac{dV}{dx} = 3x^2$ *	cso	B1	**(1)**
	(b) $\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt} = \frac{0.048}{3x^2}$		M1	
	At $x = 8$ $\frac{dx}{dt} = \frac{0.048}{3(8^2)} = 0.00025$ $\left(\text{cm s}^{-1}\right)$	2.5×10^{-4}	A1	**(2)**
	(c) $S = 6x^2 \Rightarrow \frac{dS}{dx} = 12x$		B1	
	$\frac{dS}{dt} = \frac{dS}{dx} \times \frac{dx}{dt} = 12x\left(\frac{0.048}{3x^2}\right)$		M1	
	At $x = 8$ $\frac{dS}{dt} = 0.024$ $\left(\text{cm}^2\,\text{s}^{-1}\right)$		A1	**(3)**
				[6]

Question Number	Scheme	Marks
3.	(a) $\mathrm{f}(x) = \ldots(\ldots - \ldots x)^{-\frac{1}{2}}$	M1
	$= 6 \times 9^{-\frac{1}{2}}(\ldots)$ $\quad \dfrac{6}{9^{\frac{1}{2}}}, \dfrac{6}{3}$, 2 or equivalent	B1
	$= \ldots\left(1 + \left(-\tfrac{1}{2}\right)(kx); + \dfrac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}(kx)^2 + \dfrac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!}(kx)^3 + \ldots\right)$	M1; A1ft
	$= 2\left(1 + \dfrac{2}{9}x + \ldots\right)$ $\quad$ or $2 + \dfrac{4}{9}x$	A1
	$= 2 + \dfrac{4}{9}x + \dfrac{4}{27}x^2 + \dfrac{40}{729}x^3 + \ldots$	A1 **(6)**
	(b) $\mathrm{g}(x) = 2 - \dfrac{4}{9}x + \dfrac{4}{27}x^2 - \dfrac{40}{729}x^3 + \ldots$	B1ft **(1)**
	(c) $\mathrm{h}(x) = 2 + \dfrac{4}{9}(2x) + \dfrac{4}{27}(2x)^2 + \dfrac{40}{729}(2x)^3 + \ldots$	M1 A1 **(2)**
	$\left(= 2 + \dfrac{8}{9}x + \dfrac{16}{27}x^2 + \dfrac{320}{729}x^3 + \ldots\right)$	**[9]**

Question Number	Scheme		Marks
4.	$\int y\,\mathrm{d}y = \int \frac{3}{\cos^2 x}\mathrm{d}x$ $= \int 3\sec^2 x\,\mathrm{d}x$	Can be implied. Ignore integral signs	B1
	$\frac{1}{2}y^2 = 3\tan x \quad (+C)$		M1 A1
	$y = 2,\ x = \frac{\pi}{4}$ $\frac{1}{2}2^2 = 3\tan\frac{\pi}{4} + C$		M1
	Leading to $C = -1$ $\frac{1}{2}y^2 = 3\tan x - 1$	or equivalent	A1 **(5)**
			[5]

Question Number	Scheme	Marks
5.	(a) Differentiating implicitly to obtain $\pm ay^2\frac{\mathrm{d}y}{\mathrm{d}x}$ and/or $\pm bx^2\frac{\mathrm{d}y}{\mathrm{d}x}$	M1
	$48y^2\frac{\mathrm{d}y}{\mathrm{d}x} + \ \dots\ -54\ \dots$	A1
	$9x^2y \to 9x^2\frac{\mathrm{d}y}{\mathrm{d}x} + 18xy$ or equivalent	B1
	$\left(48y^2+9x^2\right)\frac{\mathrm{d}y}{\mathrm{d}x} + 18xy - 54 = 0$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{54-18xy}{48y^2+9x^2} \quad \left(= \frac{18-6xy}{16y^2+3x^2}\right)$	A1 **(5)**
	(b) $18-6xy=0$	M1
	Using $x=\frac{3}{y}$ or $y=\frac{3}{x}$	
	$16y^3+9\left(\frac{3}{y}\right)^2 y - 54\left(\frac{3}{y}\right)=0$ or $16\left(\frac{3}{x}\right)^3+9x^2\left(\frac{3}{x}\right)-54x=0$	M1
	Leading to	
	$16y^4+81-162=0$ or $16+x^4-2x^4=0$	M1
	$y^4=\frac{81}{16}$ or $x^4=16$	
	$y=\frac{3}{2}, -\frac{3}{2}$ or $x=2, -2$	A1 A1
	Substituting either of their values into $xy=3$ to obtain a value of the other variable.	M1
	$\left(2, \frac{3}{2}\right), \left(-2, -\frac{3}{2}\right)$ both	A1 **(7)**
		[12]

Question Number	Scheme	Marks
6.	(a) $\dfrac{dx}{dt} = 2\sqrt{3}\cos 2t$	B1
	$\dfrac{dy}{dt} = -8\cos t \sin t$	M1 A1
	$\dfrac{dy}{dx} = \dfrac{-8\cos t\sin t}{2\sqrt{3}\cos 2t}$	M1
	$= -\dfrac{4\sin 2t}{2\sqrt{3}\cos 2t}$	
	$\dfrac{dy}{dx} = -\dfrac{2}{3}\sqrt{3}\tan 2t \qquad \left(k = -\dfrac{2}{3}\right)$	A1 **(5)**
	(b) When $t = \dfrac{\pi}{3}$ $\quad x = \dfrac{3}{2},\ y = 1$ can be implied	B1
	$m = -\dfrac{2}{3}\sqrt{3}\tan\left(\dfrac{2\pi}{3}\right) \quad (= 2)$	M1
	$y - 1 = 2\left(x - \dfrac{3}{2}\right)$	M1
	$y = 2x - 2$	A1 **(4)**
	(c) $x = \sqrt{3}\sin 2t = \sqrt{3} \times 2\sin t\cos t$	M1
	$x^2 = 12\sin^2 t\cos^2 t = 12\left(1 - \cos^2 t\right)\cos^2 t$	
	$x^2 = 12\left(1 - \dfrac{y}{4}\right)\dfrac{y}{4}$ or equivalent	M1 A1 **(3)**
		[12]
	Alternative to (c)	
	$y = 2\cos 2t + 2$	M1
	$\sin^2 2t + \cos^2 2t = 1$	
	$\dfrac{x^2}{3} + \dfrac{(y-2)^2}{4} = 1$	M1 A1 **(3)**

Question Number	Scheme	Marks
7.	(a)	
		x	1	2	3	4
y	ln2	$\sqrt{2}\ln 4$	$\sqrt{3}\ln 6$	2ln8
	0.6931	1.9605	3.1034	4.1589		M1
	Area $=\frac{1}{2}\times 1(\ \dots)$	B1
	$\approx\ \dots\ \left(0.6931+2\left(1.9605+3.1034\right)+4.1589\right)$	M1
	$\approx\frac{1}{2}\times 14.97989\ \dots\ \approx 7.49$ 7.49 cao	A1 **(4)**
	(b) $\int x^{\frac{1}{2}}\ln 2x\,dx=\frac{2}{3}x^{\frac{3}{2}}\ln 2x-\int\frac{2}{3}x^{\frac{3}{2}}\times\frac{1}{x}\,dx$	M1 A1
	$=\frac{2}{3}x^{\frac{3}{2}}\ln 2x-\int\frac{2}{3}x^{\frac{1}{2}}\,dx$	
	$=\frac{2}{3}x^{\frac{3}{2}}\ln 2x-\frac{4}{9}x^{\frac{3}{2}}\quad(+C)$	M1 A1 **(4)**
	(c) $\left[\frac{2}{3}x^{\frac{3}{2}}\ln 2x-\frac{4}{9}x^{\frac{3}{2}}\right]_1^4=\left(\frac{2}{3}4^{\frac{3}{2}}\ln 8-\frac{4}{9}4^{\frac{3}{2}}\right)-\left(\frac{2}{3}\ln 2-\frac{4}{9}\right)$	M1
	$=\left(16\ln 2-\ \dots\right)-\ \dots$ Using or implying $\ln 2^n=n\ln 2$	M1
	$=\frac{46}{3}\ln 2-\frac{28}{9}$	A1 **(3)**
		[11]

Question Number	Scheme	Marks
8.	(a) $\overrightarrow{AB} = \begin{pmatrix}8\\3\\4\end{pmatrix} - \begin{pmatrix}10\\2\\3\end{pmatrix} = \begin{pmatrix}-2\\1\\1\end{pmatrix}$	M1 A1 **(2)**
	(b) $\mathbf{r} = \begin{pmatrix}10\\2\\3\end{pmatrix} + t\begin{pmatrix}-2\\1\\1\end{pmatrix}$ $\mathbf{r} = \begin{pmatrix}8\\3\\4\end{pmatrix} + t\begin{pmatrix}-2\\1\\1\end{pmatrix}$	M1 A1ft **(2)**
	(c) $\overrightarrow{CP} = \begin{pmatrix}10-2t\\2+t\\3+t\end{pmatrix} - \begin{pmatrix}3\\12\\3\end{pmatrix} = \begin{pmatrix}7-2t\\t-10\\t\end{pmatrix}$	M1 A1
	$\begin{pmatrix}7-2t\\t-10\\t\end{pmatrix} . \begin{pmatrix}-2\\1\\1\end{pmatrix} = -14+4t+t-10+t=0$	M1
	Leading to $t=4$	A1
	Position vector of P is $\begin{pmatrix}10-8\\2+4\\3+4\end{pmatrix} = \begin{pmatrix}2\\6\\7\end{pmatrix}$	M1 A1 **(6)**
		[10]
	Alternative working for (c)	
	$\overrightarrow{CP} = \begin{pmatrix}8-2t\\3+t\\4+t\end{pmatrix} - \begin{pmatrix}3\\12\\3\end{pmatrix} = \begin{pmatrix}5-2t\\t-9\\t+1\end{pmatrix}$	M1 A1
	$\begin{pmatrix}5-2t\\t-9\\t+1\end{pmatrix} . \begin{pmatrix}-2\\1\\1\end{pmatrix} = -10+4t+t-9+t+1=0$	M1
	Leading to $t=3$	A1
	Position vector of P is $\begin{pmatrix}8-6\\3+3\\4+3\end{pmatrix} = \begin{pmatrix}2\\6\\7\end{pmatrix}$	M1 A1 **(6)**

Mark Scheme (Results)

January 2013

GCE Mathematics
6666 Core Mathematics 4

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January 2013
6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme	Marks
1.	$(2+3x)^{-3} = \underline{(2)^{-3}}\left(1+\frac{3x}{2}\right)^{-3} = \underline{\frac{1}{8}}\left(1+\frac{3x}{2}\right)^{-3}$ $\underline{(2)^{-3}}$ or $\underline{\frac{1}{8}}$	<u>B1</u>
	$= \left\{\frac{1}{8}\right\}\left[1+(-3)(kx)+\frac{(-3)(-4)}{2!}(kx)^2+\frac{(-3)(-4)(-5)}{3!}(kx)^3+\ldots\right]$ see notes	M1 A1
	$= \left\{\frac{1}{8}\right\}\left[\underline{1+(-3)\left(\frac{3x}{2}\right)+\frac{(-3)(-4)}{2!}\left(\frac{3x}{2}\right)^2+\frac{(-3)(-4)(-5)}{3!}\left(\frac{3x}{2}\right)^3+\ldots}\right]$	
	$= \frac{1}{8}\left[1-\frac{9}{2}x;+\frac{27}{2}x^2-\frac{135}{4}x^3+\ldots\right]$ See notes below!	
	$= \frac{1}{8}-\frac{9}{16}x;+\frac{27}{16}x^2-\frac{135}{32}x^3+\ldots$	A1; A1
		[5] **5**
	<u>B1</u>: $\underline{(2)^{-3}}$ or $\underline{\frac{1}{8}}$ outside brackets or $\underline{\frac{1}{8}}$ as constant term in the binomial expansion.	
	M1: Expands $(\ldots+kx)^{-3}$ to give any 2 terms out of 4 terms simplified or un-simplified, Eg: $1+(-3)(kx)$ or $(-3)(kx)+\frac{(-3)(-4)}{2!}(kx)^2$ or $1+\ldots\ldots+\frac{(-3)(-4)}{2!}(kx)^2$ or $\frac{(-3)(-4)}{2!}(kx)^2+\frac{(-3)(-4)(-5)}{3!}(kx)^3$ **where** $k\neq 1$ are ok for M1.	
	A1: A correct simplified or un-simplified $1+(-3)(kx)+\frac{(-3)(-4)}{2!}(kx)^2+\frac{(-3)(-4)(-5)}{3!}(kx)^3$ expansion with consistent (kx) where $k\neq 1$.	
	"Incorrect bracketing" $\left\{\frac{1}{8}\right\}\left[\underline{1+(-3)\left(\frac{3x}{2}\right)+\frac{(-3)(-4)}{2!}\left(\frac{3x^2}{2}\right)+\frac{(-3)(-4)(-5)}{3!}\left(\frac{3x^3}{2}\right)+\ldots}\right]$ is M1A0 unless recovered.	
	A1: For $\frac{1}{8}-\frac{9}{16}x$ **(simplified fractions)** or also allow $0.125-0.5625x$. **Allow Special Case A1 for either SC:** $\frac{1}{8}\left[1-\frac{9}{2}x;\ldots\right]$ or **SC:** $K\left[1-\frac{9}{2}x+\frac{27}{2}x^2-\frac{135}{4}x^3+\ldots\right]$ (where K can be 1 or omitted), with each term in the $[\ldots\ldots\ldots]$ either a simplified fraction or a decimal.	
	A1: Accept only $\frac{27}{16}x^2-\frac{135}{32}x^3$ or $1\frac{11}{16}x^2-4\frac{7}{32}x^3$ or $1.6875x^2-4.21875x^3$	

1. ctd	Candidates who write $= \frac{1}{8}\left[1 + (-3)\left(-\frac{3x}{2}\right) + \frac{(-3)(-4)}{2!}\left(-\frac{3x}{2}\right)^2 + \frac{(-3)(-4)(-5)}{3!}\left(-\frac{3x}{2}\right)^3 + \ldots \right]$ where $k = -\frac{3}{2}$ and not $\frac{3}{2}$ and achieve $\frac{1}{8} + \frac{9}{16}x + \frac{27}{16}x^2 + \frac{135}{32}x^3 + \ldots$ will get B1M1A1A0A0. ***<u>Alternative method:</u>*** Candidates can apply an alternative form of the binomial expansion. $(2+3x)^{-3} = (2)^{-3} + (-3)(2)^{-4}(3x) + \frac{(-3)(-4)}{2!}(2)^{-5}(3x)^2 + \frac{(-3)(-4)(-5)}{3!}(2)^{-6}(3x)^3$ **B1:** $\frac{1}{8}$ or $(2)^{-3}$ **M1:** Any two of four (un-simplified) terms correct. **A1:** All four (un-simplified) terms correct. **A1:** $\frac{1}{8} - \frac{9}{16}x$ **A1:** $+\frac{27}{16}x^2 - \frac{135}{32}x^3$ **Note:** The terms in C need to be evaluated, so ${}^{-3}C_0(2)^{-3} + {}^{-3}C_1(2)^{-4}(3x) + {}^{-3}C_2(2)^{-5}(3x)^2 + {}^{-3}C_3(2)^{-6}(3x)^3$ without further working is B0M0A0.

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Question Number	Scheme	
2. (a)	$\int \frac{1}{x^3}\ln x \, \mathrm{d}x$, $\left\{\begin{array}{l} u = \ln x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{x} \\ \frac{\mathrm{d}v}{\mathrm{d}x} = x^{-3} \Rightarrow v = \frac{x^{-2}}{-2} = \frac{-1}{2x^2} \end{array}\right\}$	
	In the form $\frac{\pm\lambda}{x^2}\ln x \pm \int \mu \frac{1}{x^2}.\frac{1}{x}$	M1
	$= \underline{\frac{-1}{2x^2}\ln x} - \underline{\underline{\int \frac{-1}{2x^2}.\frac{1}{x}\,\mathrm{d}x}}$ $\quad$ $\underline{\frac{-1}{2x^2}\ln x}$ simplified or un-simplified.	$\underline{\text{A1}}$
	$\underline{\underline{-\int \frac{-1}{2x^2}.\frac{1}{x}}}$ simplified or un-simplified.	$\underline{\underline{\text{A1}}}$
	$\left\{= \frac{-1}{2x^2}\ln x + \frac{1}{2}\int \frac{1}{x^3}\,\mathrm{d}x\right\}$	
	$= -\frac{1}{2x^2}\ln x + \frac{1}{2}\left(-\frac{1}{2x^2}\right)\ \{+c\}$ $\quad$ $\pm\int \mu \frac{1}{x^2}.\frac{1}{x} \to \pm\beta x^{-2}.$	dM1
	Correct answer, with/without $+c$	A1
		[5]
(b)	$\left\{\left[-\frac{1}{2x^2}\ln x - \frac{1}{4x^2}\right]_1^2\right\} = \left(-\frac{1}{2(2)^2}\ln 2 - \frac{1}{4(2)^2}\right) - \left(-\frac{1}{2(1)^2}\ln 1 - \frac{1}{4(1)^2}\right)$ $\quad$ Applies limits of 2 and 1 to their part (a) answer and subtracts the correct way round.	M1
	$= \frac{3}{16} - \frac{1}{8}\ln 2$ or $\frac{3}{16} - \ln 2^{\frac{1}{8}}$ or $\frac{1}{16}(3 - 2\ln 2)$, etc, or awrt 0.1 $\quad$ or equivalent.	A1
		[2] **7**
(a)	**M1**: Integration by parts is applied in the form $\frac{\pm\lambda}{x^2}\ln x \pm \int \mu \frac{1}{x^2}.\frac{1}{x}$ or equivalent. $\underline{\textbf{A1}}$: $\underline{\frac{-1}{2x^2}\ln x}$ simplified or un-simplified. $\underline{\underline{\textbf{A1}}}$: $\underline{\underline{-\int \frac{-1}{2x^2}.\frac{1}{x}}}$ or equivalent. You can ignore the $\mathrm{d}x$. **dM1:** Depends on the previous M1. $\pm\int \mu \frac{1}{x^2}.\frac{1}{x} \to \pm\beta x^{-2}.$ **A1:** $-\frac{1}{2x^2}\ln x + \frac{1}{2}\left(-\frac{1}{2x^2}\right)\ \{+c\}$ or $= -\frac{1}{2x^2}\ln x - \frac{1}{4x^2}\ \{+c\}$ or $\frac{x^{-2}}{-2}\ln x - \frac{x^{-2}}{4}\ \{+c\}$ or $\frac{-1-2\ln x}{4x^2}\ \{+c\}$ or equivalent. You can ignore subsequent working after a correct stated answer.	
(b)	**M1:** Some evidence of applying limits of 2 and 1 to their part (a) answer and subtracts the correct way round. **A1:** ***Two term exact answer*** of either $\frac{3}{16} - \frac{1}{8}\ln 2$ or $\frac{3}{16} - \ln 2^{\frac{1}{8}}$ or $\frac{1}{16}(3 - 2\ln 2)$ or $\frac{\ln\left(\frac{1}{4}\right) + 3}{16}$ or $0.1875 - 0.125\ln 2$. Also allow awrt 0.1. Also note the fraction terms must be combined. **Note:** Award the final A0 in part (b) for a candidate who achieves awrt 0.1 in part (b), when their answer to part (a) is incorrect.	

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2. (b) ctd	**Note:** Decimal answer is 0.100856... in part (b).		
	Alternative Solution		
	$\int \frac{1}{x^3}\ln x\,\mathrm{d}x\,,\quad \left\{\begin{array}{ll} u = x^{-3} & \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = -3x^{-4} \\ \frac{\mathrm{d}v}{\mathrm{d}x} = \ln x & \Rightarrow v = x\ln x - x \end{array}\right\}$		
	$\int \frac{1}{x^3}\ln x\,\mathrm{d}x = \frac{1}{x^3}(x\ln x - x) - \int (x\ln x - x)\frac{-3}{x^4}\,\mathrm{d}x$		
		$k\int \frac{1}{x^3}\ln x\,\mathrm{d}x = \frac{1}{x^3}(x\ln x - x) \pm \int \frac{\lambda}{x^3}\,\mathrm{d}x$ where $k \neq 1$	M1
	$-2\int \frac{1}{x^3}\ln x\,\mathrm{d}x = \frac{1}{x^3}(x\ln x - x) - \int \frac{3}{x^3}\,\mathrm{d}x$	Any one of $\frac{1}{x^3}(x\ln x - x)$ or $-\int \frac{3}{x^3}\,\mathrm{d}x$	A1
		$\frac{1}{x^3}(x\ln x - x) - \int \frac{3}{x^3}\,\mathrm{d}x$ and $k = -2$	A1
	$-2\int \frac{1}{x^3}\ln x\,\mathrm{d}x = \frac{1}{x^3}(x\ln x - x) + \frac{3}{2x^2}\ \{+c\}$	$\pm\int \mu\frac{1}{x^3} \to \pm\beta x^{-2}$.	dM1
	$\int \frac{1}{x^3}\ln x\,\mathrm{d}x = -\frac{1}{2x^3}(x\ln x - x) - \frac{3}{4x^2}\ \{+c\}$	$-\frac{1}{2x^3}(x\ln x - x) - \frac{3}{4x^2}$ or equivalent with/without $+c$.	A1
	$= -\frac{1}{2x^2}\ln x - \frac{1}{4x^2}\ \{+c\}$		

Question Number	Scheme		Marks
3.	**Method 1: Using one identity**		
	$\dfrac{9x^2+20x-10}{(x+2)(3x-1)} \equiv A + \dfrac{B}{(x+2)} + \dfrac{C}{(3x-1)}$		
	$A = 3$	their constant term $= 3$	B1
	$9x^2+20x-10 \equiv A(x+2)(3x-1) + B(3x-1) + C(x+2)$	Forming a correct identity.	B1
	Either $x^2:\ 9 = 3A,\quad x:\ 20 = 5A+3B+C$, constant: $-10 = -2A - B + 2C$ or $x=-2 \Rightarrow 36-40-10 = -7B \Rightarrow -14 = -7B \Rightarrow B = 2$	Attempts to find the value of either one of their B or their C from their identity.	M1
	$x = \frac{1}{3} \Rightarrow 1 + \frac{20}{3} - 10 = \frac{7}{3}C \Rightarrow -\frac{7}{3} = \frac{7}{3}C \Rightarrow C = -1$	Correct values for their B and their C, which are found using a correct identity.	A1
			[4]
	Method 2: Long Division		
	$\dfrac{9x^2+20x-10}{(x+2)(3x-1)} \equiv 3 + \dfrac{5x-4}{(x+2)(3x-1)}$	their constant term $= 3$	B1
	So, $\dfrac{5x-4}{(x+2)(3x-1)} \equiv \dfrac{B}{(x+2)} + \dfrac{C}{(3x-1)}$		
	$5x-4 \equiv B(3x-1) + C(x+2)$	Forming a correct identity.	B1
	Either $x:\ 5 = 3B + C$, constant: $-4 = -B + 2C$ or $x=-2 \Rightarrow -10-4 = -7B \Rightarrow -14 = -7B \Rightarrow B = 2$	Attempts to find the value of either one of their B or their C from their identity.	M1
	$x = \frac{1}{3} \Rightarrow \frac{5}{3} - 4 = \frac{7}{3}C \Rightarrow -\frac{7}{3} = \frac{7}{3}C \Rightarrow C = -1$	Correct values for their B and their C, which are found using $5x-4 \equiv B(3x-1)+C(x+2)$	A1
	So, $\dfrac{9x^2+20x-10}{(x+2)(3x-1)} \equiv 3 + \dfrac{2}{(x+2)} - \dfrac{1}{(3x-1)}$		**[4]**
			4

1st B1: Their constant term must be equal to 3 for this mark.
2nd B1 (M1 on epen): Forming a correct identity. This can be implied by later working.
M1 (A1 on epen): Attempts to find the value of either one of their B or their C from their identity. This can be achieved by ***either*** substituting values into their identity ***or*** comparing coefficients and solving the resulting equations simultaneously.
A1: Correct values for their B and their C, which are found using a correct identity.
Note : $\dfrac{9x^2+20x-10}{(x+2)(3x-1)} \equiv \dfrac{A}{(x+2)} + \dfrac{B}{(3x-1)}$, leading to $9x^2+20x-10 \equiv A(3x-1) + B(x+2)$, leading to $A = 2$ and $B = -1$ will gain a maximum of B0B0M1A0

.

3. ctd	**Note:** You can imply the 2nd B1 from either $\dfrac{9x^2+20x-10}{(x+2)(3x-1)} \equiv \dfrac{A(x+2)(3x-1)+B(3x-1)+C(x+2)}{(x+2)(3x-1)}$ or $\dfrac{5x-4}{(x+2)(3x-1)} \equiv \dfrac{B(3x-1)+C(x+2)}{(x+2)(3x-1)}$ **Alternative Method 1: Initially dividing by (x + 2)** $\dfrac{9x^2+20x-10}{"(x+2)"(3x-1)} \equiv \dfrac{9x+2}{(3x-1)} - \dfrac{14}{(x+2)(3x-1)}$ $\equiv 3 + \dfrac{5}{(3x-1)} - \dfrac{14}{(x+2)(3x-1)}$ **B1:** their constant term $= 3$ So, $\dfrac{-14}{(x+2)(3x-1)} \equiv \dfrac{B}{(x+2)} + \dfrac{C}{(3x-1)}$ $-14 \equiv B(3x-1)+C(x+2)$ **B1:** Forming a correct identity. $\Rightarrow B=2,\ C=-6$ **M1:** Attempts to find either one of their B or their C from their identity. So, $\dfrac{9x^2+20x-10}{(x+2)(3x-1)} \equiv 3 + \dfrac{5}{(3x-1)} + \dfrac{2}{(x+2)} - \dfrac{6}{(3x-1)}$ and $\dfrac{9x^2+20x-10}{(x+2)(3x-1)} \equiv 3 + \dfrac{2}{(x+2)} - \dfrac{1}{(3x-1)}$ **A1:** Correct answer in partial fractions. **Alternative Method 2: Initially dividing by (3x - 1)** $\dfrac{9x^2+20x-10}{(x+2)"(3x-1)"} \equiv \dfrac{3x+\frac{23}{3}}{(x+2)} - \dfrac{\frac{7}{3}}{(x+2)(3x-1)}$ $\equiv 3 + \dfrac{\frac{5}{3}}{(x+2)} - \dfrac{\frac{7}{3}}{(x+2)(3x-1)}$ **B1:** their constant term $= 3$ So, $\dfrac{-\frac{7}{3}}{(x+2)(3x-1)} \equiv \dfrac{B}{(x+2)} + \dfrac{C}{(3x-1)}$ $-\frac{7}{3} \equiv B(3x-1)+C(x+2)$ **B1:** Forming a correct identity. $\Rightarrow B=\dfrac{1}{3},\ C=-1$ **M1:** Attempts to find either one of their B or their C from their identity. So, $\dfrac{9x^2+20x-10}{(x+2)(3x-1)} \equiv 3 + \dfrac{\frac{5}{3}}{(x+2)} + \dfrac{\frac{1}{3}}{(x+2)} - \dfrac{1}{(3x-1)}$ and $\dfrac{9x^2+20x-10}{(x+2)(3x-1)} \equiv 3 + \dfrac{2}{(x+2)} - \dfrac{1}{(3x-1)}$ **A1:** Correct answer in partial fractions.

Question Number	Scheme		Marks
4. (a)	1.0981		B1 **cao** **[1]**
(b)	Area $\approx \frac{1}{2}\times 1\,;\times\underline{\left[0.5+2\left(0.8284+\text{their }1.0981\right)+1.3333\right]}$		B1; <u>M1</u>
	$= \frac{1}{2}\times 5.6863 = 2.84315 = 2.843$ (3 dp)	2.843 or awrt 2.843	A1 **[3]**
(c)	$\{u = 1+\sqrt{x}\} \Rightarrow \underline{\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{2}x^{-\frac{1}{2}}}$ or $\underline{\frac{\mathrm{d}x}{\mathrm{d}u} = 2(u-1)}$		<u>B1</u>
	$\left\{\int \frac{x}{1+\sqrt{x}}\,\mathrm{d}x = \right\} \int \frac{(u-1)^2}{u}.\,2(u-1)\,\mathrm{d}u$	$\int \frac{(u-1)^2}{u}......$	M1
		$\int \frac{(u-1)^2}{u}.\,2(u-1)$	A1
	$= 2\int \frac{(u-1)^3}{u}\,\mathrm{d}u \;=\{2\}\int \frac{(u^3-3u^2+3u-1)}{u}\,\mathrm{d}u$	Expands to give a "four term" cubic in u. Eg: $\pm Au^3 \pm Bu^2 \pm Cu \pm D$	M1
	$= \{2\}\int \left(u^2-3u+3-\frac{1}{u}\right)\mathrm{d}u$	An attempt to divide at least three terms in ***their cubic*** by u. See notes.	M1
	$= \{2\}\left(\frac{u^3}{3}-\frac{3u^2}{2}+3u-\ln u\right)$	$\int \frac{(u-1)^3}{u} \to \left(\frac{u^3}{3}-\frac{3u^2}{2}+3u-\ln u\right)$	A1
	$\text{Area}(R) = \left[\frac{2u^3}{3}-3u^2+6u-2\ln u\right]_2^3$		
	$= \left(\frac{2(3)^3}{3}-3(3)^2+6(3)-2\ln 3\right)-\left(\frac{2(2)^3}{3}-3(2)^2+6(2)-2\ln 2\right)$	Applies limits of 3 and 2 in u or 4 and 1 in x and subtracts either way round.	M1
	$= \frac{11}{3}+2\ln 2-2\ln 3$ or $\frac{11}{3}+2\ln\left(\frac{2}{3}\right)$ or $\frac{11}{3}-\ln\left(\frac{9}{4}\right)$, etc	Correct exact answer or equivalent.	A1 **[8]** **12**
(a)	**B1:** 1.0981 correct answer only. Look for this on the table or in the candidate's working.		
(b)	**B1**: Outside brackets $\frac{1}{2}\times 1$ or $\frac{1}{2}$		
	M1: <u>For structure of trapezium rule</u> $\left[\;\underline{............}\;\right]$		
	A1: anything that rounds to 2.843		
	<u>**Note:**</u> Working must be seen to demonstrate the use of the trapezium rule. <u>**Note**</u>: actual area is 2.85573645…		
	<u>Note:</u> Award B1M1 A1 for $\frac{1}{2}(0.5+1.3333)+\left(0.8284+\text{their }1.0981\right) = 2.84315$		
	<u>Bracketing mistake</u>: **Unless the final answer implies that the calculation has been done correctly**		
	Award B1M0A0 for $\frac{1}{2}\times 1\;+0.5+2\left(0.8284+\text{their }1.0981\right)+1.3333$ (nb: answer of 6.1863).		
	Award B1M0A0 for $\frac{1}{2}\times 1\;(0.5+1.3333)+2\left(0.8284+\text{their }1.0981\right)$ (nb: answer of 4.76965).		

4. (b) ctd

Alternative method for part (b): Adding individual trapezia

$$\text{Area} \approx 1\times\left[\frac{0.5+0.8284}{2}+\frac{0.8284+1.0981}{2}+\frac{1.0981+1.3333}{2}\right] = 2.84315$$

B1: 1 and a divisor of 2 on all terms inside brackets.
M1: First and last ordinates once and two of the middle ordinates twice inside brackets ignoring the 2.
A1: anything that rounds to 2.843

(c)

B1: $\frac{du}{dx}=\frac{1}{2}x^{-\frac{1}{2}}$ or $du=\frac{1}{2\sqrt{x}}\,dx$ or $2\sqrt{x}\,du = dx$ or $dx = 2(u-1)du$ or $\frac{dx}{du}=2(u-1)$ oe.

1st M1: $\frac{x}{1+\sqrt{x}}$ becoming $\frac{(u-1)^2}{u}$ (Ignore integral sign).

1st A1 (B1 on epen): $\frac{x}{1+\sqrt{x}}\,dx$ becoming $\frac{(u-1)^2}{u}.\,2(u-1)\{du\}$ or $\frac{(u-1)^2}{u}.\frac{2}{(u-1)^{-1}}\{du\}$.

You can ignore the integral sign and the du.

2nd M1: Expands to give a "four term" cubic in u, $\pm Au^3 \pm Bu^2 \pm Cu \pm D$
where $A\neq 0$, $B\neq 0$, $C\neq 0$ and $D\neq 0$ The cubic does not need to be simplified for this mark.

3rd M1: An attempt to divide at least three terms in ***their cubic*** by u.

Ie. $\frac{(u^3-3u^2+3u-1)}{u}\to u^2-3u+3-\frac{1}{u}$

2nd A1: $\int\frac{(u-1)^3}{u}\,du\to\left(\frac{u^3}{3}-\frac{3u^2}{2}+3u-\ln u\right)$

4th M1: Some evidence of limits of 3 and 2 in u and subtracting either way round.

3rd A1: Exact answer of $\frac{11}{3}+2\ln 2-2\ln 3$ or $\frac{11}{3}+2\ln\left(\frac{2}{3}\right)$ or $\frac{11}{3}-\ln\left(\frac{9}{4}\right)$ or $2\left(\frac{11}{6}+\ln 2-\ln 3\right)$

or $\frac{22}{6}+2\ln\left(\frac{2}{3}\right)$, etc. **Note**: that fractions must be combined to give either $\frac{11}{3}$ or $\frac{22}{6}$ or $3\frac{2}{3}$

Alternative method for 2nd M1 and 3rd M1 mark

$\{2\}\int\frac{(u-1)^2}{u}.(u-1)\,du = \{2\}\int\frac{(u^2-2u+1)}{u}.(u-1)\,du$ $=\{2\}\int\left(u-2+\frac{1}{u}\right).(u-1)\,du = \{2\}\int(u^2-...)\,du$	An attempt to expand $(u-1)^2$, then divide the result by u and then go on to multiply by $(u-1)$.	**2nd M1**
$=\{2\}\int\left(u^2-2u+1-u+2-\frac{1}{u}\right)du$ $=\{2\}\int\left(u^2-3u+3-\frac{1}{u}\right)du$	to give three out of four of $\pm Au^2, \pm Bu, \pm C$ or $\pm\frac{D}{u}$	**3rd M1**

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4. (c) ctd

Final two marks in part (c): $u = 1+\sqrt{x}$

$$\text{Area}(R) = \left[\frac{2\left(1+\sqrt{x}\right)^3}{3} - 3\left(1+\sqrt{x}\right)^2 + 6\left(1+\sqrt{x}\right) - 2\ln\left(1+\sqrt{x}\right)\right]_1^4$$

$$= \left(\frac{2\left(1+\sqrt{4}\right)^3}{3} - 3\left(1+\sqrt{4}\right)^2 + 6\left(1+\sqrt{4}\right) - 2\ln\left(1+\sqrt{4}\right)\right)$$

$$- \left(\frac{2\left(1+\sqrt{1}\right)^3}{3} - 3\left(1+\sqrt{1}\right)^2 + 6\left(1+\sqrt{1}\right) - 2\ln\left(1+\sqrt{1}\right)\right)$$

M1: Applies limits of 4 and 1 in x and subtracts either way round.

$$= \left(18 - 27 + 18 - 2\ln 3\right) - \left(\frac{16}{3} - 12 + 12 - 2\ln 2\right)$$

$$= \frac{11}{3} + 2\ln 2 - 2\ln 3 \quad \text{or} \quad \frac{11}{3} + 2\ln\left(\frac{2}{3}\right) \quad \text{or} \quad \frac{11}{3} - \ln\left(\frac{9}{4}\right), \text{ etc}$$

A1: Correct exact answer or equivalent.

Alternative method for the final 5 marks in part (b)

$$\int \frac{(u-1)^3}{u}\,\mathrm{d}u, \qquad \left\{\begin{array}{lll} "u" = u^{-1} & \Rightarrow & \frac{\mathrm{d}"u"}{\mathrm{d}x} = -u^{-2} \\ \frac{\mathrm{d}v}{\mathrm{d}x} = (u-1)^3 & \Rightarrow & v = \frac{(u-1)^4}{4} \end{array}\right\}$$

$$= \frac{(u-1)^4}{4u} - -\frac{1}{4}\int \frac{(u-1)^4}{u^2}\,\mathrm{d}u$$

$$= \frac{(u-1)^4}{4u} + \frac{1}{4}\int \frac{u^4 - 4u^3 + 6u^2 - 4u + 1}{u^2}\,\mathrm{d}u$$

M1: Applies integration by parts and expands to give a five term quartic.

$$= \frac{(u-1)^4}{4u} + \frac{1}{4}\int u^2 - 4u + 6 - \frac{4}{u} + \frac{1}{u^2}\,\mathrm{d}u$$

M1: Dividing at least 4 terms.

$$= \frac{(u-1)^4}{4u} + \frac{1}{4}\left(\frac{u^3}{3} - 2u^2 + 6u - 4\ln u - \frac{1}{u}\right)$$

A1: Correct Integration.

$$\int_2^3 \frac{(u-1)^3}{u}\,\mathrm{d}u = \left[\frac{(u-1)^4}{4u} + \frac{u^3}{12} - \frac{u^2}{2} + \frac{3u}{2} - \ln u - \frac{1}{4u}\right]_2^3$$

$$= \left(\frac{16}{12} + \frac{27}{12} - \frac{9}{2} + \frac{9}{2} - \ln 3 - \frac{1}{12}\right) - \left(\frac{1}{8} + \frac{8}{12} - \frac{4}{2} + \frac{6}{2} - \ln 2 - \frac{1}{8}\right)$$

M1

$$= \left(7 - \ln 3\right) - \left(\frac{5}{3} - \ln 2\right)$$

$$= \frac{11}{6} + \ln\frac{2}{3}$$

$$\text{Area}(R) = 2\int_2^3 \frac{(u-1)^3}{u}\,\mathrm{d}u = 2\left(\frac{11}{6} + \ln\frac{2}{3}\right)$$

A1

Question Number	Scheme		Marks
5.	***Working parametrically:***		
	$x = 1 - \frac{1}{2}t,\quad y = 2^t - 1$ or $y = e^{t\ln 2} - 1$		
(a)	$\{x = 0 \Rightarrow\}\ 0 = 1 - \frac{1}{2}t \Rightarrow t = 2$	Applies $x = 0$ to obtain a value for t.	M1
	When $t = 2$, $y = 2^2 - 1 = 3$	Correct value for y.	A1
			[2]
(b)	$\{y = 0 \Rightarrow\}\ 0 = 2^t - 1 \Rightarrow t = 0$	Applies $y = 0$ to obtain a value for t. (Must be seen in part (b)).	M1
	When $t = 0$, $x = 1 - \frac{1}{2}(0) = 1$	$x = 1$	A1
			[2]
(c)	$\frac{dx}{dt} = -\frac{1}{2}$ and either $\frac{dy}{dt} = 2^t \ln 2$ or $\frac{dy}{dt} = e^{t\ln 2}\ln 2$		B1
	$\frac{dy}{dx} = \frac{2^t \ln 2}{-\frac{1}{2}}$	Attempts their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$.	M1
	At A, $t = \text{"2"}$, so $m(\mathbf{T}) = -8\ln 2 \Rightarrow m(\mathbf{N}) = \frac{1}{8\ln 2}$	Applies $t = \text{"2"}$ and $m(\mathbf{N}) = \frac{-1}{m(\mathbf{T})}$	M1
	$y - 3 = \frac{1}{8\ln 2}(x - 0)$ or $y = 3 + \frac{1}{8\ln 2}x$ or equivalent.	See notes.	M1 A1 oe cso
			[5]
(d)	$\text{Area}(R) = \int (2^t - 1).\left(-\frac{1}{2}\right)dt$	Complete substitution for both y and dx	M1
	$x = -1 \to t = 4$ and $x = 1 \to t = 0$		B1
	$= \left\{-\frac{1}{2}\right\}\left(\frac{2^t}{\ln 2} - t\right)$	Either $2^t \to \frac{2^t}{\ln 2}$ or $(2^t - 1) \to \frac{(2^t)}{\pm\alpha(\ln 2)} - t$ or $(2^t - 1) \to \pm\alpha(\ln 2)(2^t) - t$	M1*
		$(2^t - 1) \to \frac{2^t}{\ln 2} - t$	A1
	$\left\{-\frac{1}{2}\left[\frac{2^t}{\ln 2} - t\right]_4^0\right\} = -\frac{1}{2}\left(\left(\frac{1}{\ln 2}\right) - \left(\frac{16}{\ln 2} - 4\right)\right)$	**Depends on the previous method mark.** Substitutes their changed limits in t and subtracts either way round.	dM1*
	$= \frac{15}{2\ln 2} - 2$	$\frac{15}{2\ln 2} - 2$ or equivalent.	A1
			[6]
			15

5. (a)	**M1:** Applies $x = 0$ and obtains a value of t.
	A1: For $y = 2^2 - 1 = 3$ or $y = 4 - 1 = 3$
	Alternative Solution 1:
	M1: For substituting $t = 2$ into either x or y.
	A1: $x = 1 - \frac{1}{2}(2) = 0$ **and** $y = 2^2 - 1 = 3$
	Alternative Solution 2:
	M1: Applies $y = 3$ and obtains a value of t.
	A1: For $x = 1 - \frac{1}{2}(2) = 0$ or $x = 1 - 1 = 0$.
	Alternative Solution 3:
	M1: Applies $y = 3$ or $x = 0$ and obtains a value of t.
	A1: Shows that $t = 2$ for both $y = 3$ and $x = 0$.
(b)	**M1:** Applies $y = 0$ and obtains a value of t. Working must be seen in part (b).
	A1: For finding $x = 1$.
	Note: Award M1A1 for $x = 1$.
(c)	**B1:** Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ correct. This mark can be implied by later working.
	M1: Their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ or their $\frac{dy}{dt} \times \frac{1}{\text{their}\left(\frac{dx}{dt}\right)}$. **Note:** their $\frac{dy}{dt}$ must be a function of t.
	M1: Uses their value of t found in part (a) and applies $m(\mathbf{N}) = \frac{-1}{m(\mathbf{T})}$.
	M1: $y - 3 = (\text{their normal gradient})x$ **or** $y = (\text{their normal gradient})x + 3$ or equivalent.
	A1: $y - 3 = \frac{1}{8\ln 2}(x - 0)$ or $y = 3 + \frac{1}{8\ln 2}x$ or $y - 3 = \frac{1}{\ln 256}(x - 0)$ or $(8\ln 2)y - 24\ln 2 = x$ or $\frac{y-3}{(x-0)} = \frac{1}{8\ln 2}$. You can apply isw here.
	Working in decimals is ok for the three method marks. B1, A1 require exact values.
(d)	**M1:** Complete substitution for both y and dx. So candidate should write down $\int (2^t - 1).\left(\text{their } \frac{dx}{dt}\right)$
	B1: Changes limits from $x \to t$. $x = -1 \to t = 4$ and $x = 1 \to t = 0$. Note $t = 4$ and $t = 0$ seen is B1.
	M1*: Integrates 2^t correctly to give $\frac{2^t}{\ln 2}$
	... or integrates $(2^t - 1)$ to give either $\frac{(2^t)}{\pm\alpha(\ln 2)} - t$ or $\pm\alpha(\ln 2)(2^t) - t$.
	A1: Correct integration of $(2^t - 1)$ with respect to t to give $\frac{2^t}{\ln 2} - t$.
	dM1*: **Depends upon the previous method mark.** Substitutes their limits in t and subtracts either way round.
	A1: Exact answer of $\frac{15}{2\ln 2} - 2$ or $\frac{15}{\ln 4} - 2$ or $\frac{15 - 4\ln 2}{2\ln 2}$ or $\frac{7.5}{\ln 2} - 2$ or $\frac{15}{2}\log_2 e - 2$ or equivalent.

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Question Number	Scheme		Marks
5.	***Alternative: Converting to a Cartesian equation:***		
	$t = 2 - 2x \Rightarrow \quad y = 2^{2-2x} - 1$		
(a)	$\{x = 0 \Rightarrow\}\ y = 2^2 - 1$	Applies $x = 0$ in their Cartesian equation...	M1
	$y = 3$	... to arrive at a correct answer of 3.	A1
			[2]
(b)	$\{y = 0 \Rightarrow\}\ 0 = 2^{2-2x} - 1 \Rightarrow 0 = 2 - 2x \Rightarrow x = ...$	Applies $y = 0$ to obtain a value for x. (Must be seen in part (b)).	M1
	$x = 1$	$x = 1$	A1
			[2]
(c)	$\frac{\mathrm{d}y}{\mathrm{d}x} = -2\left(2^{2-2x}\right)\ln 2$	$\pm\lambda 2^{2-2x},\ \lambda \neq 1$	M1
		$-2\left(2^{2-2x}\right)\ln 2$ or equivalent	A1
	At A, $x = 0$, so $m(\mathbf{T}) = -8\ln 2 \Rightarrow m(\mathbf{N}) = \frac{1}{8\ln 2}$	Applies $x = 0$ and $m(\mathbf{N}) = \frac{-1}{m(\mathbf{T})}$	M1
	$y - 3 = \frac{1}{8\ln 2}(x - 0)$ or $y = 3 + \frac{1}{8\ln 2}x$ or equivalent.	***As in the original scheme.***	M1 A1 oe
			[5]
(d)	$\text{Area}(R) = \int \left(2^{2-2x} - 1\right)\mathrm{d}x$	Form the integral of their Cartesian equation of C.	M1
	$= \int_{-1}^{1} \left(2^{2-2x} - 1\right)\mathrm{d}x$	For $2^{2-2x} - 1$ with limits of $x = -1$ and $x = 1$. Ie. $\int_{-1}^{1}\left(2^{2-2x} - 1\right)$	B1
	$= \left(\frac{2^{2-2x}}{-2\ln 2} - x\right)$	Either $2^{2-2x} \to \frac{2^{2-2x}}{-2\ln 2}$ or $\left(2^{2-2x} - 1\right) \to \frac{2^{2-2x}}{\pm\alpha(\ln 2)} - x$ or $\left(2^{2-2x} - 1\right) \to \pm\alpha(\ln 2)(2^{2-2x}) - x$	M1*
		$\left(2^{2-2x} - 1\right) \to \frac{2^{2-2x}}{-2\ln 2} - x$	A1
	$\left\{\left[\frac{2^{2-2x}}{-2\ln 2} - x\right]_{-1}^{1}\right\} = \left(\left(\frac{1}{-2\ln 2} - 1\right) - \left(\frac{16}{-2\ln 2} + 1\right)\right)$	**Depends on the previous method mark.** Substitutes limits of -1 and their x_B and subtracts either way round.	dM1*
	$= \frac{15}{2\ln 2} - 2$	$\frac{15}{2\ln 2} - 2$ or equivalent.	A1
			[6] **15**
(d)	**Alternative method: In Cartesian and applying $u = 2 - 2x$**		

	Area$(R) = \int \left(2^u - 1\right)\{\mathrm{d}x\}$, where $u = 2 - 2x$ $= \int_4^0 \left(2^u - 1\right)\left(-\tfrac{1}{2}\right)\{\mathrm{d}u\}$	**M0**: Unless a candidate ***writes*** $\int \left(2^{2-2x} - 1\right)\{\mathrm{d}x\}$ Then apply the "working parametrically" mark scheme.

Question Number	Scheme		Marks
5. (d)	***Alternative method: For substitution*** $\mathbf{u = 2^t}$		
	$\text{Area}(R) = \int \left(2^t - 1\right).\left(-\frac{1}{2}\right)\text{d}t$	Complete substitution for both y and $\text{d}x$	M1
	where $u = 2^t \Rightarrow \frac{\text{d}u}{\text{d}t} = 2^t \ln 2 \Rightarrow \frac{\text{d}u}{\text{d}t} = u \ln 2$		
	$x = -1 \rightarrow t = 4 \rightarrow u = 16$ and $x = 1 \rightarrow t = 0 \rightarrow u = 1$	Both correct limits in t or both correct limits in u.	B1
	So $\text{area}(R) = -\frac{1}{2}\int \frac{u-1}{u\ln 2}\text{d}u$	If not awarded above, you can award M1 for this integral	
	$= -\frac{1}{2}\int \frac{1}{\ln 2} - \frac{1}{u\ln 2}\text{d}u$		
	$= \left\{-\frac{1}{2}\right\}\left(\frac{u}{\ln 2} - \frac{\ln u}{\ln 2}\right)$	Either $2^t \rightarrow \frac{u}{\ln 2}$ or $\left(2^t - 1\right) \rightarrow \frac{u}{\pm\alpha(\ln 2)} - \frac{\ln u}{\ln 2}$ or $\left(2^t - 1\right) \rightarrow \pm\alpha(\ln 2)(u) - \frac{\ln u}{\ln 2}$	M1*
		$\left(2^t - 1\right) \rightarrow \frac{u}{\ln 2} - \frac{\ln u}{\ln 2}$	A1
	$\left\{-\frac{1}{2}\left[\frac{u}{\ln 2} - \frac{\ln u}{\ln 2}\right]_{16}^{1}\right\} = -\frac{1}{2}\left(\left(\frac{1}{\ln 2}\right) - \left(\frac{16}{\ln 2} - \frac{\ln 16}{\ln 2}\right)\right)$	**Depends on the previous method mark.** Substitutes their changed limits ***in u*** and subtracts either way round.	dM1*
	$= \frac{15}{2\ln 2} - \frac{\ln 16}{2\ln 2}$ or $\frac{15}{2\ln 2} - 2$	$\frac{15}{2\ln 2} - \frac{\ln 16}{2\ln 2}$ or $\frac{15}{2\ln 2} - 2$ or equivalent.	A1
			[6]

Question Number	Scheme		Marks
6. (a)	$\{y = 0 \Rightarrow\}\ 1 - 2\cos x = 0$	$1 - 2\cos x = 0$, seen or implied.	M1
	$\Rightarrow x = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$	At least one correct value of x. (See notes).	A1
		Both $\dfrac{\pi}{3}$ and $\dfrac{5\pi}{3}$	A1 **cso**
			[3]
(b)	$V = \pi\displaystyle\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2\cos x)^2\ \mathrm{d}x$	For $\pi\displaystyle\int(1 - 2\cos x)^2$. Ignore limits and $\mathrm{d}x$	B1
	$\left\{\displaystyle\int(1 - 2\cos x)^2\ \mathrm{d}x\right\} = \displaystyle\int(1 - 4\cos x + 4\cos^2 x)\mathrm{d}x$		
	$= \displaystyle\int 1 - 4\cos x + 4\left(\frac{1 + \cos 2x}{2}\right)\mathrm{d}x$	$\cos 2x = 2\cos^2 x - 1$ See notes.	M1
	$= \displaystyle\int\left(3 - 4\cos x + 2\cos 2x\right)\mathrm{d}x$		
	$= 3x - 4\sin x + \dfrac{2\sin 2x}{2}$	Attempts $\int y^2$ to give any two of $\pm A \to \pm Ax$, $\pm B\cos x \to \pm B\sin x$ or $\pm\lambda\cos 2x \to \pm\mu\sin 2x$.	M1
		Correct integration.	A1
	$V = \{\pi\}\left(\left(3\left(\frac{5\pi}{3}\right) - 4\sin\left(\frac{5\pi}{3}\right) + \dfrac{2\sin\left(\frac{10\pi}{3}\right)}{2}\right) - \left(3\left(\frac{\pi}{3}\right) - 4\sin\left(\frac{\pi}{3}\right) + \dfrac{2\sin\left(\frac{2\pi}{3}\right)}{2}\right)\right)$	Applying limits the correct way round. Ignore π.	ddM1
	$= \pi\left(\left(5\pi + 2\sqrt{3} - \dfrac{\sqrt{3}}{2}\right) - \left(\pi - 2\sqrt{3} + \dfrac{\sqrt{3}}{2}\right)\right)$		
	$= \pi\left((18.3060...) - (0.5435...)\right) = 17.7625\pi = 55.80$		
	$= \pi\left(4\pi + 3\sqrt{3}\right)$ or $4\pi^2 + 3\pi\sqrt{3}$	Two term exact answer.	A1
			[6] **9**

6. (a)	**M1:** $1-2\cos x=0$. This can be implied by either $\cos x=\frac{1}{2}$ or any one of the correct values for x in radians or in degrees. **1st A1:** Any one of either $\frac{\pi}{3}$ or $\frac{5\pi}{3}$ or 60 or 300 or awrt 1.05 or 5.23 or awrt 5.24 . **2nd A1:** Both $\frac{\pi}{3}$ and $\frac{5\pi}{3}$.
(b)	**B1:** (M1 on epen) For $\pi\int(1-2\cos x)^2$. Ignore limits and $\mathrm{d}x$. **1st M1:** Any correct form of $\cos 2x=2\cos^2 x-1$ used or written down in the same variable. This can be implied by $\cos^2 x=\frac{1+\cos 2x}{2}$ or $4\cos^2 x \to 2+2\cos 2x$ or $\cos 2A=2\cos^2 A-1$. **2nd M1:** Attempts $\int y^2$ to give any two of $\pm A\to\pm Ax$, $\pm B\cos x\to\pm B\sin x$ or $\pm\lambda\cos 2x\to\pm\mu\sin 2x$. Do not worry about the signs when integrating $\cos x$ or $\cos 2x$ for this mark. Note: $\int(1-2\cos x)^2=\int 1+4\cos^2 x$ is ok for an attempt at $\int y^2$. **1st A1:** Correct integration. Eg. $3x-4\sin x+\frac{2\sin 2x}{2}$ or $x-4\sin x+\frac{2\sin 2x}{2}+2x$ oe. **3rd ddM1:** Depends on both of the two previous method marks. (Ignore π). Some evidence of substituting their $x=\frac{5\pi}{3}$ and their $x=\frac{\pi}{3}$ and subtracting the correct way round. You will need to use your calculator to check for correct substitution of their limits into their integrand if a candidate does not explicitly give **some evidence.** **Note:** For correct integral and limits decimals gives: $\pi\left((18.3060...)-(0.5435...)\right)=17.7625\pi=55.80$ **2nd A1:** ***Two term*** exact answer of either $\pi\left(4\pi+3\sqrt{3}\right)$ or $4\pi^2+3\pi\sqrt{3}$ or equivalent. **Note:** The π in the volume formula is only required for the B1 mark and the final A1 mark. **Note:** Decimal answer of 58.802... without correct exact answer is A0. **Note:** Applying $\int(1-2\cos x)\,\mathrm{d}x$ will usually be given no marks in this part.

Question Number	Scheme		Marks
7. (a)	**i:** $9+\lambda = 2+2\mu$ **(1)** **j:** $13+4\lambda = -1+\mu$ **(2)** **k:** $-3-2\lambda = 1+\mu$ **(3)**	Any two equations. (Allow one slip).	M1
	Eg: **(2) − (3)**: $16+6\lambda = -2$ or **(2) − 4(1)**: $-23 = -9-7\mu$	An attempt to eliminate one of the parameters.	dM1
	Leading to $\lambda = -3$ or $\mu = 2$	Either $\lambda = -3$ or $\mu = 2$	A1
	$l_1 : \mathbf{r} = \begin{pmatrix} 9 \\ 13 \\ -3 \end{pmatrix} - 3\begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \\ 3 \end{pmatrix}$ or $l_2 : \mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} + 2\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \\ 3 \end{pmatrix}$	See notes	ddM1 A1
			[5]
(b)	$\mathbf{d_1} = \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}$, $\mathbf{d}_2 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$	Realisation that the dot product is required between $\pm A\mathbf{d_1}$ and $\pm B\mathbf{d}_2$.	M1
	$\cos\theta = \pm\left(\dfrac{2+4-2}{\sqrt{(1)^2+(4)^2+(-2)^2}\,.\sqrt{(2)^2+(1)^2+(1)^2}}\right)$	Correct **equation**.	A1
	$\cos\theta = \dfrac{4}{\sqrt{21}.\sqrt{6}} \Rightarrow \theta = 69.1238974... = 69.1$ (1 dp)	awrt 69.1	A1
			[3]
(c)	$\overrightarrow{OA} = \begin{pmatrix} 4 \\ 16 \\ -3 \end{pmatrix}$, $\overrightarrow{OP} = \begin{pmatrix} 9 \\ 13 \\ -3 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 9+\lambda \\ 13+4\lambda \\ -3-2\lambda \end{pmatrix}$		
	$\overrightarrow{AP} = \begin{pmatrix} 9+\lambda \\ 13+4\lambda \\ -3-2\lambda \end{pmatrix} - \begin{pmatrix} 4 \\ 16 \\ -3 \end{pmatrix} = \begin{pmatrix} \lambda+5 \\ 4\lambda-3 \\ -2\lambda \end{pmatrix}$		M1 A1
	$\overrightarrow{AP} \bullet \mathbf{d_1} = 0 \Rightarrow \begin{pmatrix} \lambda+5 \\ 4\lambda-3 \\ -2\lambda \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = \lambda+5+16\lambda-12+4\lambda = 0$		dM1
	leading to $\{21\lambda - 7 = 0 \Rightarrow\}\ \lambda = \dfrac{1}{3}$	$\lambda = \dfrac{1}{3}$	A1
	Position vector $\overrightarrow{OP} = \begin{pmatrix} 9 \\ 13 \\ -3 \end{pmatrix} + \dfrac{1}{3}\begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 9\frac{1}{3} \\ 14\frac{1}{3} \\ -3\frac{2}{3} \end{pmatrix}$ or $\begin{pmatrix} \frac{28}{3} \\ \frac{43}{3} \\ -\frac{11}{3} \end{pmatrix}$		ddM1 A1
			[6] **14**

7. (a)	**M1:** Writes down any two equations. Allow one slip. **dM1:** Attempts to eliminate either λ or μ to form an equation in one parameter only. **A1:** For either $\lambda = -3$ or $\mu = 2$. **Note**: candidates only need to find one of the parameters. **ddM1:** For either substituting their value of λ into l_1 or their μ into l_2. **2nd A1:** For either $\begin{pmatrix}6\\1\\3\end{pmatrix}$ or $6\mathbf{i}+\mathbf{j}+3\mathbf{k}$ or $(6\quad 1\quad 3)$. **Note:** Each of the method marks in this part are dependent upon the previous method marks.
(b)	**M1:** Realisation that the dot product is required between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$. Allow one slip in $\mathbf{d}_1 = \mathbf{i}+4\mathbf{j}-2\mathbf{k}$. **A1:** Correct application of the dot product formula $\mathbf{d}_1 \bullet \mathbf{d}_2 = \pm\lvert\mathbf{d}_1\rvert\lvert\mathbf{d}_2\rvert\cos\theta$ or $\cos\theta = \pm\left(\dfrac{\mathbf{d}_1 \bullet \mathbf{d}_2}{\lvert\mathbf{d}_1\rvert\lvert\mathbf{d}_2\rvert}\right)$ The dot product must be correctly applied and the square roots although they can be un-simplified must be correctly applied. **A1:** awrt 69.1 . This can be also be achieved by $180 - 110.876 = \text{awrt } 69.1$. $\theta = 1.2064...^c$ is A0. **Common response:** $\cos\theta = \left(\dfrac{-12-24+12}{\sqrt{(-3)^2+(-12)^2+(6)^2}.\sqrt{(4)^2+(2)^2+(2)^2}}\right) = \dfrac{-24}{\sqrt{189}.\sqrt{24}}$ is M1A1... ***<u>Alternative Method: Vector Cross Product</u>*** **Only apply this scheme if it is clear that a candidate is applying a vector cross product method.** $\mathbf{d}_1\times\mathbf{d}_2 = \begin{pmatrix}1\\4\\-2\end{pmatrix}\times\begin{pmatrix}2\\1\\1\end{pmatrix} = \left\{\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\1&4&-2\\2&1&1\end{vmatrix} = 6\mathbf{i}-5\mathbf{j}-7\mathbf{k}\right\}$ <u>**M1**</u>**:** Realisation that the vector cross product is required between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$. Allow one slip in $\mathbf{d}_1 = \mathbf{i}+4\mathbf{j}-2\mathbf{k}$. $\sin\theta = \dfrac{\sqrt{(6)^2+(5)^2+(-7)^2}}{\sqrt{(1)^2+(4)^2+(-2)^2}.\sqrt{(2)^2+(1)^2+(1)^2}}$ **A1:** Correct applied **equation**. $\sin\theta = \dfrac{\sqrt{110}}{\sqrt{21}.\sqrt{6}} \Rightarrow \theta = 69.1238974... = 69.1 \text{ (1 dp)}$ **A1:** awrt 69.1
(c)	**M1:** Attempts to find $\overrightarrow{AP}$ in terms of the parameter by subtracting the components of $\overrightarrow{OP}$ from l_1 and $\overrightarrow{OA}$. Ignore the direction of subtraction and ignore any confusion between $\overrightarrow{OP}$ and $\overrightarrow{PO}$ or between $\overrightarrow{OA}$ and $\overrightarrow{AO}$. The correct subtraction of two components is enough to establish that subtraction is intended. The coordinates or position vector of *P* must be given in terms of a parameter. Taking $P:(x, y, z)$ gains no marks although this can be recovered later. See **Additional Solutions**. **A1: (M1 on epen)** A correct expression for $\overrightarrow{AP}$. Again accept the reverse direction. **dM1**: Depends on the previous M. Taking the scalar product of their expression for $\overrightarrow{AP}$ with $\mathbf{d}_1$ or a multiple of $\mathbf{d}_1$ and equating to 0 and obtaining an equation for λ. The equation must derive from an expression of the form $x_1x_2 + y_1y_2 + z_1z_2 = 0$. Differentiation can be used. See **Additional Solutions**. **A1:** Solving to find $\lambda = \frac{1}{3}$. **ddM1:** Depends on both previous Ms. Substitutes their value of the parameter into their expression for $\overrightarrow{OP}$. Substituting into $\overrightarrow{AP}$ is a common error which loses the mark. **Note:** Needs 2 correct co-ordinates if $\lambda = \frac{1}{3}$ found and then *P* stated without method to gain ddM1.

	A1: $9\frac{1}{3}\mathbf{i}+14\frac{1}{3}\mathbf{j}-3\frac{2}{3}\mathbf{k}$. Accept vector notation or coordinates. ***Must be exact.***

7. (c)

<u>Additional Solution 1:</u>

Taking $\overrightarrow{OP} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$, in itself, can gain no marks but this may be converted to a parameter at a later stage in the solution and, at that stage, any relevant marks can be awarded.

For example, $\overrightarrow{AP} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 4 \\ 16 \\ -3 \end{pmatrix} = \begin{pmatrix} x-4 \\ y-16 \\ z+3 \end{pmatrix}$

leading to: $\begin{pmatrix} x-4 \\ y-16 \\ z+3 \end{pmatrix} \square \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = x - 4 + 4y - 64 - 2z - 6 = 0$ — No marks gained at this stage.

Using, $\overrightarrow{OP} = \begin{pmatrix} 9 \\ 13 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 9+\lambda \\ 13+4\lambda \\ -3-2\lambda \end{pmatrix}$ on $x + 4y - 2z = 74$

which gives: $9 + \lambda + 4(13 + 4\lambda) - 2(-3 - 2\lambda) = 74$ — At this stage award **M1A1** and **dM1** (which is implied by an equation)

$\Rightarrow \quad 21\lambda + 67 = 74 \Rightarrow \underline{\lambda = \frac{1}{3}}$ — A1: Solving to find $\lambda = \frac{1}{3}$.

Position vector

$\overrightarrow{OP} = \begin{pmatrix} 9 \\ 13 \\ -3 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 9\frac{1}{3} \\ 14\frac{1}{3} \\ -3\frac{2}{3} \end{pmatrix}$ or $\begin{pmatrix} \frac{28}{3} \\ \frac{43}{3} \\ -\frac{11}{3} \end{pmatrix}$ — ddM1 A1

<u>Additional Solution 2: Using Differentiation</u>

$\overrightarrow{AP} = \begin{pmatrix} 9+\lambda \\ 13+4\lambda \\ -3-2\lambda \end{pmatrix} - \begin{pmatrix} 4 \\ 16 \\ -3 \end{pmatrix} = \begin{pmatrix} \lambda+5 \\ 4\lambda-3 \\ -2\lambda \end{pmatrix}$ — **M1A1:** As main scheme

$AP^2 = (\lambda + 5)^2 + (4\lambda - 3)^2 + (-2\lambda)^2 = \{21\lambda^2 - 14\lambda + 34\}$

$\frac{\mathrm{d}}{\mathrm{d}\lambda}\left(AP^2\right) = 42\lambda - 14 = 0$ — **M1**

leading to $\lambda = \frac{1}{3}$ — **A1:** Solving to find $\lambda = \frac{1}{3}$.

... then apply the main scheme.

Question Number	Scheme		Marks
8. (a)	$\left\{\dfrac{d\theta}{dt} = \dfrac{(3-\theta)}{125}\right\} \Rightarrow \int \dfrac{1}{3-\theta}\,d\theta = \int \dfrac{1}{125}\,dt$ or $\int \dfrac{125}{3-\theta}\,d\theta = \int dt$		B1
	$-\ln(\theta-3) = \dfrac{1}{125}t\ \{+c\}$ or $-\ln(3-\theta) = \dfrac{1}{125}t\ \{+c\}$	See notes.	M1 A1
	$\ln(\theta-3) = -\dfrac{1}{125}t + c$		
	$\theta - 3 = e^{-\frac{1}{125}t + c}$ or $e^{-\frac{1}{125}t}e^{c}$	***Correct*** completion to $\theta = Ae^{-0.008t} + 3$.	
	$\theta = Ae^{-0.008t} + 3$ *		A1
			[4]
(b)	$\{t = 0,\ \theta = 16 \Rightarrow\}\quad 16 = Ae^{-0.008(0)} + 3;\ \Rightarrow \underline{A = 13}$	***See notes.***	M1; A1
	$10 = 13e^{-0.008t} + 3$	Substitutes $\theta = 10$ into an equation **of the form** $\theta = Ae^{-0.008t} + 3$, or equivalent. ***See notes.***	M1
	$e^{-0.008t} = \dfrac{7}{13} \Rightarrow -0.008t = \ln\left(\dfrac{7}{13}\right)$	Correct algebra to $-0.008t = \ln k$, where k is a positive value. ***See notes.***	M1
	$\left\{t = \dfrac{\ln\left(\frac{7}{13}\right)}{(-0.008)}\right\} = 77.3799... = 77$ (nearest minute)	awrt 77	A1
			[5]
			9

8. (a)

B1: (M1 on epen) Separates variables as shown. $d\theta$ and dt should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.

M1: Both $\pm\lambda\ln(3-\theta)$ or $\pm\lambda\ln(\theta-3)$ **and** $\pm\mu t$ where λ and μ are constants.

A1: For $-\ln(\theta-3) = \dfrac{1}{125}t$ or $-\ln(3-\theta) = \dfrac{1}{125}t$ or $-125\ln(\theta-3) = t$ or $-125\ln(3-\theta) = t$

Note: $+c$ is not needed for this mark.

A1: Correct completion to $\theta = Ae^{-0.008t} + 3$. **Note:** $+c$ is needed for this mark.

Note: $\ln(\theta-3) = -\dfrac{1}{125}t + c$ leading to $\theta - 3 = e^{-\frac{1}{125}t} + e^{c}$ or $\theta - 3 = e^{-\frac{1}{125}t} + A$, would be final A0.

Note: From $-\ln(\theta-3) = \dfrac{1}{125}t + c$, then $\ln(\theta-3) = -\dfrac{1}{125}t + c$

$\Rightarrow \theta - 3 = e^{-\frac{1}{125}t + c}$ **or** $\theta - 3 = e^{-\frac{1}{125}t}e^{c} \Rightarrow \theta = Ae^{-0.008t} + 3$ is required for A1.

Note: From $-\ln(3-\theta) = \dfrac{1}{125}t + c$, then $\ln(3-\theta) = -\dfrac{1}{125}t + c$

$\Rightarrow 3 - \theta = e^{-\frac{1}{125}t + c}$ **or** $3 - \theta = e^{-\frac{1}{125}t}e^{c} \Rightarrow \theta = Ae^{-0.008t} + 3$ is sufficient for A1.

Note: The jump from $3 - \theta = Ae^{-\frac{1}{125}t}$ to $\theta = Ae^{-0.008t} + 3$ is fine.

	Note: $\ln(\theta - 3) = -\frac{1}{125}t + c \Rightarrow \theta - 3 = Ae^{-\frac{1}{125}t}$, where candidate writes $A = e^{c}$ is also acceptable.

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8. (b)

M1: (B1 on epen) Substitutes $\theta = 16,\ t = 0$, into either their equation containing an unknown constant or the printed equation. **Note:** You can imply this method mark.

A1: (M1 on epen) $A = 13$. **Note:** $\theta = 13\mathrm{e}^{-0.008t} + 3$ without any working implies the first two marks, M1A1.

M1: Substitutes $\theta = 10$ into an equation **of the form** $\theta = A\mathrm{e}^{-0.008t} + 3$, or equivalent. where A is a positive or negative numerical value and A can be equal to 1 or -1.

M1: Uses correct algebra to rearrange **their equation** into the form $-0.008t = \ln k$, where k is ***a positive numerical value***.

A1: awrt 77 or awrt 1 hour 17 minutes.

<u>Alternative Method 1 for part (b)</u>

$$\int \frac{1}{3-\theta}\,\mathrm{d}\theta = \int \frac{1}{125}\,\mathrm{d}t \Rightarrow -\ln(\theta - 3) = \frac{1}{125}t + c$$

$$\{t = 0\ ,\ \theta = 16 \Rightarrow\}\ -\ln(16-3) = \frac{1}{125}(0) + c \Rightarrow c = -\ln 13$$

M1: Substitutes $t = 0, \theta = 16$, into $-\ln(\theta - 3) = \frac{1}{125}t + c$

A1: $c = -\ln 13$

$$-\ln(\theta - 3) = \frac{1}{125}t - \ln 13 \quad \text{or} \quad \ln(\theta - 3) = -\frac{1}{125}t + \ln 13$$

$$-\ln(10-3) = \frac{1}{125}t - \ln 13$$

M1: Substitutes $\theta = 10$ into an equation **of the form** $\pm\lambda\ln(\theta - 3) = \pm\frac{1}{125}t \pm \mu$ where λ, μ are numerical values.

$$\ln 13 - \ln 7 = \frac{1}{125}t$$

M1: Uses correct algebra to rearrange **their equation** into the form $\pm 0.008t = \ln C - \ln D$, where C, D are ***positive numerical values***.

$$t = 77.3799... = 77\ (\text{nearest minute})$$

A1: awrt 77.

<u>Alternative Method 2 for part (b)</u>

$$\int \frac{1}{3-\theta}\,\mathrm{d}\theta = \int \frac{1}{125}\,\mathrm{d}t \Rightarrow -\ln|3 - \theta| = \frac{1}{125}t + c$$

$$\{t = 0\ ,\ \theta = 16 \Rightarrow\}\ -\ln|3-16| = \frac{1}{125}(0) + c \Rightarrow c = -\ln 13$$

M1: Substitutes $t = 0, \theta = 16$, into $-\ln(3 - \theta) = \frac{1}{125}t + c$

A1: $c = -\ln 13$

$$-\ln|3 - \theta| = \frac{1}{125}t - \ln 13 \quad \text{or} \quad \ln|3 - \theta| = -\frac{1}{125}t + \ln 13$$

$$-\ln(3-10) = \frac{1}{125}t - \ln 13$$

M1: Substitutes $\theta = 10$ into an equation **of the form** $\pm\lambda\ln(3 - \theta) = \pm\frac{1}{125}t \pm \mu$ where λ, μ are numerical values.

$$\ln 13 - \ln 7 = \frac{1}{125}t$$

M1: Uses correct algebra to rearrange **their equation** into the form $\pm 0.008t = \ln C - \ln D$,

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	$t = 77.3799... = 77$ (nearest minute)	where C, D are ***positive numerical values***. **A1:** awrt 77.

8. (b)	***<u>Alternative Method 3 for part (b)</u>***	
	$\int_{16}^{10} \frac{1}{3-\theta}\, d\theta = \int_{0}^{t} \frac{1}{125}\, dt$	
	$= \left[-\ln\lvert 3-\theta\rvert\right]_{16}^{10} = \left[\frac{1}{125}t\right]_{0}^{t}$	
		M1A1: $\ln 13$
		M1: Substitutes limit of $\theta = 10$ correctly.
	$-\ln 7 - -\ln 13 = \frac{1}{125}t$	**M1:** Uses correct algebra to rearrange **their own equation** into the form $\pm 0.008t = \ln C - \ln D$, where C, D are ***positive numerical values***.
	$t = 77.3799... = 77$ (nearest minute)	**A1:** awrt 77.
	<u>Alternative Method 4 for part (b)</u>	
	$\{\theta = 16 \Rightarrow\}\quad 16 = Ae^{-0.008t} + 3$	**M1*:** Writes down a pair of equations in A and t, for $\theta = 16$ and $\theta = 10$ with either A unknown or A being a positive or negative value.
	$\{\theta = 10 \Rightarrow\}\quad 10 = Ae^{-0.008t} + 3$	**A1:** Two equations with an unknown A.
	$-0.008t = \ln\left(\frac{13}{A}\right)$ or $-0.008t = \ln\left(\frac{7}{A}\right)$	**M1:** Uses ***correct algebra*** to solve both of **their equations** leading to answers of the form $-0.008t = \ln k$, where k is ***a positive numerical value***.
	$t_{(1)} = \frac{\ln\left(\frac{13}{A}\right)}{-0.008}$ and $t_{(2)} = \frac{\ln\left(\frac{7}{A}\right)}{-0.008}$	
	$t = t_{(1)} - t_{(2)} = \frac{\ln\left(\frac{13}{A}\right)}{-0.008} - \frac{\ln\left(\frac{7}{A}\right)}{-0.008}$	**M1:** Finds difference between the two times. (either way round).
	$\left\{t = \frac{\ln\left(\frac{7}{13}\right)}{(-0.008)}\right\} = 77.3799... = 77$ (nearest minute)	**A1:** awrt 77. Correct solution only.

Mark Scheme (Results)

Summer 2013

GCE Core Mathematics 4 (6666/01R)

PEARSON

Question Number	Scheme		Marks
1.	$\dfrac{5x+3}{(2x+1)(x+1)^2} \equiv \dfrac{A}{(2x+1)} + \dfrac{B}{(x+1)} + \dfrac{C}{(x+1)^2}$	At least one of "A" or "C" are correct.	B1
	$A = 2, \quad C = 2$	Breaks up their partial fraction correctly into three terms **and** both $"A" = 2$ and $"C" = 2$.	B1 **cso**
	$5x + 3 \equiv A(x+1)^2 + B(2x+1)(x+1) + C(2x+1)$ $x = -1 \Rightarrow -2 = -C \Rightarrow C = 2$ $x = -\dfrac{1}{2} \Rightarrow -\dfrac{5}{2} + 3 = \dfrac{1}{4}A \Rightarrow \dfrac{1}{2} = \dfrac{1}{4}A \Rightarrow A = 2$	Writes down ***a correct identity*** and attempts to find the value of either one "A" or "B" or "C".	M1
	Either $x^2: 0 = A + 2B$, constant: $3 = A + B + C$ $x: 5 = 2A + 3B + 2C$		
	leading to $B = -1$	Correct value for "B" which is found using a correct identity and follows from their partial fraction decomposition.	A1 **cso**
			[4]
	So, $\dfrac{5x+3}{(2x+1)(x+1)^2} \equiv \dfrac{2}{(2x+1)} - \dfrac{1}{(x+1)} + \dfrac{2}{(x+1)^2}$		**4**

Notes for Question 1

BE CAREFUL! Candidates will assign ***their own*** "A, B and C" for this question.

B1: At least one of "A" or "C" are correct.

B1: Breaks up their partial fraction correctly into three terms **and** both $"A" = 2$ and $"C" = 2$.

M1: Writes down ***a correct identity*** (although this can be implied) and attempts to find the value of either one of "A" or "B" or "C".
This can be achieved by ***either*** substituting values into their identity ***or*** comparing coefficients and solving the resulting equations simultaneously.

A1: Correct value for "B" which is found using a correct identity and follows from their partial fraction decomposition.
Note: If a candidate does not give partial fraction decomposition then:
- the 2nd B1 mark can follow from a correct identity.
- the final A1 mark can be awarded for a correct "B" if a candidate goes writes out their partial fractions at the end.

Note: The correct partial fraction from no working scores B1B1M1A1.

Note: A number of candidates will start this problem by writing out the correct identity and then attempt to find "A" or "B" or "C". Therefore the B1 marks can be awarded from this method.

Question Number	Scheme		Marks
2.	$3^{x-1} + xy - y^2 + 5 = 0$		
		$3^{x-1} \rightarrow 3^{x-1}\ln 3$	B1 oe
	$\left\{\cancel{\frac{dy}{dx}} = \right\}$ (ignore) $3^{x-1}\ln 3 + \left(y + x\frac{dy}{dx}\right) - 2y\frac{dy}{dx} = 0$	Differentiates implicitly to include either $\pm\lambda x\frac{dy}{dx}$ or $\pm ky\frac{dy}{dx}$.	M1*
		$xy \rightarrow +y + x\frac{dy}{dx}$	B1
		$\ldots + y + x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$	A1
	$\{(1, 3) \Rightarrow\}\ 3^{(1-1)}\ln 3 + 3 + (1)\frac{dy}{dx} - 2(3)\frac{dy}{dx} = 0$	Substitutes $x = 1,\ y = 3$ into their differentiated equation or expression.	dM1*
	$\ln 3 + 3 + \frac{dy}{dx} - 6\frac{dy}{dx} = 0 \Rightarrow 3 + \ln 3 = 5\frac{dy}{dx}$		
	$\frac{dy}{dx} = \frac{3 + \ln 3}{5}$		dM1*
	$\frac{dy}{dx} = \frac{1}{5}\left(\ln e^3 + \ln 3\right) = \frac{1}{5}\ln\left(3e^3\right)$	**Uses** $3 = \ln e^3$ to achieve $\frac{dy}{dx} = \frac{1}{5}\ln\left(3e^3\right)$	A1 **cso**
			[7] **7**

Notes for Question 2

B1: Correct differentiation of 3^{x-1}. I.e. $3^{x-1} \rightarrow 3^{x-1}\ln 3$ or $3^{x-1} = \frac{1}{3}\left(3^x\right) \rightarrow \frac{1}{3}\left(3^x\right)\ln 3$

or $3^{x-1} = e^{(x-1)\ln 3} \rightarrow \ln 3\, e^{(x-1)\ln 3}$ or $3^{x-1} = \frac{1}{3}\left(3^x\right) = \frac{1}{3}e^{x\ln 3} \rightarrow \frac{1}{3}(\ln 3)e^{x\ln 3}$

M1: Differentiates implicitly to include either $\pm\lambda x\frac{dy}{dx}$ or $\pm ky\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$).

B1: $xy \rightarrow +y + x\frac{dy}{dx}$

1st A1: $\ldots + y + x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$ **Note:** The 1st A0 follows from an award of the 2nd B0.

Note: The "$= 0$" can be implied by rearrangement of their equation.

ie: $3^{x-1}\ln 3 + y + x\frac{dy}{dx} - 2y\frac{dy}{dx}$ leading to $3^{x-1}\ln 3 + y = 2y\frac{dy}{dx} - x\frac{dy}{dx}$ will get A1 (implied).

2nd M1: **Note:** This method mark is dependent upon the 1st M1* mark being awarded.
Substitutes $x = 1,\ y = 3$ into their differentiated equation or expression. Allow one slip.

3rd M1: **Note:** This method mark is dependent upon the 1st M1* mark being awarded.

Candidate has two differentiated terms in $\frac{dy}{dx}$ and rearranges to make $\frac{dy}{dx}$ the subject.

Note: It is possible to gain the 3rd M1 mark before the 2nd M1 mark.

Eg: Candidate may write $\frac{dy}{dx} = \frac{y + 3^{x-1}\ln 3}{2y - x}$ before substituting in $x = 1$ and $y = 3$

2nd A1: **cso.** Uses $3 = \ln e^3$ to achieve $\frac{dy}{dx} = \frac{1}{5}\ln\left(3e^3\right)$, $\left(= \frac{1}{\lambda}\ln\left(\mu e^3\right),\ \lambda = 5 \text{ and } \mu = 3\right)$

Note: $3 = \ln e^3$ needs to be seen in their proof.

Notes for Question 2 Continued

Question	Scheme		Marks
2.	**Alternative Method:** Multiplying both sides by 3		
	$3^{x-1} + xy - y^2 + 5 = 0$		
	$3^x + 3xy - 3y^2 + 15 = 0$		
		$3^x \rightarrow 3^x \ln 3$	B1
Aliter **Way 2**	$\left\{\cancel{\frac{\mathrm{d}y}{\mathrm{d}x}} \cancel{=}\right\}\ 3^x \ln 3 + \left(3y + 3x\frac{\mathrm{d}y}{\mathrm{d}x}\right) - 6y\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ (ignore)	Differentiates implicitly to include either $\pm\lambda x\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\pm ky\frac{\mathrm{d}y}{\mathrm{d}x}$.	M1*
		$3xy \rightarrow +3y + 3x\frac{\mathrm{d}y}{\mathrm{d}x}$	B1
		$... + 3y + 3x\frac{\mathrm{d}y}{\mathrm{d}x} - 6y\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	A1
	$\{(1, 3) \Rightarrow\}\ 3^1 \ln 3 + 3(3) + (3)(1)\frac{\mathrm{d}y}{\mathrm{d}x} - 6(3)\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	Substitutes $x = 1$, $y = 3$ into their differentiated equation or expression.	dM1*
	$3\ln 3 + 9 + 3\frac{\mathrm{d}y}{\mathrm{d}x} - 18\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow 9 + 3\ln 3 = 15\frac{\mathrm{d}y}{\mathrm{d}x}$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{9 + 3\ln 3}{15}\ \left\{= \frac{3 + \ln 3}{5}\right\}$		dM1*
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{5}\left(\ln e^3 + \ln 3\right)$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{5}\left(\ln e^3 + \ln 3\right) = \frac{1}{5}\ln\left(3e^3\right)$	**Uses** $3 = \ln e^3$ to achieve $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{5}\ln\left(3e^3\right)$	A1 **cso**
			[7] **7**
	NOTE: Only apply this scheme if the candidate has multiplied both sides of their equation by 3. **NOTE:** For reference, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3y + 3^x \ln 3}{6y - 3x}$ **NOTE:** If the candidate applies this method then $3xy \rightarrow +3y + 3x\frac{\mathrm{d}y}{\mathrm{d}x}$ must be seen for the 2nd B1 mark.		

<table>
<tr><th>Question Number</th><th>Scheme</th><th></th><th>Marks</th></tr>
<tr><td>3.</td><td>$\displaystyle\int_0^4 \frac{1}{2+\sqrt{(2x+1)}}\,\mathrm{d}x\ ,\ \ u = 2+\sqrt{(2x+1)}$</td><td></td><td></td></tr>
<tr><td></td><td>$\dfrac{\mathrm{d}u}{\mathrm{d}x} = (2x+1)^{-\frac{1}{2}}$ or $\dfrac{\mathrm{d}x}{\mathrm{d}u} = u-2$</td><td>Either $\dfrac{\mathrm{d}u}{\mathrm{d}x} = \pm K(2x+1)^{-\frac{1}{2}}$ or $\dfrac{\mathrm{d}x}{\mathrm{d}u} = \pm\lambda(u-2)$</td><td>M1</td></tr>
<tr><td></td><td></td><td>Either $\dfrac{\mathrm{d}u}{\mathrm{d}x} = (2x+1)^{-\frac{1}{2}}$ or $\dfrac{\mathrm{d}x}{\mathrm{d}u} = (u-2)$</td><td>A1</td></tr>
<tr><td></td><td>$\left\{\displaystyle\int \frac{1}{2+\sqrt{(2x+1)}}\,\mathrm{d}x\right\} = \displaystyle\int \frac{1}{u}(u-2)\,\mathrm{d}u$</td><td>Correct substitution
(Ignore integral sign and $\mathrm{d}u$).</td><td>A1</td></tr>
<tr><td></td><td>$= \displaystyle\int \left(1-\frac{2}{u}\right)\mathrm{d}u$</td><td>An attempt to divide each term by u.</td><td>dM1</td></tr>
<tr><td></td><td></td><td>$\pm Au \pm B\ln u$</td><td>ddM1</td></tr>
<tr><td></td><td>$= u - 2\ln u$</td><td>$u - 2\ln u$</td><td>A1 ft</td></tr>
<tr><td></td><td>$\left\{\text{So } \left[u-2\ln u\right]_3^5\right\} = (5-2\ln 5) - (3-2\ln 3)$</td><td>Applies limits of 5 and 3 in u
or 4 and 0 in x in their integrated function
and subtracts the correct way round.</td><td>M1</td></tr>
<tr><td></td><td>$= 2 + 2\ln\left(\dfrac{3}{5}\right)$</td><td>$2 + 2\ln\left(\dfrac{3}{5}\right)$</td><td>A1
cao cso</td></tr>
<tr><td></td><td></td><td></td><td>[8]
8</td></tr>
<tr><td colspan="4">Notes for Question 3</td></tr>
<tr><td></td><td colspan="3">
M1: Also allow $\mathrm{d}u = \pm\lambda\dfrac{1}{(u-2)}\mathrm{d}x$ or $(u-2)\mathrm{d}u = \pm\lambda\,\mathrm{d}x$
Note: The expressions must contain $\mathrm{d}u$ and $\mathrm{d}x$. They can be simplified or un-simplified.
A1: Also allow $\mathrm{d}u = \dfrac{1}{(u-2)}\mathrm{d}x$ or $(u-2)\mathrm{d}u = \pm\lambda\,\mathrm{d}x$
Note: The expressions must contain $\mathrm{d}u$ and $\mathrm{d}x$. They can be simplified or un-simplified.
A1: $\displaystyle\int \frac{1}{u}(u-2)\,\mathrm{d}u$. (Ignore integral sign and $\mathrm{d}u$).
dM1: An attempt to divide each term by u.
Note that this mark is dependent on the previous M1 mark being awarded.
Note that this mark can be implied by later working.
ddM1: $\pm Au \pm B\ln u$, $A \neq 0$, $B \neq 0$
Note that this mark is dependent on the two previous M1 marks being awarded.
A1ft: $u - 2\ln u$ or $\pm Au \pm B\ln u$ being correctly followed through, $A \neq 0$, $B \neq 0$
M1: Applies limits of 5 and 3 in u or 4 and 0 in x in their integrated function and subtracts the correct way round.
A1: cso and cao. $2 + 2\ln\left(\dfrac{3}{5}\right)$ or $2 + 2\ln(0.6)$, $\left(= A + 2\ln B, \text{ so } A = 2,\ B = \dfrac{3}{5}\right)$
Note: $2 - 2\ln\left(\dfrac{3}{5}\right)$ is A0.
</td></tr>
</table>

Notes for Question 3 Continued	
3. ctd	**Note:** $\int \frac{1}{u}(u-2)\,\mathrm{d}u = u - 2\ln u$ with no working is 2nd M1, 3rd M1, 3rd A1. but **Note:** $\int \frac{1}{u}(u-2)\,\mathrm{d}u = (u-2)\ln u$ with no working is 2nd M0, 3rd M0, 3rd A0.

Question Number	Scheme		Marks
4. (a)	$\left\{\sqrt[3]{(8-9x)}\right\} = (8-9x)^{\frac{1}{3}}$	Power of $\frac{1}{3}$	M1
	$= \underline{(8)^{\frac{1}{3}}}\left(1-\frac{9x}{8}\right)^{\frac{1}{3}} = \underline{2}\left(1-\frac{9x}{8}\right)^{\frac{1}{3}}$	$\underline{(8)^{\frac{1}{3}}}$ or $\underline{2}$	<u>B1</u>
	$=\{2\}\left[1+\left(\frac{1}{3}\right)(kx)+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(kx)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(kx)^3+\ldots\right]$	see notes	M1 A1
	$=\{2\}\left[\underline{1+\left(\frac{1}{3}\right)\left(\frac{-9x}{8}\right)+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}\left(\frac{-9x}{8}\right)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}\left(\frac{-9x}{8}\right)^3+\ldots}\right]$		
	$= 2\left[1-\frac{3}{8}x;-\frac{9}{64}x^2-\frac{45}{512}x^3+\ldots\right]$	See notes below!	
	$= 2-\frac{3}{4}x;-\frac{9}{32}x^2-\frac{45}{256}x^3+\ldots$		A1; A1
			[6]
(b)	$\left\{\sqrt[3]{7100} = 10\sqrt[3]{71} = 10\sqrt[3]{(8-9x)},\right\}$ so $x = 0.1$	Writes down or uses $x = 0.1$	B1
	When $x = 0.1$, $\sqrt[3]{(8-9x)} \approx 2-\frac{3}{4}(0.1)-\frac{9}{32}(0.1)^2-\frac{45}{256}(0.1)^3+\ldots$ $= 2-0.075-0.0028125-0.00017578125$ $= 1.922011719$		M1
	So, $\sqrt[3]{7100} = 19.220117919\ldots = \underline{19.2201}$ (4 dp)	19.2201 **cso**	A1 **cao**
			[3] **9**

Notes for Question 4

(a)	**M1:** Writes or uses $\frac{1}{3}$. This mark can be implied by a constant term of $8^{\frac{1}{3}}$ or 2. **<u>B1</u>**: $\underline{(8)^{\frac{1}{3}}}$ or $\underline{2}$ outside brackets or $\underline{2}$ as candidate's constant term in their binomial expansion. **M1:** Expands $(\ldots+kx)^{\frac{1}{3}}$ to give any 2 terms out of 4 terms simplified or un-simplified, Eg: $1+\left(\frac{1}{3}\right)(kx)$ or $\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(kx)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(kx)^3$ or $1+\ \ldots\ldots+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(kx)^2$ or $\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(kx)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(kx)^3$ **where** $k \neq 1$ are fine for M1. **A1:** A correct simplified or un-simplified $1+\left(\frac{1}{3}\right)(kx)+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(kx)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(kx)^3$ expansion with consistent (kx). Note that (kx) must be consistent (on the RHS, not necessarily the LHS) in a candidate's expansion. **Note** that $k \neq 1$. You would award B1M1A0 for $2\left[\underline{1+\left(\frac{1}{3}\right)\left(\frac{-9x}{8}\right)+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(-9x)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}\left(\frac{-9x}{8}\right)^3+\ldots}\right]$ because (kx) is not consistent.

Notes for Question 4 Continued	
4. (a) ctd	***"Incorrect bracketing"*** $=\{2\}\left[\underline{1+\left(\frac{1}{3}\right)\left(\frac{-9x}{8}\right)+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}\left(\frac{-9x^2}{8}\right)+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}\left(\frac{-9x^3}{8}\right)+\ldots}\right]$ is M1A0 unless recovered. **A1:** For $2-\frac{3}{4}x$ **(simplified please)** or also allow $2-0.75x$. **Allow Special Case A1A0 for either SC:** $=2\left[1-\frac{3}{8}x;\ \ldots\right]$ **or SC**: $K\left[1-\frac{3}{8}x-\frac{9}{64}x^2-\frac{45}{512}x^3+\ldots\right]$ (where K can be 1 or omitted), with each term in the $[\ldots\ldots\ldots]$ either a simplified fraction or a decimal. **A1:** Accept only $-\frac{9}{32}x^2-\frac{45}{256}x^3$ or $-0.28125x^2-0.17578125x^3$ Candidates who write $=2\left[\underline{1+\left(\frac{1}{3}\right)\left(\frac{9x}{8}\right)+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}\left(\frac{9x}{8}\right)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}\left(\frac{9x}{8}\right)^3+\ldots}\right]$ where $k=\frac{9}{8}$ and not $-\frac{9}{8}$ and achieve $2+\frac{3}{4}x;-\frac{9}{32}x^2+\frac{45}{256}x^3+\ldots$ will get B1M1A1A0A0. **<u>Note for final two marks:</u>** $2\left[1-\frac{3}{8}x;-\frac{9}{64}x^2-\frac{45}{512}x^3+\ldots\right]=2+\frac{3}{4}x-\frac{9}{32}x^2-\frac{45}{256}x^3+\ldots$ scores final A0A1. $2\left[1-\frac{3}{8}x;-\frac{9}{64}x^2-\frac{45}{512}x^3+\ldots\right]=2-\frac{3}{4}-\frac{9}{32}x^2-\frac{45}{256}x^3+\ldots$ scores final A0A1 ***<u>Alternative method:</u>*** Candidates can apply an alternative form of the binomial expansion. $\left\{\sqrt[3]{(8-9x)}\right\}=(8-9x)^{\frac{1}{3}}=(8)^{\frac{1}{3}}+\left(\frac{1}{3}\right)(8)^{-\frac{2}{3}}(-9x)+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(8)^{-\frac{5}{3}}(-9x)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(8)^{-\frac{8}{3}}(-9x)^3$ **B1:** $(8)^{\frac{1}{3}}$ or 2 **M1:** Any two of four (un-simplified or simplified) terms correct. **A1:** All four (un-simplified or simplified) terms correct. **A1:** $2-\frac{3}{4}x$ **A1:** $-\frac{9}{32}x^2-\frac{45}{256}x^3$ **Note:** The terms in C need to be evaluated, so ${}^{\frac{1}{3}}C_0(8)^{\frac{1}{3}}+{}^{\frac{1}{3}}C_1(8)^{-\frac{2}{3}}(-9x)+{}^{\frac{1}{3}}C_2(8)^{-\frac{5}{3}}(-9x)^2+{}^{\frac{1}{3}}C_3(8)^{-\frac{8}{3}}(-9x)^3$ without further working is B0M0A0.
(b)	**B1:** Writes down or uses $x=0.1$ **M1:** Substitutes their x, where $\lvert x\rvert<\frac{8}{9}$ into at least two terms of their binomial expansion. **A1:** 19.2201 **cao** **Be Careful!** The binomial answer is 19.22011719 and the calculated $\sqrt[3]{7100}$ is 19.21997343... which is 19.2200 to 4 decimal places.

Question Number	Scheme		Marks
5. (a)	$6.248046798... = 6.248$ (3dp)	6.248 or awrt 6.248	B1
			[1]
(b)	Area $\approx \frac{1}{2}\times 2\,;\times\underline{\left[3+2\left(7.107+7.218+\text{their } 6.248\right)+5.223\right]}$		B1; <u>M1</u>
	$= 49.369 = 49.37$ (2 dp)	49.37 or awrt 49.37	A1
			[3]
(c)	$\left\{\int (4t\,\mathrm{e}^{-\frac{1}{3}t}+3)\,\mathrm{d}t\right\} = -12t\,\mathrm{e}^{-\frac{1}{3}t} - \int -12\mathrm{e}^{-\frac{1}{3}t}\,\{\mathrm{d}t\}$	$\pm At\,\mathrm{e}^{-\frac{1}{3}t} \pm B\int \mathrm{e}^{-\frac{1}{3}t}\{\mathrm{d}t\}$, $A\neq 0$, $B\neq 0$	M1
		See notes.	A1
	$+\,3t$	$3 \to 3t$	B1
	$= -12t\,\mathrm{e}^{-\frac{1}{3}t} - 36\mathrm{e}^{-\frac{1}{3}t}\;\{+3t\}$	$-12t\,\mathrm{e}^{-\frac{1}{3}t} - 36\mathrm{e}^{-\frac{1}{3}t}$	A1
	$\left[-12t\,\mathrm{e}^{-\frac{1}{3}t} - 36\mathrm{e}^{-\frac{1}{3}t} + 3t\right]_0^8 =$		
	$= \left(-12(8)\mathrm{e}^{-\frac{1}{3}(8)} - 36\mathrm{e}^{-\frac{1}{3}(8)} + 3(8)\right) - \left(-12(0)\mathrm{e}^{-\frac{1}{3}(0)} - 36\mathrm{e}^{-\frac{1}{3}(0)} + 3(0)\right)$	Substitutes limits of 8 and 0 into an integrated function of the form of either $\pm\lambda t\mathrm{e}^{-\frac{1}{3}t} \pm \mu\mathrm{e}^{-\frac{1}{3}t}$ or $\pm\lambda t\,\mathrm{e}^{-\frac{1}{3}t} \pm \mu\mathrm{e}^{-\frac{1}{3}t} + Bt$ and subtracts the correct way round.	dM1
	$= \left(-96\mathrm{e}^{-\frac{8}{3}} - 36\mathrm{e}^{-\frac{8}{3}} + 24\right) - \left(0-36+0\right)$		
	$= 60 - 132\mathrm{e}^{-\frac{8}{3}}$	$60 - 132\mathrm{e}^{-\frac{8}{3}}$	A1
			[6]
(d)	Difference $= \left\lvert 60 - 132\mathrm{e}^{-\frac{8}{3}} - 49.37\right\rvert = 1.458184439... = 1.46$ (2 dp)	1.46 or awrt 1.46	B1
			[1]
			11

Notes for Question 5

(a)	**B1:** 6.248 or awrt 6.248. Look for this on the table or in the candidate's working.
(b)	**B1**: Outside brackets $\frac{1}{2}\times 2$ or 1
	M1: <u>For structure of trapezium rule</u>$[\;\ldots\ldots\ldots\ldots\;]$. Allow one miscopy of their values.
	A1: 49.37 or anything that rounds to 49.37
	Note: It can be possible to award : (a) B0 (b) B1M1A1 (awrt 49.37)
	<u>**Note:**</u> Working must be seen to demonstrate the use of the trapezium rule. <u>**Note**</u>: actual area is 50.828…
	<u>Bracketing mistake</u>: **Unless the final answer implies that the calculation has been done correctly,**
	Award B1M0A0 for $1+3+2\left(7.107+7.218+\text{their } 6.248\right)+5.223$ (nb: answer of 50.369).

Notes for Question 5 Continued

5. (b) ctd

<u>*Alternative method for part (b): Adding individual trapezia*</u>

$$\text{Area} \approx 2\times\left[\frac{3+7.107}{2}+\frac{7.107+7.218}{2}+\frac{7.218+6.248}{2}+\frac{6.248+5.223}{2}\right]= 49.369$$

B1: 2 and a divisor of 2 on all terms inside brackets.

M1: First and last ordinates once and two of the middle ordinates twice inside brackets ignoring the 2.

A1: anything that rounds to 49.37

(c)

M1: For $4t\,e^{-\frac{1}{3}t} \to \pm At\,e^{-\frac{1}{3}t} \pm B\int e^{-\frac{1}{3}t}\{dt\}$, $A \neq 0,\ B \neq 0$

A1: For $t\,e^{-\frac{1}{3}t} \to \left(-3t\,e^{-\frac{1}{3}t} - \int -3e^{-\frac{1}{3}t}\right)$ (some candidates lose the 4 and this is fine for the first A1 mark).

or $4t\,e^{-\frac{1}{3}t} \to 4\left(-3t\,e^{-\frac{1}{3}t} - \int -3e^{-\frac{1}{3}t}\right)$ or $-12t\,e^{-\frac{1}{3}t} - \int -12e^{-\frac{1}{3}t}$ or $12\left(-t\,e^{-\frac{1}{3}t} - \int -e^{-\frac{1}{3}t}\right)$

These results can be implied. They can be simplified or un-simplified.

B1: $3 \to 3t$ or $3 \to 3x$ (bod) .

Note: Award B0 for 3 integrating to $12t$ (implied), which is a common error when taking out a factor of 4.

Be careful some candidates will factorise out 4 and have $4\left(....+\frac{3}{4}\right) \to 4\left(....+\frac{3}{4}t\right)$

which would then be fine for B1.

Note: Allow B1 for $\int_0^8 3\,dt = 24$

A1: For correct integration of $4t\,e^{-\frac{1}{3}t}$ to give $-12t\,e^{-\frac{1}{3}t} - 36e^{-\frac{1}{3}t}$ or $4\left(-3t\,e^{-\frac{1}{3}t} - 9e^{-\frac{1}{3}t}\right)$ or equivalent.

This can be simplified or un-simplified.

dM1: Substitutes limits of 8 and 0 into an integrated function of the form of either $\pm\lambda t e^{-\frac{1}{3}t} \pm \mu e^{-\frac{1}{3}t}$ or

$\pm\lambda t e^{-\frac{1}{3}t} \pm \mu e^{-\frac{1}{3}t} + Bt$ and subtracts the correct way round.

Note: Evidence of a proper consideration of the limit of 0 (as detailed in the scheme) is needed for dM1. So, just subtracting zero is M0.

A1: An exact answer of $60 - 132e^{-\frac{8}{3}}$. A decimal answer of 50.82818444... without a correct answer is A0.

Note: A decimal answer of 50.82818444... without a correct exact answer is A0.

Note: If a candidate gains M1A1B1A1 and then writes down 50.8 or awrt 50.8 with no method for substituting limits of 8 and 0, then award the final M1A0.

IMPORTANT: that is fine for candidates to work in terms of x rather than t in part (c).

Note: The "$3t$" is needed for B1 and the final A1 mark.

(d)

B1: 1.46 or awrt 1.46 or -1.46 or awrt -1.46.

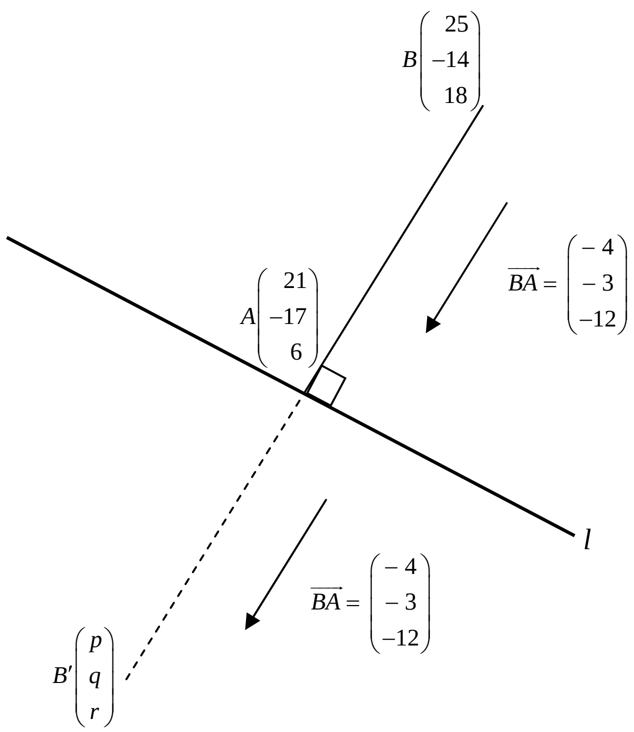
Candidates may give correct decimal answers of 1.458184439... or 1.459184439...

Note: You can award this mark whether or not the candidate has answered part (c) correctly.

Question Number	Scheme		Marks
6.	$l: \mathbf{r} = \begin{pmatrix} a \\ b \\ 10 \end{pmatrix} + \lambda\begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix}, \quad \overrightarrow{OA} = \begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 25 \\ -14 \\ 18 \end{pmatrix}$		
(a)	A is on l, so $\begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix} = \begin{pmatrix} a \\ b \\ 10 \end{pmatrix} + \lambda\begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix}$		
	$\{\mathbf{k}:\ 10 - \lambda = 6 \Rightarrow\}\ \lambda = 4$	$\lambda = 4$	B1
	$\{\mathbf{i}:\ a + 6\lambda = 21 \Rightarrow\}\ a + 6(4) = 21$	Substitutes their value of λ into $a + 6\lambda = 21$	M1
	$a = -3$	$a = -3$	A1 **cao**
			[3]
(b)	$\{\overrightarrow{AB}\} = \begin{pmatrix} 25 \\ -14 \\ 18 \end{pmatrix} - \begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix}$ | $\{\overrightarrow{BA}\} = \begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix} - \begin{pmatrix} 25 \\ -14 \\ 18 \end{pmatrix}$	Finds the difference between $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Ignore labelling.	M1
	$\{\overrightarrow{AB}\} = \begin{pmatrix} 4 \\ 3 \\ 12 \end{pmatrix}$ | $\{\overrightarrow{BA}\} = \begin{pmatrix} -4 \\ -3 \\ -12 \end{pmatrix}$		
	$\left\{\overrightarrow{AB} \perp l \Rightarrow \overrightarrow{AB} \bullet \mathbf{d} = 0\right\} \Rightarrow \begin{pmatrix} 4 \\ 3 \\ 12 \end{pmatrix} \bullet \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix} = 24 + 3c - 12 = 0; \Rightarrow c = -4$	See notes.	M1; A1 ft
	$\{\mathbf{j}:\ b + c\lambda = -17 \Rightarrow\}\ b + (-4)(4) = -17\ ; \Rightarrow\ b = -1$	See notes.	ddM1; A1 **cso cao**
			[5]
(c)	$\lvert AB\rvert = \sqrt{4^2 + 3^2 + 12^2}$ or $\lvert AB\rvert = \sqrt{(-4)^2 + (-3)^2 + (-12)^2}$	See notes.	M1
	So, $\lvert AB\rvert = 13$		A1 **cao**
			[2]
(d)	$\overrightarrow{OB'}\ \left\{= \overrightarrow{OA} + \overrightarrow{BA}\right\} = \begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix} + \begin{pmatrix} -4 \\ -3 \\ -12 \end{pmatrix}; = \begin{pmatrix} 17 \\ -20 \\ -6 \end{pmatrix}$	See notes for alternative methods.	M1;A1 **cao**
			[2]
			12

Notes for Question 6

(a)	**B1:** $\lambda = 4$ seen or implied. **M1:** Substitutes their value of λ into $a + 6\lambda = 21$ **A1:** $a = -3$. Note: Award B1M1A1 if the candidate states $a = -3$ from no working. ***<u>Alternative Method Using Simultaneous equations for part (a).</u>*** **B1:** For $60 - 6\lambda = 36$ **M1:** $60 - 6\lambda = 36$ and $a + 6\lambda = 21$ solved simultaneously to give $a = ...$ **A1:** $a = -3$, **cao**.

Notes for Question 6 Continued	
6. (b) ctd	**M1:** Finds the difference between $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Ignore labelling. If no "subtraction" seen, you can award M1 for 2 out of 3 correct components of the difference. **M1:** ***Applies*** the formula $\overrightarrow{AB} \bullet \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix}$ or $\overrightarrow{BA} \bullet \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix}$ correctly to give a linear equation in c which is set equal to zero. **Note:** The dot product can also be with $\pm k\begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix}$. **A1ft:** $c = -4$ **or** for finding a correct follow through c. **ddM1:** Substitutes their value of λ and their value of c into $b + c\lambda = -17$ **Note** that this mark is dependent on the two previous method marks being awarded. **A1:** $b = -1$
(c)	**M1:** An attempt to apply a three term Pythagoras in order to find $\lvert AB \rvert$, so taking the square root is required here. **A1:** 13 **cao** **Note:** Don't recover work for part (b) in part (c).
(d)	**M1:** For a full ***applied*** method of finding the coordinates of B'. **Note:** You can give M1 for 2 out of 3 correct components of B'. **A1:** For either $\begin{pmatrix} 17 \\ -20 \\ -6 \end{pmatrix}$ or $17\mathbf{i} - 20\mathbf{j} - 6\mathbf{k}$ or $(17, -20, -6)$ **cao**. ***Helpful diagram!*** 

Notes for Question 6 Continued	
	Acceptable Methods for the Method mark in part (d)
Way 1	$\overrightarrow{OB'}\left\{= \overrightarrow{OA} + \overrightarrow{BA}\right\} = \begin{pmatrix}21\\-17\\6\end{pmatrix} + \begin{pmatrix}-4\\-3\\-12\end{pmatrix}$ (using their $\overrightarrow{BA}$)
Way 2	$\overrightarrow{OB'}\left\{= \overrightarrow{OA} - \overrightarrow{AB}\right\} = \begin{pmatrix}21\\-17\\6\end{pmatrix} - \begin{pmatrix}4\\3\\12\end{pmatrix}$ (using their $\overrightarrow{AB}$)
Way 3	$\overrightarrow{OB'}\left\{= \overrightarrow{OB} + 2\overrightarrow{BA}\right\} = \begin{pmatrix}25\\-14\\18\end{pmatrix} + 2\begin{pmatrix}-4\\-3\\-12\end{pmatrix}$ (using their $\overrightarrow{BA}$)
Way 4	$\overrightarrow{OB'}\left\{= \overrightarrow{OB} - 2\overrightarrow{AB}\right\} = \begin{pmatrix}25\\-14\\18\end{pmatrix} - 2\begin{pmatrix}4\\3\\12\end{pmatrix}$ (using their $\overrightarrow{AB}$)
Way 5	$\begin{pmatrix}25\\-14\\18\end{pmatrix} \to \begin{pmatrix}\text{Minus } 4\\\text{Minus } 3\\\text{Minus } 12\end{pmatrix} \to \begin{pmatrix}21\\-17\\6\end{pmatrix} \to \begin{pmatrix}\text{Minus } 4\\\text{Minus } 3\\\text{Minus } 12\end{pmatrix} \left\{\to \begin{pmatrix}17\\-20\\-6\end{pmatrix}\right\}$, so $\overrightarrow{OA}$ + their $\overrightarrow{BA}$
Way 6	$\overrightarrow{OB'}\left\{= 2\overrightarrow{OA} - \overrightarrow{OB}\right\} = 2\begin{pmatrix}21\\-17\\6\end{pmatrix} - \begin{pmatrix}25\\-14\\18\end{pmatrix}$
Way 7	$\overrightarrow{OB} = 25\mathbf{i} - 14\mathbf{j} + 18\mathbf{k}$, $\overrightarrow{OA} = 21\mathbf{i} - 17\mathbf{j} + 6\mathbf{k}$ and $\overrightarrow{OB'} = p\mathbf{i} + q\mathbf{j} + r\mathbf{k}$, $(21, -17, 6) = \left(\frac{25+p}{2}, \frac{-14+q}{2}, \frac{18+r}{2}\right)$ $p = 21(2) - 25 = 17$ $q = -17(2) + 14 = -20$ $r = 6(2) - 18 = -6$ **M1:** Writing down any two equations correctly and an attempt to find at least two of p, q or r.

Question Number	Scheme		Marks
7.	$x = 27\sec^3 t,\quad y = 3\tan t,\quad 0 \leqslant t \leqslant \dfrac{\pi}{3}$		
(a)	$\dfrac{dx}{dt} = 81\sec^2 t\sec t\tan t\,,\quad \dfrac{dy}{dt} = 3\sec^2 t$	At least one of $\dfrac{dx}{dt}$ or $\dfrac{dy}{dt}$ correct.	B1
		Both $\dfrac{dx}{dt}$ and $\dfrac{dy}{dt}$ are correct.	B1
	$\dfrac{dy}{dx} = \dfrac{3\sec^2 t}{81\sec^3 t\tan t}\left\{= \dfrac{1}{27\sec t\tan t} = \dfrac{\cos t}{27\tan t} = \dfrac{\cos^2 t}{27\sin t}\right\}$	Applies their $\dfrac{dy}{dt}$ divided by their $\dfrac{dx}{dt}$	M1;
	At $t = \dfrac{\pi}{6},\ \dfrac{dy}{dx} = \dfrac{3\sec^2\left(\frac{\pi}{6}\right)}{81\sec^3\left(\frac{\pi}{6}\right)\tan\left(\frac{\pi}{6}\right)} = \dfrac{4}{72}\left\{= \dfrac{3}{54} = \dfrac{1}{18}\right\}$	$\dfrac{4}{72}$	A1 **cao cso**
			[4]
(b)	$\{1 + \tan^2 t = \sec^2 t\} \Rightarrow 1 + \left(\dfrac{y}{3}\right)^2 = \left(\sqrt[3]{\left(\dfrac{x}{27}\right)}\right)^2 = \left(\dfrac{x}{27}\right)^{\frac{2}{3}}$		M1
	$\Rightarrow 1 + \dfrac{y^2}{9} = \dfrac{x^{\frac{2}{3}}}{9} \Rightarrow\ 9 + y^2 = x^{\frac{2}{3}} \Rightarrow y = \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}$ *		A1 * **cso**
	$a = 27$ and $b = 216$ or $27 \leqslant x \leqslant 216$	$a = 27$ and $b = 216$	B1
			[3]
(c)	$V = \pi\displaystyle\int_{27}^{125}\left(\left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}\right)^2 dx$ or $\pi\displaystyle\int_{27}^{125}\left(x^{\frac{2}{3}} - 9\right)dx$	For $\pi\displaystyle\int\left(\left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}\right)^2$ or $\pi\displaystyle\int\left(x^{\frac{2}{3}} - 9\right)$ Ignore limits and dx. Can be implied.	B1
	$= \{\pi\}\left[\dfrac{3}{5}x^{\frac{5}{3}} - 9x\right]_{27}^{125}$	Either $\pm Ax^{\frac{5}{3}} \pm Bx$ or $\dfrac{3}{5}x^{\frac{5}{3}}$ oe	M1
		$\dfrac{3}{5}x^{\frac{5}{3}} - 9x$ oe	A1
	$= \{\pi\}\left(\left(\dfrac{3}{5}(125)^{\frac{5}{3}} - 9(125)\right) - \left(\dfrac{3}{5}(27)^{\frac{5}{3}} - 9(27)\right)\right)$	Substitutes limits of 125 and 27 into an integrated function and subtracts the correct way round.	dM1
	$= \{\pi\}\left((1875 - 1125) - (145.8 - 243)\right)$		
	$= \dfrac{4236\pi}{5}$ or 847.2π	$\dfrac{4236\pi}{5}$ or 847.2π	A1
			[5] **12**

Notes for Question 7

(a)	**B1:** At least one of $\dfrac{dx}{dt}$ or $\dfrac{dy}{dt}$ correct. **Note:** that this mark can be implied from their working.
	B1: Both $\dfrac{dx}{dt}$ and $\dfrac{dy}{dt}$ are correct. **Note:** that this mark can be implied from their working.
	M1: Applies their $\dfrac{dy}{dt}$ divided by their $\dfrac{dx}{dt}$, where both $\dfrac{dy}{dt}$ and $\dfrac{dx}{dt}$ are trigonometric functions of t.
	A1: $\dfrac{4}{72}$ or any equivalent correct rational answer not involving surds.
	Allow $0.0\dot{5}$ with the recurring symbol.

Notes for Question 7 Continued

Note: Please check that their $\frac{dx}{dt}$ is differentiated correctly.

Eg. Note that $x = 27\sec^3 t = 27(\cos t)^{-3} \Rightarrow \frac{dx}{dt} = -81(\cos t)^{-2}(-\sin t)$ is correct.

(b)

M1: Either:

- Applying a correct trigonometric identity (usually $1 + \tan^2 t = \sec^2 t$) to give a Cartesian equation in x and y only.
- Starting from the RHS and goes on to achieve $\sqrt{9\tan^2 t}$ by using a correct trigonometric identity.
- Starts from the LHS and goes on to achieve $\sqrt{9\sec^2 t - 9}$ by using a correct trigonometric identity.

A1*: For a correct proof of $y = \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}$.

Note this result is printed on the Question Paper, so no incorrect working is allowed.

B1: Both $a = 27$ and $b = 216$. **Note** that $27 \leqslant x \leqslant 216$ is also fine for B1.

(c)

B1: For a correct statement of $\pi\int\left(\left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}\right)^2$ or $\pi\int\left(x^{\frac{2}{3}} - 9\right)$. Ignore limits and dx. Can be implied.

M1: Either integrates to give $\pm Ax^{\frac{5}{3}} \pm Bx$, $A \neq 0$, $B \neq 0$ or integrates $x^{\frac{2}{3}}$ correctly to give $\frac{3}{5}x^{\frac{5}{3}}$ oe

A1: $\frac{3}{5}x^{\frac{5}{3}} - 9x$ or. $\frac{x^{\frac{5}{3}}}{\left(\frac{5}{3}\right)} - 9x$ oe.

dM1: Substitutes limits of 125 and 27 into an integrated function and subtracts the correct way round.

Note: that this mark is dependent upon the previous method mark being awarded.

A1: A correct exact answer of $\frac{4236\pi}{5}$ or 847.2π.

Note: The π in the volume formula is only required for the B1 mark and the final A1 mark.

Note: A decimal answer of 2661.557... without a correct exact answer is A0.

Note: If a candidate gains the first B1M1A1 and then writes down 2661 or awrt 2662 with no method for substituting limits of 125 and 27, then award the final M1A0.

(a) ***Alternative response using the Cartesian equation in part (a)***

Way 2

$\left\{y = \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}} \Rightarrow\right\} \frac{dy}{dx} = \frac{1}{2}\left(x^{\frac{2}{3}} - 9\right)^{-\frac{1}{2}}\left(\frac{2}{3}x^{-\frac{1}{3}}\right)$	$\frac{dy}{dx} = \pm K x^{-\frac{1}{3}}\left(x^{\frac{2}{3}} - 9\right)^{-\frac{1}{2}}$	M1
	$\frac{dy}{dx} = \frac{1}{2}\left(x^{\frac{2}{3}} - 9\right)^{-\frac{1}{2}}\left(\frac{2}{3}x^{-\frac{1}{3}}\right)$ oe	A1
At $t = \frac{\pi}{6}$, $x = 27\sec^3\left(\frac{\pi}{6}\right) = 24\sqrt{3}$ $\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\left(24\sqrt{3}\right)^{\frac{2}{3}} - 9\right)^{-\frac{1}{2}}\left(\frac{2}{3}\left(24\sqrt{3}\right)^{-\frac{1}{3}}\right)$	Uses $t = \frac{\pi}{6}$ to find x and substitutes their x into an expression for $\frac{dy}{dx}$.	dM1
So, $\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{3\sqrt{3}}\right) = \frac{1}{18}$	$\frac{1}{18}$	A1 **cao cso**

Note: Way 2 is marked as M1 A1 dM1 A1

Note: For way 2 the second M1 mark is dependent on the first M1 being gained.

Notes for Question 7 Continued			
7. (b) **Way 2**	***Alternative responses for M1A1 in part (b)*: STARTING FROM THE RHS**		
	$\{\text{RHS} =\}\ \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}} = \sqrt{\left(27\sec^3 t\right)^{\frac{2}{3}} - 9} = \sqrt{9\sec^2 t - 9} = \sqrt{9\tan^2 t}$	For applying $1 + \tan^2 t = \sec^2 t$ oe to achieve $\sqrt{9\tan^2 t}$	M1
	$= 3\tan t = y\ \{= \text{LHS}\}$ **cso**	Correct proof from $\left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}$ to y.	A1*
	M1: Starts from the RHS and goes on to achieve $\sqrt{9\tan^2 t}$ by using a correct trigonometric identity.		
7. (b) **Way 3**	***Alternative responses for M1A1 in part (b)*: STARTING FROM THE LHS**		
	$\{\text{LHS} =\}\ y = 3\tan t = \sqrt{\left(9\tan^2 t\right)} = \sqrt{9\sec^2 t - 9}$	For applying $1 + \tan^2 t = \sec^2 t$ oe to achieve $\sqrt{9\sec^2 t - 9}$	M1
	$= \sqrt{9\left(\dfrac{x}{27}\right)^{\frac{2}{3}} - 9} = \sqrt{9\left(\dfrac{x^{\frac{2}{3}}}{9}\right) - 9} = \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}$ **cso**	Correct proof from y to $\left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}$.	A1*
	M1: Starts from the LHS and goes on to achieve $\sqrt{9\sec^2 t - 9}$ by using a correct trigonometric identity.		
7. (c) **Way 2**	***Alternative response for part (c) using parametric integration***		
	$V = \pi \int 9\tan^2 t\left(81\sec^2 t \sec t \tan t\right) \mathrm{d}t$	$\pi \int 3\tan t\left(81\sec^2 t \sec t \tan t\right) \mathrm{d}t$ Ignore limits and $\mathrm{d}x$. Can be implied.	B1
	$= \{\pi\} \int 729\sec^2 t \tan^2 t \sec t \tan t \ \mathrm{d}t$		
	$= \{\pi\} \int 729\sec^2 t\left(\sec^2 t - 1\right)\sec t \tan t \ \mathrm{d}t$		
	$= \{\pi\} \int 729\left(\sec^4 t - \sec^2 t\right)\sec t \tan t \ \mathrm{d}t$		
	$= \{\pi\} \int 729\left(\sec^4 t - \sec^2 t\right)\sec t \tan t \ \mathrm{d}t$		
	$= \{\pi\}\left[729\left(\dfrac{1}{5}\sec^5 t - \dfrac{1}{3}\sec^3 t\right)\right]$	$\pm A\sec^5 t \pm B\sec^3 t$	M1
		$729\left(\dfrac{1}{5}\sec^5 t - \dfrac{1}{3}\sec^3 t\right)$	A1
	$V = \{\pi\}\left[729\left(\dfrac{1}{5}\left(\dfrac{5}{3}\right)^5 - \dfrac{1}{3}\left(\dfrac{5}{3}\right)^3\right) - 729\left(\dfrac{1}{5}1^5 - \dfrac{1}{3}1^3\right)\right]$	Substitutes $\sec t = \dfrac{5}{3}$ and $\sec t = 1$ into an integrated function and subtracts the correct way round.	dM1
	$= 729\pi\left[\left(\dfrac{250}{243}\right) - \left(-\dfrac{2}{15}\right)\right]$		
	$= \dfrac{4236\pi}{5}$ or 847.2π	$\dfrac{4236\pi}{5}$ or 847.2π	A1
			[5]

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>8.</td><td>$\frac{dx}{dt} = k(M - x)$, where M is a constant</td><td></td></tr>
<tr><td>(a)</td><td>$\frac{dx}{dt}$ is the <u>rate of increase</u> of the <u>mass of waste</u> products. — Any one correct explanation.</td><td>B1</td></tr>
<tr><td></td><td>M is the <u>total mass</u> of <u>unburned fuel</u> and <u>waste fuel</u>
(or the <u>initial mass</u> of <u>unburned fuel</u>) — Both explanations are correct.</td><td>B1
[2]</td></tr>
<tr><td>(b)</td><td>$\int \frac{1}{M-x}\,dx = \int k\,dt$ or $\int \frac{1}{k(M-x)}\,dx = \int dt$</td><td>B1</td></tr>
<tr><td></td><td>$-\ln(M - x) = kt \ \{+ c\}$ or $-\frac{1}{k}\ln(M - x) = t \ \{+ c\}$ — See notes</td><td>M1 A1</td></tr>
<tr><td></td><td>$\{t = 0 , x = 0 \Rightarrow\} \ -\ln(M - 0) = k(0) + c$ — See notes</td><td>M1</td></tr>
<tr><td></td><td>$c = -\ln M \Rightarrow -\ln(M - x) = kt - \ln M$</td><td></td></tr>
<tr><td></td><td>then either...
$-kt = \ln(M - x) - \ln M$
$-kt = \ln\left(\frac{M - x}{M}\right)$
$e^{-kt} = \frac{M - x}{M}$

or...
$kt = \ln M - \ln(M - x)$
$kt = \ln\left(\frac{M}{M - x}\right)$
$e^{kt} = \frac{M}{M - x}$</td><td>ddM1</td></tr>
<tr><td></td><td>either: $Me^{-kt} = M - x$
or: $(M - x)e^{kt} = M$
$M - x = Me^{-kt}$
leading to $x = M - Me^{-kt}$ or $x = M(1 - e^{-kt})$ oe</td><td>A1 * cso
[6]</td></tr>
<tr><td>(c)</td><td>$\left\{x = \frac{1}{2}M , t = \ln 4 \Rightarrow\right\} \ \frac{1}{2}M = M(1 - e^{-k\ln 4})$</td><td>M1</td></tr>
<tr><td></td><td>$\Rightarrow \frac{1}{2} = 1 - e^{-k\ln 4} \Rightarrow e^{-k\ln 4} = \frac{1}{2} \Rightarrow -k\ln 4 = -\ln 2$</td><td></td></tr>
<tr><td></td><td>So $k = \frac{1}{2}$</td><td>A1</td></tr>
<tr><td></td><td>$x = M\left(1 - e^{-\frac{1}{2}\ln 9}\right)$</td><td>dM1</td></tr>
<tr><td></td><td>$x = \frac{2}{3}M$ — $x = \frac{2}{3}M$</td><td>A1 cso
[4]
12</td></tr>
</table>

Notes for Question 8 Continued

8. (a)

B1: At least one explanation correct.
B1: Both explanations are correct.

$\frac{dx}{dt}$ is the <u>rate of increase</u> of the <u>mass of waste</u> products.
or the <u>rate of change</u> of the <u>mass of waste</u> products.

M is the <u>total mass</u> of <u>unburned fuel</u> and <u>waste fuel</u>
or the <u>initial mass</u> of <u>unburned fuel</u>
or the <u>total mass</u> of <u>rocket fuel</u> and <u>waste fuel</u>
or the <u>initial mass</u> of <u>rocket fuel</u>
or the <u>initial mass</u> of <u>fuel</u>
or the <u>total mass of waste</u> and <u>unburned</u> products.

(b)

B1: Separates variables as shown. dx and dt should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.

M1: Both $\pm\lambda\ln(M-x)$ or $\pm\lambda\ln(x-M)$ **and** $\pm\mu t$ where λ and μ are any constants.

A1: For $-\ln(M-x)=kt$ or $-\ln(x-M)=kt$ or $-\frac{1}{k}\ln(M-x)=t$ or $-\frac{1}{k}\ln(x-M)=t$
or $-\frac{1}{k}\ln(kM-kx)=t$ or $-\frac{1}{k}\ln(kx-kM)=t$

Note: $+c$ is not needed for this mark.
IMPORTANT: $+c$ can be on either side of their equation for the 1st A1 mark.

M1: Substitutes $t=0$ AND $x=0$ in an integrated or changed equation containing c (or A or $\ln A$, etc.)
Note that this mark can be implied by the correct value of c.

ddM1: Uses their value of c which must be a $\ln$ term, and uses fully correct method to eliminate their logarithms. **Note:** This mark is dependent on both previous method marks being awarded.

A1: $x=M-Me^{-kt}$ or $x=M(1-e^{-kt})$ or $x=\frac{M(e^{kt}-1)}{e^{kt}}$ or equivalent where x is the subject.
Note: Please check their working as incorrect working can lead to a correct answer.

Note: $\left\{\frac{dx}{dt}=k(M-x)\Rightarrow\frac{dx}{dt}=\frac{1}{kM-kx}\Rightarrow\right\}$ $x=-\frac{1}{k}\ln(kM-kx)\ \{+c\}$ is B1(Implied) M1A1.

(c)

M1: Substitutes $x=\frac{1}{2}M$ and $t=\ln 4$ into one of their earlier equations connecting x and t.

A1: $k=\frac{1}{2}$, which can be an un-simplified equivalent numerical value. i.e. $k=\frac{\ln 2}{\ln 4}$ is fine for A1.

dM1: Substitutes $t=\ln 4$ and their evaluated k (which must be a numerical value) into one of their earlier equations connecting x and t.
Note: that the 2nd Method mark is dependent on the 1st Method mark being awarded in part (c).

A1: $x=\frac{2}{3}M$ **cso**.
Note: Please check their working as incorrect working can lead to a correct answer.

Notes for Question 8 Continued		
***Aliter* 8. (b) Way 2**	$\int \frac{1}{M-x}\,\mathrm{d}x = \int k\,\mathrm{d}t$	B1
	$-\ln(M-x) = kt\ \{+c\}$ See notes	M1 A1
	$\ln(M-x) = -kt + c$	
	$M - x = A\mathrm{e}^{-kt}$	
	$\{t=0,\ x=0 \Rightarrow\}\ M - 0 = A\mathrm{e}^{-k(0)}$	M1
	$\Rightarrow M = A$	
	$M - x = M\mathrm{e}^{-kt}$	ddM1
	So, $x = M - M\mathrm{e}^{-kt}$	A1
		[6]
(b)	**B1M1A1:** Mark as in the original scheme. **M1:** Substitutes $t=0$ AND $x=0$ in an integrated equation containing their constant of integration which could be c or A. **Note** that this mark can be implied by the correct value of c or A. **ddM1:** Uses a fully correct method to eliminate their logarithms and writes down an equation containing their evaluated constant of integration. **Note:** This mark is dependent on both previous method marks being awarded. **Note:** $\ln(M-x) = -kt + c$ leading to $\ln(M-x) = \mathrm{e}^{-kt} + \mathrm{e}^{c}$ or $\ln(M-x) = \mathrm{e}^{-kt} + A$ would be dddM0. **A1:** Same as the original scheme.	
***Aliter* 8. (b) Way 3**	$\int_0^x \frac{1}{M-x}\,\mathrm{d}x = \int_0^t k\,\mathrm{d}t$	B1
	$\left[-\ln(M-x)\right]_0^x = \left[kt\right]_0^t$	M1 A1
	$-\ln(M-x) - (-\ln M) = kt$ Applies limits of	M1
	$-\ln(M-x) + \ln M = kt$	
	and then follows the original scheme.	
(a)	**B1M1A1:** Mark as in the original scheme (ignoring the limits). **ddM1:** Applies limits 0 and x on their integrated LHS and limits of 0 and t. **M1A1:** Same as the original scheme.	

Notes for Question 8 Continued

Aliter 8. (b) Way 4

Working	Notes	Marks
$\int \frac{1}{M-x}\,dx = \int k\,dt \quad \left\{\Rightarrow \int \frac{-1}{x-M}\,dx = \int k\,dt\right\}$		B1
$-\ln\lvert x-M\rvert = kt + c$	*Modulus not required for 1st A1.*	M1 A1
$\{t=0,\ x=0 \Rightarrow\}\ -\ln\lvert 0-M\rvert = k(0)+c$	*Modulus not required here!*	M1
$\Rightarrow c = -\ln M \Rightarrow -\ln\lvert x-M\rvert = kt - \ln M$		
then either... $-kt = \ln\lvert x-M\rvert - \ln M$; $-kt = \ln\left\lvert\frac{x-M}{M}\right\rvert$ ***or...*** $kt = \ln M - \ln\lvert x-M\rvert$; $kt = \ln\left\lvert\frac{M}{x-M}\right\rvert$		
As $x < M$: $-kt = \ln\left(\frac{M-x}{M}\right)$; $e^{-kt} = \frac{M-x}{M}$ **or** $kt = \ln\left(\frac{M}{M-x}\right)$; $e^{kt} = \frac{M}{M-x}$	*Understanding of modulus is required here!*	**ddM1**
$Me^{-kt} = M-x$ **or** $(M-x)e^{kt} = M$; $M-x = Me^{-kt}$		
leading to $x = M - Me^{-kt}$ or $x = M(1-e^{-kt})$ oe		**A1 * cso**
		[6]

B1: Mark as in the original scheme.
M1A1M1: Mark as in the original scheme ignoring the modulus.
ddM1: Mark as in the original scheme **AND** the candidate must demonstrate that they have converted $\ln\lvert x-M\rvert$ to $\ln(M-x)$ in their working.

Note: This mark is dependent on both the previous method marks being awarded.

A1: Mark as in the original scheme.

Aliter 8. (b) Way 5

Use of an integrating factor **(I.F.)**

Working	Marks
$\frac{dx}{dt} = k(M-x) \Rightarrow \frac{dx}{dt} + kx = kM$	
$\text{I.F.} = e^{kt}$	B1
$\frac{d}{dt}\left(e^{kt}x\right) = kMe^{kt}$,	
$e^{kt}x = Me^{kt} + c$	M1A1
$x = M + ce^{-kt}$	
$\{t=0,\ x=0 \Rightarrow\}\ 0 = M + ce^{-k(0)}$	M1
$\Rightarrow c = -M$	
$x = M - Me^{-kt}$	ddM1A1

Mark Scheme (Results)

June 2013

GCE Core Mathematics 4 (6666/01)

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$$(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c|, \text{ leading to x =}$$

$$(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where } |pq| = |c| \text{ and } |mn| = |a|, \text{ leading to x =}$$

2. Formula

Attempt to use correct formula (with values for *a*, *b* and *c*).

3. Completing the square

$$\text{Solving } x^2 + bx + c = 0: \quad \left(x \pm \frac{b}{2}\right)^2 \pm q \pm c, \quad q \neq 0, \quad \text{leading to x =...}$$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n \rightarrow x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n \rightarrow x^{n+1})$

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.
Normal marking procedure is as follows:
Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Question Number	Scheme		Marks
1. (a)	$\int x^2 e^x \, dx$, 1st Application: $\left\{ \begin{array}{ll} u = x^2 \Rightarrow \frac{du}{dx} = 2x \\ \frac{dv}{dx} = e^x \Rightarrow v = e^x \end{array} \right\}$, 2nd Application: $\left\{ \begin{array}{ll} u = x \Rightarrow \frac{du}{dx} = 1 \\ \frac{dv}{dx} = e^x \Rightarrow v = e^x \end{array} \right\}$		
	$= x^2 e^x - \int 2xe^x dx$	$x^2 e^x - \int \lambda x e^x \{dx\}$, $\lambda > 0$	M1
		$x^2 e^x - \int 2xe^x \{dx\}$	A1 oe
	$= x^2 e^x - 2\left(xe^x - \int e^x dx\right)$	**Either** $\pm Ax^2 e^x \pm Bxe^x \pm C\int e^x \{dx\}$ **or** for $\pm K\int xe^x \{dx\} \to \pm K\left(xe^x - \int e^x \{dx\}\right)$	M1
	$= x^2 e^x - 2(xe^x - e^x)\ \{+ c\}$	$\pm Ax^2 e^x \pm Bxe^x \pm C e^x$	M1
		Correct answer, with/without $+ c$	A1
			[5]
(b)	$\left\{\left[x^2 e^x - 2(xe^x - e^x)\right]_0^1\right\}$ $= \left(1^2 e^1 - 2(1e^1 - e^1)\right) - \left(0^2 e^0 - 2(0e^0 - e^0)\right)$	Applies limits of 1 and 0 to an expression of the form $\pm Ax^2 e^x \pm Bxe^x \pm Ce^x$, $A \neq 0$, $B \neq 0$ and $C \neq 0$ and subtracts the correct way round.	M1
	$= e - 2$	$e - 2$ **cso**	A1 oe
			[2]
			7

Notes for Question 1

(a)	**M1:** Integration by parts is applied in the form $x^2 e^x - \int \lambda x e^x \{dx\}$, where $\lambda > 0$. (must be in this form). **A1:** $x^2 e^x - \int 2xe^x \{dx\}$ or equivalent. **M1:** **Either** achieving a result in the form $\pm Ax^2 e^x \pm Bxe^x \pm C\int e^x \{dx\}$ (can be implied) (where $A \neq 0$, $B \neq 0$ and $C \neq 0$) **or** for $\pm K\int xe^x \{dx\} \to \pm K\left(xe^x - \int e^x \{dx\}\right)$ **M1:** $\pm Ax^2 e^x \pm Bxe^x \pm C e^x$ (where $A \neq 0$, $B \neq 0$ and $C \neq 0$) **A1:** $x^2 e^x - 2(xe^x - e^x)$ or $x^2 e^x - 2xe^x + 2e^x$ or $(x^2 - 2x + 2)e^x$ or equivalent with/without $+ c$.
(b)	**M1:** **Complete method** of applying limits of 1 and 0 to their part (a) answer in the form $\pm Ax^2 e^x \pm Bxe^x \pm Ce^x$, (where $A \neq 0$, $B \neq 0$ and $C \neq 0$) and subtracting the correct way round. Evidence of a proper consideration of the limit of 0 (as detailed above) is needed for M1. So, just subtracting zero is M0. **A1:** $e - 2$ or $e^1 - 2$ or $-2 + e$. Do not allow $e - 2e^0$ unless simplified to give $e - 2$. **Note:** that 0.718... without seeing $e - 2$ or equivalent is A0. **WARNING:** Please note that this A1 mark is for correct solution only. So incorrect $[.......]_0^1$ leading to $e - 2$ is A0. **Note:** If their part (a) is correct candidates can get M1A1 in part (b) for $e - 2$ from no working. **Note:** 0.718... from no working is M0A0

Question Number	Scheme		Marks
2. (a)	$\left\{\sqrt{\left(\frac{1+x}{1-x}\right)}\right\} = (1+x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$	$(1+x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$	B1
	$= \left(1+\left(\frac{1}{2}\right)x+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2+\dots\right)\times\left(1+\left(-\frac{1}{2}\right)(-x)+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-x)^2+\dots\right)$	See notes	M1 A1 A1
	$= \left(1+\frac{1}{2}x-\frac{1}{8}x^2+\dots\right)\times\left(1+\frac{1}{2}x+\frac{3}{8}x^2+\dots\right)$		
	$= 1+\frac{1}{2}x+\frac{3}{8}x^2+\frac{1}{2}x+\frac{1}{4}x^2-\frac{1}{8}x^2+\dots$	See notes	M1
	$= 1+x+\frac{1}{2}x^2$	*Answer is given in the question.*	A1 *
			[6]
(b)	$\sqrt{\left(\frac{1+(\frac{1}{26})}{1-(\frac{1}{26})}\right)} = 1+\left(\frac{1}{26}\right)+\frac{1}{2}\left(\frac{1}{26}\right)^2$		M1
	ie: $\frac{3\sqrt{3}}{5} = \frac{1405}{1352}$		B1
	so, $\sqrt{3} = \frac{7025}{4056}$	$\frac{7025}{4056}$	A1 **cao**
			[3] **9**

Notes for Question 2

(a)

B1: $(1+x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$ or $\sqrt{(1+x)}(1-x)^{-\frac{1}{2}}$ seen or implied. (Also allow $\left((1+x)(1-x)^{-1}\right)^{\frac{1}{2}}$).

M1: Expands $(1+x)^{\frac{1}{2}}$ to give any 2 out of 3 terms simplified or un-simplified,

Eg: $1+\frac{1}{2}x$ or $+\left(\frac{1}{2}\right)x+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2$ or $1+ \dots + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2$

or expands $(1-x)^{-\frac{1}{2}}$ to give any 2 out of 3 terms simplified or un-simplified,

Eg: $1+\left(-\frac{1}{2}\right)(-x)$ or $+\left(-\frac{1}{2}\right)(-x)+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-x)^2$ or $1+\dots+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-x)^2$

Also allow: $1+\dots+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(x)^2$ for M1.

A1: At least one binomial expansion correct (either un-simplified or simplified). (ignore x^3 and x^4 terms)
A1: Two binomial expansions are correct (either un-simplified or simplified). (ignore x^3 and x^4 terms)
Note: Candidates can give decimal equivalents when expanding out their binomial expansions.
M1: Multiplies out to give 1, exactly two terms in x and exactly three terms in x^2.
A1: Candidate achieves the result on the exam paper. Make sure that their working is sound.

Special Case: Award SC FINAL M1A1 for ***a correct*** $\left(1+\frac{1}{2}x-\frac{1}{8}x^2+\dots\right)\times\left(1+\frac{1}{2}x+\frac{3}{8}x^2+\dots\right)$

multiplied out with no errors to give either $1+x+\frac{3}{8}x^2+\frac{1}{4}x^2-\frac{1}{8}x^2$ or $1+\frac{1}{2}x+\frac{3}{8}x^2+\frac{1}{2}x+\frac{1}{8}x^2$ or

$1+\frac{1}{2}x+\frac{1}{4}x^2+\frac{1}{2}x+\frac{1}{4}x^2$ or $1+\frac{1}{2}x+\frac{5}{8}x^2+\frac{1}{2}x-\frac{1}{8}x^2$ leading to the correct answer of $1+x+\frac{1}{2}x^2$.

Notes for Question 2 Continued		
2. (a) ctd	**Note:** If a candidate writes down either $(1+x)^{\frac{1}{2}} = 1+\frac{1}{2}x-\frac{1}{8}x^2+...$ **or** $(1-x)^{-\frac{1}{2}} = 1+\frac{1}{2}x+\frac{3}{8}x^2+...$ with no working then you can award 1st M1, 1st A1. **Note:** If a candidate writes down both correct binomial expansions with no working, then you can award 1st M1, 1st A1, 2nd A1.	
(b)	**M1:** Substitutes $x=\frac{1}{26}$ into **both** sides of $\sqrt{\left(\frac{1+x}{1-x}\right)}$ and $1+x+\frac{1}{2}x^2$ **B1:** For sight of $\sqrt{\frac{27}{25}}$ (or better) and $\frac{1405}{1352}$ or equivalent fraction Eg: $\frac{3\sqrt{3}}{5}$ and $\frac{1405}{1352}$ or $0.6\sqrt{3}$ and $\frac{1405}{1352}$ or $\frac{3\sqrt{3}}{5}$ and $1\frac{53}{1352}$ or $\sqrt{3}$ and $\frac{5}{3}\left(\frac{1405}{1352}\right)$ are fine for B1. **A1:** $\frac{7025}{4056}$ **or any equivalent fraction**, eg: $\frac{14050}{8112}$ or $\frac{182650}{105456}$ etc. **Special Case:** Award SC: M1B1A0 for $\sqrt{3} \approx 1.732001972..$ or truncated 1.732001 or awrt 1.732002. **Note** that $\frac{7025}{4056} = 1.732001972...$ and $\sqrt{3} = 1.732050808...$	
Aliter 2. (a) Way 2	$\left\{\sqrt{\left(\frac{1+x}{1-x}\right)} = \sqrt{\frac{(1+x)(1-x)}{(1+x)(1-x)}} = \sqrt{\frac{(1-x^2)}{(1-x)^2}} = \right\} = (1-x^2)^{\frac{1}{2}}(1-x)^{-1}$ $\quad (1-x^2)^{\frac{1}{2}}(1-x)^{-1}$	B1
	$= \left(1+\left(\frac{1}{2}\right)(-x^2)+...\right)\times\left(1+(-1)(-x)+\frac{(-1)(-2)}{2!}(-x)^2+...\right)$ See notes	M1A1A1
	$= \left(1-\frac{1}{2}x^2+...\right)\times\left(1+x+x^2+...\right)$	
	$= 1+x+x^2-\frac{1}{2}x^2$ See notes	M1
	$= 1+x+\frac{1}{2}x^2$ *Answer is given in the question.*	A1 *
		[6]
Aliter 2. (a) Way 2	**B1**: $(1-x^2)^{\frac{1}{2}}(1-x)^{-1}$ seen or implied. **M1:** Expands $(1-x^2)^{\frac{1}{2}}$ to give both terms simplified or un-simplified, $1+\left(\frac{1}{2}\right)(-x^2)$ **or** expands $(1-x)^{-1}$ to give any 2 out of 3 terms simplified or un-simplified, Eg: $1+(-1)(-x)$ or $...+(-1)(-x)+\frac{(-1)(-2)}{2!}(-x)^2$ or $1+....+\frac{(-1)(-2)}{2!}(-x)^2$ **A1:** At least one binomial expansion correct (either un-simplified or simplified). (ignore x^3 and x^4 terms) **A1:** Two binomial expansions are correct (either un-simplified or simplified). (ignore x^3 and x^4 terms) **M1:** Multiplies out to give 1, exactly one term in x and exactly two terms in x^2. **A1:** Candidate achieves the result on the exam paper. Make sure that their working is sound.	

Notes for Question 2 Continued

Aliter 2. (a) Way 3	$\left\{\sqrt{\left(\frac{1+x}{1-x}\right)} = \sqrt{\frac{(1+x)(1+x)}{(1-x)(1+x)}} = \right\} = (1+x)(1-x^2)^{-\frac{1}{2}}$ $\quad (1+x)(1-x^2)^{-\frac{1}{2}}$	B1
	$= (1+x)\left(1+\frac{1}{2}x^2+...\right)$ Must follow on from above.	M1A1A1
	$= 1+x+\frac{1}{2}x^2$	dM1A1
	Note: The final M1 mark is dependent on the previous method mark for Way 3.	
Aliter 2. (a) Way 4	**Assuming the result on the Question Paper. (You need to be convinced that a candidate is applying this method before you apply the Mark Scheme for Way 4).**	
	$\left\{\sqrt{\left(\frac{1+x}{1-x}\right)} = \frac{\sqrt{(1+x)}}{\sqrt{(1-x)}} = 1+x+\frac{1}{2}x^2\right\} \Rightarrow (1+x)^{\frac{1}{2}} = \left(1+x+\frac{1}{2}x^2\right)(1-x)^{\frac{1}{2}}$	B1
	$(1+x)^{\frac{1}{2}} = 1+\left(\frac{1}{2}\right)x+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2+... \left\{= 1+\frac{1}{2}x-\frac{1}{8}x^2+...\right\}$, $(1-x)^{\frac{1}{2}} = 1+\left(\frac{1}{2}\right)(-x)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(-x)^2+... \left\{= 1-\frac{1}{2}x-\frac{1}{8}x^2+...\right\}$	M1A1A1
	$\text{RHS} = \left(1+x+\frac{1}{2}x^2\right)(1-x)^{\frac{1}{2}} = \left(1+x+\frac{1}{2}x^2\right)\left(1-\frac{1}{2}x-\frac{1}{8}x^2+...\right)$	
	$=1-\frac{1}{2}x-\frac{1}{8}x^2+x-\frac{1}{2}x^2+\frac{1}{2}x^2$ See notes	M1
	$=1+\frac{1}{2}x-\frac{1}{8}x^2$	
	So, $\text{LHS} = 1+\frac{1}{2}x-\frac{1}{8}x^2 = \text{RHS}$	A1 *
		[6]
	B1: $(1+x)^{\frac{1}{2}} = \left(1+x+\frac{1}{2}x^2\right)(1-x)^{\frac{1}{2}}$ seen or implied. **M1: For Way 4, this M1 mark is dependent on the first B1 mark.** Expands $(1+x)^{\frac{1}{2}}$ to give any 2 out of 3 terms simplified or un-simplified, Eg: $1+\frac{1}{2}x$ or $+\left(\frac{1}{2}\right)x+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2$ or $1+....+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2$ **or** expands $(1-x)^{\frac{1}{2}}$ to give any 2 out of 3 terms simplified or un-simplified, Eg: $1+\left(\frac{1}{2}\right)(-x)$ or $+\left(\frac{1}{2}\right)(-x)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(-x)^2$ or $1+....+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(-x)^2$ **A1:** At least one binomial expansion correct (either un-simplified or simplified). (ignore x^3 and x^4 terms) **A1:** Two binomial expansions are correct (either un-simplified or simplified). (ignore x^3 and x^4 terms) **M1: For Way 4, this M1 mark is dependent on the first B1 mark.** Multiplies out RHS to give 1, exactly two terms in x and exactly three terms in x^2. **A1:** Candidate achieves the result on the exam paper. Candidate needs to have correctly processed both the LHS and RHS of $(1+x)^{\frac{1}{2}} = \left(1+x+\frac{1}{2}x^2\right)(1-x)^{\frac{1}{2}}$.	

Question Number	Scheme		Marks
3. (a)	1.154701		B1 **cao** **[1]**
(b)	Area $\approx \frac{1}{2}\times\frac{\pi}{6}\,;\times\underline{\left[1+2\left(1.035276 + \text{their } 1.154701\right)+1.414214\right]}$		B1; <u>M1</u>
	$= \frac{\pi}{12}\times 6.794168 \;=\; 1.778709023... = 1.7787 \;\; (4\text{ dp})$	1.7787 or awrt 1.7787	A1 **[3]**
(c)	$V = \pi\int_0^{\frac{\pi}{2}}\left(\sec\left(\frac{x}{2}\right)\right)^2 \mathrm{d}x$	For $\pi\int\left(\sec\left(\frac{x}{2}\right)\right)^2$. Ignore limits and $\mathrm{d}x$. Can be implied.	B1
	$= \{\pi\}\left[2\tan\left(\frac{x}{2}\right)\right]_0^{\frac{\pi}{2}}$	$\pm\lambda\tan\left(\frac{x}{2}\right)$	M1
		$2\tan\left(\frac{x}{2}\right)$ or equivalent	A1
	$= 2\pi$	2π	A1 **cao cso** **[4]** **8**

Notes for Question 3

(a)	**B1:** 1.154701 correct answer only. Look for this on the table or in the candidate's working.
(b)	**B1**: Outside brackets $\frac{1}{2}\times\frac{\pi}{6}$ or $\frac{\pi}{12}$ or awrt 0.262
	M1: <u>For structure of trapezium rule</u>[<u>............</u>]
	A1: anything that rounds to 1.7787
	Note: It can be possible to award : (a) B0 (b) B1M1A1 (awrt 1.7787)
	<u>Note:</u> Working must be seen to demonstrate the use of the trapezium rule. **<u>Note</u>**: actual area is 1.762747174…
	<u>Note:</u> Award B1M1A1 for $\frac{\pi}{12}(1+1.414214)+\frac{\pi}{6}\left(1.035276 + \text{their } 1.154701\right) = 1.778709023...$
	<u>Bracketing mistake</u>: **Unless the final answer implies that the calculation has been done correctly,**
	Award B1M0A0 for $\frac{1}{2}\times\frac{\pi}{6}+1+2\left(1.035276 + \text{their } 1.154701\right)+1.414214$ (nb: answer of 7.05596...).
	Award B1M0A0 for $\frac{1}{2}\times\frac{\pi}{6}\;(1+1.414214)+2\left(1.035276 + \text{their } 1.154701\right)$ (nb: answer of 5.01199...).
	<u>*Alternative method for part (b): Adding individual trapezia*</u>
	Area $\approx \frac{\pi}{6}\times\left[\frac{1+1.035276}{2}+\frac{1.035276+1.154701}{2}+\frac{1.154701+1.414214}{2}\right] = 1.778709023...$
	B1: $\frac{\pi}{6}$ and a divisor of 2 on all terms inside brackets.
	M1: First and last ordinates once and two of the middle ordinates twice inside brackets ignoring the 2.
	A1: anything that rounds to 1.7787

Notes for Question 3 Continued	
3. (c)	**B1:** For a correct statement of $\pi\int\left(\sec\left(\frac{x}{2}\right)\right)^2$ or $\pi\int\sec^2\left(\frac{x}{2}\right)$ or $\pi\int\frac{1}{\left(\cos\left(\frac{x}{2}\right)\right)^2}\{dx\}$. Ignore limits and $\mathrm{d}x$. Can be implied. Note: Unless a correct expression stated $\pi\int\sec\left(\frac{x^2}{4}\right)$ would be B0. **M1:** $\pm\lambda\tan\left(\frac{x}{2}\right)$ from any working. **A1:** $2\tan\left(\frac{x}{2}\right)$ or $\frac{1}{\left(\frac{1}{2}\right)}\tan\left(\frac{x}{2}\right)$ from any working. **A1:** 2π **from a correct solution only.** **Note:** The π in the volume formula is only required for the B1 mark and the final A1 mark. **Note:** Decimal answer of $6.283...$ without correct exact answer is A0. **Note:** The B1 mark can be implied by later working – as long as it is clear that the candidate has applied $\pi\int y^2$ in their working. **Note:** Writing the correct formula of $V=\pi\int y^2\{\mathrm{d}x\}$, but incorrectly applying it is B0.

Question Number	Scheme		Marks
4.	$x = 2\sin t,\quad y = 1 - \cos 2t \quad \{= 2\sin^2 t\},\qquad -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$		
(a)	$\frac{dx}{dt} = 2\cos t,\quad \frac{dy}{dt} = 2\sin 2t$ or $\frac{dy}{dt} = 4\sin t\cos t$	At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct.	B1
		Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct.	B1
	So, $\frac{dy}{dx} = \frac{2\sin 2t}{2\cos t}\ \left\{= \frac{4\cos t\sin t}{2\cos t} = 2\sin t\right\}$	Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$	M1;
	At $t = \frac{\pi}{6}$, $\frac{dy}{dx} = \frac{2\sin\left(\frac{2\pi}{6}\right)}{2\cos\left(\frac{\pi}{6}\right)}$; $= 1$	and substitutes $t = \frac{\pi}{6}$ into their $\frac{dy}{dx}$.	
		Correct value for $\frac{dy}{dx}$ of 1	A1 **cao cso**
			[4]
(b)	$y = 1 - \cos 2t = 1 - (1 - 2\sin^2 t)$		M1
	$= 2\sin^2 t$		
	So, $y = 2\left(\frac{x}{2}\right)^2$ or $y = \frac{x^2}{2}$ or $y = 2 - 2\left(1 - \left(\frac{x}{2}\right)^2\right)$	$y = \frac{x^2}{2}$ or equivalent.	A1 **cso isw**
	Either $k = 2$ or $-2 \leqslant x \leqslant 2$		B1
			[3]
(c)	Range: $0 \leqslant f(x) \leqslant 2$ or $0 \leqslant y \leqslant 2$ or $0 \leqslant f \leqslant 2$	See notes	B1 B1
			[2]
			9

Notes for Question 4

(a)

B1: At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct. **Note:** that this mark can be implied from their working.

B1: Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct. **Note:** that this mark can be implied from their working.

M1: Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ **and** attempts to substitute $t = \frac{\pi}{6}$ into their expression for $\frac{dy}{dx}$.

This mark may be implied by their final answer.

Ie. $\frac{dy}{dx} = \frac{\sin 2t}{2\cos t}$ followed by an answer of $\frac{1}{2}$ would be M1 (implied).

A1: For an answer of 1 ***by correct solution only.***

Note: Don't just look at the answer! A number of candidates are finding $\frac{dy}{dx} = 1$ from incorrect methods.

Note: Applying $\frac{dx}{dt}$ divided by their $\frac{dy}{dt}$ is M0, even if they state $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$.

Special Case: Award SC: B0B0M1A1 for $\frac{dx}{dt} = -2\cos t$, $\frac{dy}{dt} = -2\sin 2t$ leading to $\frac{dy}{dx} = \frac{-2\sin 2t}{-2\cos t}$

which after substitution of $t = \frac{\pi}{6}$, yields $\frac{dy}{dx} = 1$

Note: It is possible for you to mark part(a), part (b) and part (c) together. Ignore labelling!

Notes for Question 4 Continued

4. (b)

M1: Uses the **correct** double angle formula $\cos 2t = 1 - 2\sin^2 t$ or $\cos 2t = 2\cos^2 t - 1$ or $\cos 2t = \cos^2 t - \sin^2 t$ in an attempt to get y in terms of $\sin^2 t$ or get y in terms of $\cos^2 t$ or get y in terms of $\sin^2 t$ and $\cos^2 t$. Writing down $y = 2\sin^2 t$ is fine for M1.

A1: Achieves $y = \frac{x^2}{2}$ or un-simplified equivalents **in the form $y = f(x)$**. For example:

$$y = \frac{2x^2}{4} \quad \text{or} \quad y = 2\left(\frac{x}{2}\right)^2 \quad \text{or} \quad y = 2 - 2\left(1 - \left(\frac{x}{2}\right)^2\right) \quad \text{or} \quad y = 1 - \frac{4 - x^2}{4} + \frac{x^2}{4}$$

and you can ignore subsequent working if a candidate states a correct version of the Cartesian equation.
IMPORTANT: Please check working as this result can be fluked from an incorrect method.
Award A0 if there is a $+c$ added to their answer.

B1: Either $k = 2$ or a candidate writes down $-2 \leqslant x \leqslant 2$. Note: $-2 \leqslant k \leqslant 2$ unless k stated as 2 is B0.

(c)

Note: The values of 0 and/or 2 need to be evaluated in this part

B1: Achieves an inclusive upper **or** lower limit, using acceptable notation. Eg: $f(x) \geqslant 0$ or $f(x) \leqslant 2$

B1: $0 \leqslant f(x) \leqslant 2$ or $0 \leqslant y \leqslant 2$ or $0 \leqslant f \leqslant 2$

Special Case: SC: B1B0 for either $0 < f(x) < 2$ or $0 < f < 2$ or $0 < y < 2$ or $(0, 2)$

Special Case: SC: B1B0 for $0 \leqslant x \leqslant 2$.

IMPORTANT: Note that: Therefore candidates can use either y or f in place of $f(x)$

Examples:

$0 \leqslant x \leqslant 2$ is SC: B1B0	$0 < x < 2$ is B0B0
$x \geqslant 0$ is B0B0	$x \leqslant 2$ is B0B0
$f(x) > 0$ is B0B0	$f(x) < 2$ is B0B0
$x > 0$ is B0B0	$x < 2$ is B0B0
$0 \geqslant f(x) \geqslant 2$ is B0B0	$0 < f(x) \leqslant 2$ is B1B0
$0 \leqslant f(x) < 2$ is B1B0.	$f(x) \geqslant 0$ is B1B0
$f(x) \leqslant 2$ is B1B0	$f(x) \geqslant 0$ and $f(x) \leqslant 2$ is B1B1. Must state AND {or} $\cap$
$2 \leqslant f(x) \leqslant 2$ is B0B0	$f(x) \geqslant 0$ or $f(x) \leqslant 2$ is B1B0.
$\lvert f(x) \rvert \leqslant 2$ is B1B0	$\lvert f(x) \rvert \geqslant 2$ is B0B0
$1 \leqslant f(x) \leqslant 2$ is B1B0	$1 < f(x) < 2$ is B0B0
$0 \leqslant f(x) \leqslant 4$ is B1B0	$0 < f(x) < 4$ is B0B0
$0 \leqslant$ Range $\leqslant 2$ is B1B0	Range is in between 0 and 2 is B1B0
$0 <$ Range < 2 is B0B0.	Range $\geqslant 0$ is B1B0
Range $\leqslant 2$ is B1B0	Range $\geqslant 0$ and Range $\leqslant 2$ is B1B0.
$[0, 2]$ is B1B1	$(0, 2)$ is SC B1B0

Aliter 4. (a) Way 2		
	$\frac{dx}{dt} = 2\cos t$, $\frac{dy}{dt} = 2\sin 2t$,	So B1, B1.
	At $t = \frac{\pi}{6}$, $\frac{dx}{dt} = 2\cos\left(\frac{\pi}{6}\right) = \sqrt{3}$, $\frac{dy}{dt} = 2\sin\left(\frac{2\pi}{6}\right) = \sqrt{3}$	
	Hence $\frac{dy}{dx} = 1$	So implied M1, A1.

Notes for Question 4 Continued		
Aliter 4. (a) Way 3	$y = \frac{1}{2}x^2 \Rightarrow \frac{dy}{dx} = x$ Correct differentiation of their Cartesian equation. Finds $\frac{dy}{dx} = x$, using the correct Cartesian equation only.	B1ft B1
	At $t = \frac{\pi}{6}$, $\frac{dy}{dx} = 2\sin\left(\frac{\pi}{6}\right)$ Finds the value of "x" when $t = \frac{\pi}{6}$ and substitutes this into their $\frac{dy}{dx}$	M1
	$= 1$ Correct value for $\frac{dy}{dx}$ of 1	A1
Aliter 4. (b) Way 2	$y = 1 - \cos 2t = 1 - (2\cos^2 t - 1)$ $y = 2 - 2\cos^2 t \Rightarrow \cos^2 t = \frac{2-y}{2} \Rightarrow 1 - \sin^2 t = \frac{2-y}{2}$ $1 - \left(\frac{x}{2}\right)^2 = \frac{2-y}{2}$ $y = 2 - 2\left(1 - \left(\frac{x}{2}\right)^2\right)$	M1 (Must be in the form $y = f(x)$). A1
Aliter 4. (b) Way 3	$x = 2\sin t \Rightarrow t = \sin^{-1}\left(\frac{x}{2}\right)$ So, $y = 1 - \cos\left(2\sin^{-1}\left(\frac{x}{2}\right)\right)$ Rearranges to make t the subject and substitutes the result into y. $y = 1 - \cos\left(2\sin^{-1}\left(\frac{x}{2}\right)\right)$	M1 A1 **oe**
Aliter 4. (b) Way 4	$y = 1 - \cos 2t \Rightarrow \cos 2t = 1 - y \Rightarrow t = \frac{1}{2}\cos^{-1}(1-y)$ So, $x = \pm 2\sin\left(\frac{1}{2}\cos^{-1}(1-y)\right)$ Rearranges to make t the subject and substitutes the result into y. So, $y = 1 - \cos\left(2\sin^{-1}\left(\frac{x}{2}\right)\right)$ $y = 1 - \cos\left(2\sin^{-1}\left(\frac{x}{2}\right)\right)$	M1 A1 **oe**
Aliter 4. (b) Way 5	$\frac{dy}{dx} = 2\sin t = x \Rightarrow y = \frac{1}{2}x^2 + c$ $\frac{dy}{dx} = x \Rightarrow y = \frac{1}{2}x^2 + c$ Eg: when eg: $t = 0$ (nb: $-\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$), $x = 0$, $y = 1 - 1 = 0 \Rightarrow c = 0 \Rightarrow y = \frac{1}{2}x^2$ Full method of finding $y = \frac{1}{2}x^2$ using a value of t: $-\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$ **Note:** $\frac{dy}{dx} = 2\sin t = x \Rightarrow y = \frac{1}{2}x^2$, with no attempt to find c is M1A0.	M1 A1

Question Number	Scheme		Marks
5. (a)	$\left\{x = u^2 \Rightarrow\right\} \dfrac{dx}{du} = 2u$ or $\dfrac{du}{dx} = \dfrac{1}{2}x^{-\frac{1}{2}}$ or $\dfrac{du}{dx} = \dfrac{1}{2\sqrt{x}}$		B1
	$\left\{\displaystyle\int \frac{1}{x(2\sqrt{x}-1)}\,dx\right\} = \displaystyle\int \frac{1}{u^2(2u-1)}\,2u\,du$		M1
	$= \displaystyle\int \frac{2}{u(2u-1)}\,du$		A1 * **cso**
			[3]
(b)	$\dfrac{2}{u(2u-1)} \equiv \dfrac{A}{u} + \dfrac{B}{(2u-1)} \Rightarrow 2 \equiv A(2u-1) + Bu$	See notes	M1 A1
	$u = 0 \Rightarrow 2 = -A \Rightarrow A = -2$		
	$u = \frac{1}{2} \Rightarrow 2 = \frac{1}{2}B \Rightarrow B = 4$		
	So $\displaystyle\int \frac{2}{u(2u-1)}\,du = \int \frac{-2}{u} + \frac{4}{(2u-1)}\,du$	Integrates $\dfrac{M}{u} + \dfrac{N}{(2u-1)}$, $M \neq 0$, $N \neq 0$ to obtain any one of $\pm\lambda\ln u$ or $\pm\mu\ln(2u-1)$	M1
	$= -2\ln u + 2\ln(2u-1)$	At least one term correctly followed through	A1 ft
		$-2\ln u + 2\ln(2u-1)$.	A1 **cao**
	So, $\left[-2\ln u + 2\ln(2u-1)\right]_1^3$		
	$= \left(-2\ln 3 + 2\ln(2(3)-1)\right) - \left(-2\ln 1 + 2\ln(2(1)-1)\right)$	Applies limits of 3 and 1 in u or 9 and 1 in x in their integrated function and subtracts the correct way round.	M1
	$= -2\ln 3 + 2\ln 5 - (0)$		
	$= 2\ln\left(\dfrac{5}{3}\right)$	$2\ln\left(\dfrac{5}{3}\right)$	A1 **cso cao**
			[7]
			10

Notes for Question 5

(a)	**B1:** $\dfrac{dx}{du} = 2u$ or $dx = 2u\,du$ or $\dfrac{du}{dx} = \dfrac{1}{2}x^{-\frac{1}{2}}$ or $\dfrac{du}{dx} = \dfrac{1}{2\sqrt{x}}$ or $du = \dfrac{dx}{2\sqrt{x}}$
	M1: A full substitution producing an integral in u only (including the du) (Integral sign not necessary). The candidate needs to deal with the "x", the "$(2\sqrt{x}-1)$" and the "dx" and converts from an integral term in x to an integral in u. (Remember the integral sign is not necessary for M1).
	A1*: leading to the result printed on the question paper (including the du). (Integral sign is needed).
(b)	**M1:** Writing $\dfrac{2}{u(2u-1)} \equiv \dfrac{A}{u} + \dfrac{B}{(2u-1)}$ or writing $\dfrac{1}{u(2u-1)} \equiv \dfrac{P}{u} + \dfrac{Q}{(2u-1)}$ and a complete method for finding the value of at least one of their A or their B (or their P or their Q).
	A1: Both their $A = -2$ **and** their $B = 4$. (Or their $P = -1$ **and** their $Q = 2$ with the multiplying factor of 2 in front of the integral sign).
	M1: Integrates $\dfrac{M}{u} + \dfrac{N}{(2u-1)}$, $M \neq 0$, $N \neq 0$ (i.e. ***a two term partial fraction***) to obtain any one of $\pm\lambda\ln u$ or $\pm\mu\ln(2u-1)$ or $\pm\mu\ln\left(u - \frac{1}{2}\right)$
	A1ft: At least one term correctly followed through from their A or from their B (or their P and their Q).
	A1: $-2\ln u + 2\ln(2u-1)$

Notes for Question 5 Continued

5. (b) ctd	**M1:** Applies limits of 3 and 1 in u or 9 and 1 in x in their (i.e. any) changed function and subtracts the

	correct way round. **Note:** If a candidate just writes $\left(-2\ln 3 + 2\ln(2(3)-1)\right)$ oe , this is ok for M1. **A1:** $2\ln\left(\frac{5}{3}\right)$ **correct answer only.** (Note: $a = 5,\ b = 3$). **Important note: Award M0A0M1A1A0** for a candidate who writes $\int \frac{2}{u(2u-1)}\,\mathrm{d}u = \int \frac{2}{u} + \frac{2}{(2u-1)}\,\mathrm{d}u = 2\ln u + \ln(2u-1)$ **AS EVIDENCE OF WRITING** $\frac{2}{u(2u-1)}$ **AS PARTIAL FRACTIONS IS GIVEN.** **Important note: Award M0A0M0A0A0** for a candidate who writes down either $\int \frac{2}{u(2u-1)}\,\mathrm{d}u = 2\ln u + 2\ln(2u-1)$ **or** $\int \frac{2}{u(2u-1)}\,\mathrm{d}u = 2\ln u + \ln(2u-1)$ **WITHOUT ANY EVIDENCE OF WRITING** $\frac{2}{u(2u-1)}$ as partial fractions. **Important note: Award M1A1M1A1A1** for a candidate who writes down $\int \frac{2}{u(2u-1)}\,\mathrm{d}u = -2\ln u + 2\ln(2u-1)$ **WITHOUT ANY EVIDENCE OF WRITING** $\frac{2}{u(2u-1)}$ as partial fractions. **Note:** In part (b) if they lose the "2" and find $\int \frac{1}{u(2u-1)}\,\mathrm{d}u$ we can allow a maximum of M1A0 M1A1ftA0 M1A0.

Question Number	Scheme		Marks
6.	$\dfrac{d\theta}{dt} = \lambda(120-\theta)\,, \quad \theta \leqslant 100$		
(a)	$\displaystyle\int \frac{1}{120-\theta}\,d\theta = \int \lambda\,dt$ or $\displaystyle\int \frac{1}{\lambda(120-\theta)}\,d\theta = \int dt$		B1
	$-\ln(120-\theta);\ = \lambda t + c$ or $-\dfrac{1}{\lambda}\ln(120-\theta);\ = t + c$	See notes	M1 A1; M1 A1
	$\{t=0\,,\ \theta=20 \Rightarrow\}\ -\ln(120-20) = \lambda(0) + c$	See notes	M1
	$c = -\ln 100 \Rightarrow -\ln(120-\theta) = \lambda t - \ln 100$		
	then either... $-\lambda t = \ln(120-\theta) - \ln 100$ $-\lambda t = \ln\left(\dfrac{120-\theta}{100}\right)$ $e^{-\lambda t} = \dfrac{120-\theta}{100}$ ***or...*** $\lambda t = \ln 100 - \ln(120-\theta)$ $\lambda t = \ln\left(\dfrac{100}{120-\theta}\right)$ $e^{\lambda t} = \dfrac{100}{120-\theta}$		dddM1
	$100e^{-\lambda t} = 120-\theta$ *or* $(120-\theta)e^{\lambda t} = 100$ $\Rightarrow 120-\theta = 100e^{-\lambda t}$ leading to $\theta = 120 - 100e^{-\lambda t}$		A1 *
			[8]
(b)	$\{\lambda = 0.01\,,\ \theta = 100 \Rightarrow\}\quad 100 = 120 - 100e^{-0.01t}$		M1
	$\Rightarrow 100e^{-0.01t} = 120-100 \Rightarrow -0.01t = \ln\left(\dfrac{120-100}{100}\right)$ $t = \dfrac{1}{-0.01}\ln\left(\dfrac{120-100}{100}\right)$	Uses correct order of operations by moving from $100 = 120 - 100e^{-0.01t}$ to give $t = \ldots$ and $t = A\ln B$, where $B > 0$	dM1
	$\left\{t = \dfrac{1}{-0.01}\ln\left(\dfrac{1}{5}\right) = 100\ln 5\right\}$		
	$t = 160.94379\ldots = 161$ (s) (nearest second)	awrt 161	A1
			[3] **11**

<table>
<tr><th colspan="3">Notes for Question 6</th></tr>
<tr><td>(a)</td><td colspan="2">B1: Separates variables as shown. $\mathrm{d}\theta$ and $\mathrm{d}t$ should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.
Either | or
M1: $\int \frac{1}{120-\theta}\,\mathrm{d}\theta \to \pm A\ln(120-\theta)$ | $\int \frac{1}{\lambda(120-\theta)}\,\mathrm{d}\theta \to \pm A\ln(120-\theta)$, A is a constant.
A1: $\int \frac{1}{120-\theta}\,\mathrm{d}\theta \to -\ln(120-\theta)$ | $\int \frac{1}{\lambda(120-\theta)}\,\mathrm{d}\theta \to -\frac{1}{\lambda}\ln(120-\theta)$ or $-\frac{1}{\lambda}\ln(120\lambda-\lambda\theta)$,
M1: $\int \lambda\,\mathrm{d}t \to \lambda t$ | $\int 1\,\mathrm{d}t \to t$
A1: $\int \lambda\,\mathrm{d}t \to \lambda t + c$ | or $\int 1\,\mathrm{d}t \to t + c$ The $+c$ can appear on either side of the equation.
IMPORTANT: $+c$ can be on either side of their equation for the 2nd A1 mark.
M1: Substitutes $t=0$ AND $\theta=20$ in an integrated or changed equation containing c (or A or $\ln A$).
Note that this mark can be implied by the correct value of c. { Note that $-\ln 100 = -4.60517...$ }.

dddM1: Uses their value of c which must be a $\ln$ term, and uses fully correct method to eliminate their logarithms. Note: This mark is dependent on all three previous method marks being awarded.
A1*: This is a given answer. All previous marks must have been scored and there must not be any errors in the candidate's working. Do not accept huge leaps in working at the end. So a minimum of either:
(1): $\mathrm{e}^{-\lambda t} = \frac{120-\theta}{100} \Rightarrow 100\mathrm{e}^{-\lambda t} = 120-\theta \Rightarrow \theta = 120-100\mathrm{e}^{-\lambda t}$
or (2): $\mathrm{e}^{\lambda t} = \frac{100}{120-\theta} \Rightarrow (120-\theta)\mathrm{e}^{\lambda t} = 100 \Rightarrow 120-\theta = 100\mathrm{e}^{-\lambda t} \Rightarrow \theta = 120-100\mathrm{e}^{-\lambda t}$
is required for A1.
Note: $\int \frac{1}{(120\lambda-\lambda\theta)}\,\mathrm{d}\theta \to -\frac{1}{\lambda}\ln(120\lambda-\lambda\theta)$ is ok for the first M1A1 in part (a).</td></tr>
<tr><td>(b)</td><td colspan="2">M1: Substitutes $\lambda = 0.01$ and $\theta = 100$ into the printed equation or one of their earlier equations connecting θ and t. This mark can be implied by subsequent working.
dM1: Candidate uses correct order of operations by moving from $100 = 120 - 100\mathrm{e}^{-0.01t}$ to $t = ...$
Note: that the 2nd Method mark is dependent on the 1st Method mark being awarded in part (b).
A1: awrt 161 or "awrt" 2 minutes 41 seconds. (Ignore incorrect units).</td></tr>
<tr><td>Aliter
6. (a)
Way 2</td><td>$\int \frac{1}{120-\theta}\,\mathrm{d}\theta = \int \lambda\,\mathrm{d}t$

$-\ln(120-\theta) = \lambda t + c$ See notes

$-\ln(120-\theta) = \lambda t + c$
$\ln(120-\theta) = -\lambda t + c$
$120-\theta = A\mathrm{e}^{-\lambda t}$
$\theta = 120 - A\mathrm{e}^{-\lambda t}$
$\{t=0,\ \theta=20 \Rightarrow\}\ 20 = 120 - A\mathrm{e}^{0}$
$A = 120-20 = 100$
So, $\theta = 120 - 100\mathrm{e}^{-\lambda t}$</td><td>B1

M1 A1;
M1 A1

M1

dddM1 A1 *
[8]</td></tr>
</table>

Notes for Question 6 Continued

(a)

B1M1A1M1A1: Mark as in the original scheme.

M1: Substitutes $t = 0$ AND $\theta = 20$ in an integrated equation containing their constant of integration which could be c or A. **Note** that this mark can be implied by the correct value of c or A.

dddM1: Uses a fully correct method to eliminate their logarithms and writes down an equation containing their evaluated constant of integration.

Note: This mark is dependent on all three previous method marks being awarded.

Note: $\ln(120-\theta) = -\lambda t + c$ leading to $120 - \theta = \mathrm{e}^{-\lambda t} + \mathrm{e}^{c}$ or $120 - \theta = \mathrm{e}^{-\lambda t} + A$, would be dddM0.

A1*: Same as the original scheme.

Note: The jump from $\ln(120-\theta) = -\lambda t + c$ to $120 - \theta = A\mathrm{e}^{-\lambda t}$ ***with no incorrect working*** is condoned in part (a).

Aliter 6. (a) Way 3

Working	Notes	Marks
$\int \frac{1}{120-\theta}\,\mathrm{d}\theta = \int \lambda\,\mathrm{d}t \quad \left\{\Rightarrow \int \frac{-1}{\theta-120}\,\mathrm{d}\theta = \int \lambda\,\mathrm{d}t\right\}$		B1
$-\ln\lvert\theta-120\rvert = \lambda t + c$	*Modulus required for 1st A1.*	M1 **A1** M1 A1
$\{t=0,\ \theta=20 \Rightarrow\}\ -\ln\lvert 20-120\rvert = \lambda(0) + c$	*Modulus not required here!*	M1
$\Rightarrow c = -\ln 100 \Rightarrow -\ln\lvert\theta-120\rvert = \lambda t - \ln 100$		

then either...	***or...***
$-\lambda t = \ln\lvert\theta-120\rvert - \ln 100$	$\lambda t = \ln 100 - \ln\lvert\theta-120\rvert$
$-\lambda t = \ln\left\lvert\frac{\theta-120}{100}\right\rvert$	$\lambda t = \ln\left\lvert\frac{100}{\theta-120}\right\rvert$
As $\theta \leqslant 100$	
$-\lambda t = \ln\left(\frac{120-\theta}{100}\right)$	$\lambda t = \ln\left(\frac{100}{120-\theta}\right)$
$\mathrm{e}^{-\lambda t} = \frac{120-\theta}{100}$	$\mathrm{e}^{\lambda t} = \frac{100}{120-\theta}$
$100\mathrm{e}^{-\lambda t} = 120 - \theta$	$(120-\theta)\mathrm{e}^{\lambda t} = 100$ $\Rightarrow 120 - \theta = 100\mathrm{e}^{-\lambda t}$
leading to $\theta = 120 - 100\mathrm{e}^{-\lambda t}$	

Understanding of modulus is required here! — **dddM1**

A1 *

[8]

B1: Mark as in the original scheme.

M1: Mark as in the original scheme ignoring the modulus.

A1: $\int \frac{1}{120-\theta}\,\mathrm{d}\theta \to -\ln\lvert\theta-120\rvert$. ***(The modulus is required here).***

M1A1: Mark as in the original scheme.

M1: Substitutes $t = 0$ AND $\theta = 20$ in an integrated equation containing their constant of integration which could be c or A. Mark as in the original scheme ignoring the modulus.

dddM1: Mark as in the original scheme **AND** the candidate must demonstrate that they have converted $\ln\lvert\theta-120\rvert$ to $\ln(120-\theta)$ in their working. **Note:** This mark is dependent on all three previous method marks being awarded.

A1: Mark as in the original scheme.

Notes for Question 6 Continued		
Aliter 6. (a) Way 4	***Use of an integrating factor***	
	$\dfrac{d\theta}{dt} = \lambda(120 - \theta) \Rightarrow \dfrac{d\theta}{dt} + \lambda\theta = 120\lambda$	
	$\text{IF} = e^{\lambda t}$	B1
	$\dfrac{d}{dt}\left(e^{\lambda t}\theta\right) = 120\lambda e^{\lambda t}$,	M1A1
	$e^{\lambda t}\theta = 120\lambda e^{\lambda t} + k$	M1A1
	$\theta = 120 + Ke^{-\lambda t}$	M1
	$\{t = 0 , \theta = 20 \Rightarrow\} -100 = K$	
	$\theta = 120 - 100e^{-\lambda t}$	M1A1

Question Number	Scheme		Marks
7.	$x^2 + 4xy + y^2 + 27 = 0$		
(a)	$\left\{\cancel{\frac{dy}{dx}}\,\cancel{\times}\right\}\quad \underline{2x} + \underline{\underline{\left(4y + 4x\frac{dy}{dx}\right)}} \underline{+ 2y\frac{dy}{dx}} = \underline{0}$		M1 $\underline{\text{A1}}$ $\underline{\underline{\text{B1}}}$
	$2x + 4y + (4x + 2y)\frac{dy}{dx} = 0$		dM1
	$\frac{dy}{dx} = \frac{-2x - 4y}{4x + 2y}\ \left\{= \frac{-x - 2y}{2x + y}\right\}$		A1 **cso oe**
			[5]
(b)	$4x + 2y = 0$		M1
	$y = -2x$	$x = -\frac{1}{2}y$	A1
	$x^2 + 4x(-2x) + (-2x)^2 + 27 = 0$	$\left(-\frac{1}{2}y\right)^2 + 4\left(-\frac{1}{2}y\right)y + y^2 + 27 = 0$	M1*
	$-3x^2 + 27 = 0$	$-\frac{3}{4}y^2 + 27 = 0$	
	$x^2 = 9$	$y^2 = 36$	dM1*
	$x = -3$	$y = 6$	A1
	When $x = -3$, $y = -2(-3)$	When $y = 6$, $x = -\frac{1}{2}(6)$	ddM1*
	$y = 6$	$x = -3$	A1 **cso**
			[7]
			12

Notes for Question 7

(a)

M1: Differentiates implicitly to include either $4x\frac{dy}{dx}$ or $\pm ky\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$).

A1: $\left(x^2\right) \to \left(\underline{2x}\right)$ and $\left(\ldots + y^2 + 27 = 0 \to \underline{+ 2y\frac{dy}{dx} = 0}\right)$.

Note: If an extra term appears then award A0.
Note: The "$= 0$" can be implied by rearrangement of their equation.

i.e.: $2x + 4y + 4x\frac{dy}{dx} + 2y\frac{dy}{dx}$ leading to $4x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x - 4y$ will get A1 (implied).

B1: $4y + 4x\frac{dy}{dx}$ or $4\left(y + x\frac{dy}{dx}\right)$ or equivalent

dM1: An attempt to factorise out $\frac{dy}{dx}$ as long as there are at least two terms in $\frac{dy}{dx}$.

ie. $\ldots + (4x + 2y)\frac{dy}{dx} = \ldots$ or $\ldots + 2(2x + y)\frac{dy}{dx} = \ldots$

Note: This mark is dependent on the previous method mark being awarded.

A1: For $\frac{-2x - 4y}{4x + 2y}$ or equivalent. Eg: $\frac{+2x + 4y}{-4x - 2y}$ or $\frac{-2(x + 2y)}{4x + 2y}$ or $\frac{-x - 2y}{2x + y}$

cso: If the candidate's solution is not completely correct, then do not give this mark.

Notes for Question 7 Continued	
(b)	**M1:** Sets the denominator of their $\frac{dy}{dx}$ equal to zero (or the numerator of their $\frac{dx}{dy}$ equal to zero) oe. **A1:** Rearranges to give either $y = -2x$ or $x = -\frac{1}{2}y$. (correct solution only). The first two marks can be implied from later working, i.e. for a correct substitution of either $y = -2x$ into y^2 or for $x = -\frac{1}{2}y$ into $4xy$. **M1*:** Substitutes $y = \pm\lambda x$ or or $x = \pm\mu y$ or $y = \pm\lambda x \pm a$ or $x = \pm\mu y \pm b$ $(\lambda \neq 0,\ \mu \neq 0)$ into $x^2 + 4xy + y^2 + 27 = 0$ to form an equation in one variable. **dM1*:** leading to at least either $x^2 = A,\ A > 0$ or $y^2 = B,\ B > 0$ **Note:** This mark is dependent on the previous method mark (M1*) being awarded. **A1:** For $x = -3$ (ignore $x = 3$) or if y was found first, $y = 6$ (ignore $y = -6$) (correct solution only). **ddM1*** Substitutes their value of x into $y = \pm\lambda x$ to give $y =$ value or substitutes their value of x into $x^2 + 4xy + y^2 + 27 = 0$ to give $y =$ value. **Alternatively, s**ubstitutes their value of y into $x = \pm\mu y$ to give $x =$ value or substitutes their value of y into $x^2 + 4xy + y^2 + 27 = 0$ to give $x =$ value **Note:** This mark is dependent on the two previous method marks (M1* and dM1*) being awarded. **A1:** $(-3, 6)$ **cso**. **Note:** If a candidate offers two sets of coordinates without either rejecting the incorrect set or accepting the correct set then award A0. **DO NOT APPLY ISW ON THIS OCCASION.** **Note:** $x = -3$ followed later in working by $y = 6$ is fine for A1. **Note:** $y = 6$ followed later in working by $x = -3$ is fine for A1. **Note:** $x = -3$, 3 followed later in working by $y = 6$ is A0, unless candidate indicates that they are rejecting $x = 3$ **Note:** Candidates who set the numerator of $\frac{dy}{dx}$ equal to 0 (or the denominator of their $\frac{dx}{dy}$ equal to zero) can ***only achieve a maximum of 3 marks*** in this part. They can only achieve the 2nd, 3rd and 4th Method marks to give a maximum marking profile of M0A0M1M1A0M1A0. They will usually find $(-6, 3)$ **{** or even $(6, -3)$ **}.** **Note:** Candidates who set ***the numerator*** or ***the denominator*** of $\frac{dy}{dx}$ equal to $\pm k$ (usually $k = 1$) can ***only achieve a maximum of 3 marks*** in this part. They can only achieve the 2nd, 3rd and 4th Method marks to give a marking profile of M0A0M1M1A0M1A0. **Special Case:** It is possible for a candidate who does not achieve full marks in part (a), (but has a correct denominator for $\frac{dy}{dx}$) to gain all 7 marks in part (b). Eg: An incorrect part (a) answer of $\frac{dy}{dx} = \frac{2x - 4y}{4x + 2y}$ can lead to a correct $(-3, 6)$ in part (b) and 7 marks.

Question Number	Scheme		Marks
8.	$l:\ \mathbf{r}=\begin{pmatrix}13\\8\\1\end{pmatrix}+\lambda\begin{pmatrix}2\\2\\-1\end{pmatrix},\quad A(3,-2,6),\quad \overrightarrow{OP}=\begin{pmatrix}-p\\0\\2p\end{pmatrix}$		
(a)	$\left\{\overrightarrow{PA}\right\}=\begin{pmatrix}3\\-2\\6\end{pmatrix}-\begin{pmatrix}-p\\0\\2p\end{pmatrix}$ | $\left\{\overrightarrow{AP}\right\}=\begin{pmatrix}-p\\0\\2p\end{pmatrix}-\begin{pmatrix}3\\-2\\6\end{pmatrix}$	Finds the difference between $\overrightarrow{OA}$ and $\overrightarrow{OP}$. Ignore labelling.	M1
	$=\begin{pmatrix}3+p\\-2\\6-2p\end{pmatrix}$ | $=\begin{pmatrix}-3-p\\2\\2p-6\end{pmatrix}$	Correct difference.	A1
	$\begin{pmatrix}3+p\\-2\\6-2p\end{pmatrix}\bullet\begin{pmatrix}2\\2\\-1\end{pmatrix}=6+2p-4-6+2p=0$	See notes.	M1
	$p=1$		A1 **cso**
			[4]
(b)	$\lvert AP\rvert=\sqrt{4^2+(-2)^2+4^2}$ or $\lvert AP\rvert=\sqrt{(-4)^2+2^2+(-4)^2}$	See notes.	M1
	So, PA or $AP=\sqrt{36}$ or 6 **cao**		A1 **cao**
	It follows that, $AB="6"\ \{=PA\}$ or $PB="6\sqrt{2}"\ \{=\sqrt{2}\,PA\}$	See notes.	B1 ft
	{Note that $AB="6"=2$(the modulus of the direction vector of l) }		
	$\overrightarrow{OB}=\begin{pmatrix}3\\-2\\6\end{pmatrix}\pm2\begin{pmatrix}2\\2\\-1\end{pmatrix}$ or $\overrightarrow{OB}=\begin{pmatrix}13\\8\\1\end{pmatrix}-3\begin{pmatrix}2\\2\\-1\end{pmatrix}$ and $\overrightarrow{OB}=\begin{pmatrix}13\\8\\1\end{pmatrix}-7\begin{pmatrix}2\\2\\-1\end{pmatrix}$	Uses a correct method in order to find both possible sets of coordinates of *B*.	M1
	$=\begin{pmatrix}7\\2\\4\end{pmatrix}$ and $\begin{pmatrix}-1\\-6\\8\end{pmatrix}$	Both coordinates are correct.	A1 **cao**
			[5] **9**

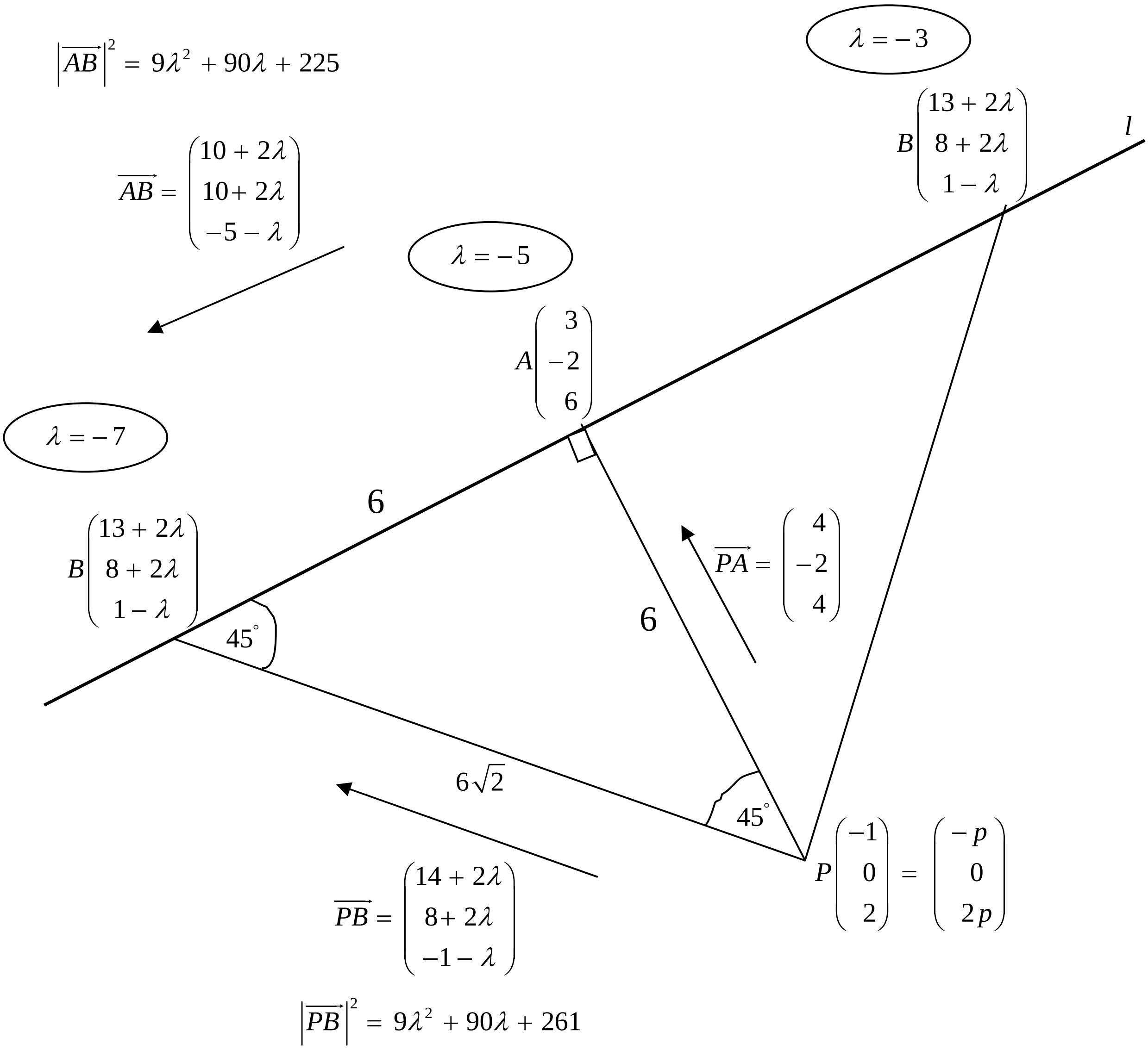
Notes for Question 8

8. (a)	**M1:** Finds the difference between $\overrightarrow{OA}$ and $\overrightarrow{OP}$. Ignore labelling. If no "subtraction" seen, you can award M1 for 2 out of 3 correct components of the difference. **A1:** Accept any of $\begin{pmatrix}3+p\\-2\\6-2p\end{pmatrix}$ or $(3+p)\mathbf{i}-2\mathbf{j}+(6-2p)\mathbf{k}$ or $\begin{pmatrix}-3-p\\2\\2p-6\end{pmatrix}$ or $(-3-p)\mathbf{i}+2\mathbf{j}+(2p-6)\mathbf{k}$

Notes for Question 8 Continued	
8. (a)	**M1:** ***Applies*** the formula $\overrightarrow{PA}\bullet\begin{pmatrix}2\\2\\-1\end{pmatrix}$ or $\overrightarrow{AP}\bullet\begin{pmatrix}2\\2\\-1\end{pmatrix}$ correctly to give a linear equation in *p* which is set equal to zero. **Note:** The dot product can also be with $\pm k\begin{pmatrix}2\\2\\-1\end{pmatrix}$. Eg: Some candidates may find $\begin{pmatrix}13\\8\\1\end{pmatrix}-\begin{pmatrix}3\\-2\\6\end{pmatrix}=\begin{pmatrix}10\\10\\-5\end{pmatrix}$, for instance, and use this in their dot product which is fine for M1. **A1:** Finds $p=1$ from ***a correct solution only.*** **Note:** The direction of subtraction is not important in part (a).
(b)	**M1:** Uses their value of *p* and Pythagoras to obtain a numerical expression for either *AP* or *PA* or AP^2 or PA^2. Eg: *PA* or *AP* $=\sqrt{4^2+(-2)^2+4^2}$ or $\sqrt{(-4)^2+2^2+(-4)^2}$ or $\sqrt{4^2+2^2+4^2}$ or PA^2 or AP^2 $=4^2+(-2)^2+4^2$ or $(-4)^2+2^2+(-4)^2$ or $4^2+2^2+4^2$ **A1:** *AP* or *PA* $=\sqrt{36}$ or 6 **cao** or $AP^2=36$ **cao** **B1ft:** States or it is clear from their working that $AB=\text{"6"}$ $\{=\text{their evaluated } PA\}$ or $PB=\text{"6"}\sqrt{2}$ $\{=\sqrt{2}\ (\text{their evaluated } PA)\}$. **Note:** So a correct follow length is required here for either *AB* or *PB* using their evaluated *PA*. **Note:** This mark may be found on a diagram. **Note:** If a candidate states that $\lvert\overrightarrow{AP}\rvert=\lvert\overrightarrow{AB}\rvert$ and then goes on to find $\lvert\overrightarrow{AP}\rvert=6$ then the B1 mark can be implied. **IMPORTANT:** This mark may be implied as part of expressions such as: $\{AB=\}\sqrt{(10+2\lambda)^2+(10+2\lambda)^2+(-5-\lambda)^2}=\mathbf{6}$ or $\{AB^2=\}\ (10+2\lambda)^2+(10+2\lambda)^2+(-5-\lambda)^2=\mathbf{36}$ or $\{PB=\}\sqrt{(14+2\lambda)^2+(8+2\lambda)^2+(-1-\lambda)^2}=\mathbf{6\sqrt{2}}$ or $\{PB^2=\}\ (14+2\lambda)^2+(8+2\lambda)^2+(-1-\lambda)^2=\mathbf{72}$ **M1:** Uses a full method in order to find **both** possible sets of coordinates of *B*: Eg 1: $\overrightarrow{OB}=\begin{pmatrix}3\\-2\\6\end{pmatrix}\pm2\begin{pmatrix}2\\2\\-1\end{pmatrix}$ Eg 2: $\overrightarrow{OB}=\begin{pmatrix}13\\8\\1\end{pmatrix}-3\begin{pmatrix}2\\2\\-1\end{pmatrix}$ and $\overrightarrow{OB}=\begin{pmatrix}13\\8\\1\end{pmatrix}-7\begin{pmatrix}2\\2\\-1\end{pmatrix}$ **Note:** If a candidate achieves at least one of the correct $(7, 2, 4)$ or $(-1, -6, 8)$ then award SC M1 here. **Note:** $\overrightarrow{OB}=\begin{pmatrix}3\\-2\\6\end{pmatrix}-3\begin{pmatrix}2\\2\\-1\end{pmatrix}$ and $\overrightarrow{OB}=\begin{pmatrix}3\\-2\\6\end{pmatrix}-7\begin{pmatrix}2\\2\\-1\end{pmatrix}$ is M0. **A1:** For both $(7, 2, 4)$ and $(-1, -6, 8)$. Accept vector notation or **i**, **j**, **k** notation. **Note:** All the marks are accessible in part (b) if $p=1$ is found from incorrect working in part (a). **Note: Imply M1A1B1 and award M1** for candidates who write: $\overrightarrow{OB}=\begin{pmatrix}3\\-2\\6\end{pmatrix}\pm2\begin{pmatrix}2\\2\\-1\end{pmatrix}$, with little or no earlier working.

Notes for Question 8 Continued

8. ***Helpful Diagram!***



8. (b) **Way 2:** Setting $AB = "6"$ or $AB^2 = "36"$ **Note:** It is possible for you to apply the main scheme for Way 2.

$\left\{AB = "6" \Rightarrow AB^2 = "36" \Rightarrow\right\}$ $(10+2\lambda)^2 + (10+2\lambda)^2 + (-5-\lambda)^2 = "36"$ **B1ft** could be implied here.

$$9\lambda^2 + 90\lambda + 225 = 36 \Rightarrow 9\lambda^2 + 90\lambda + 189 = 0$$
$$\lambda^2 + 10\lambda + 21 = 0 \Rightarrow (\lambda+3)(\lambda+7) = 0$$
$$\lambda = -3, -7$$

Then apply final M1 A1 as in the original scheme. | ... M1 A1

8. (b) **Way 3:** Setting $PB = "6\sqrt{2}"$ or $PB^2 = "72"$ **Note:** It is possible for you to apply the main scheme for Way 3.

$\left\{PB = "6"\sqrt{2} \Rightarrow PB^2 = "72" \Rightarrow\right\}$ $(14+2\lambda)^2 + (8+2\lambda)^2 + (-1-\lambda)^2 = "72"$ **B1ft** could be implied here.

$$9\lambda^2 + 90\lambda + 261 = 72 \Rightarrow 9\lambda^2 + 90\lambda + 189 = 0$$
$$\lambda^2 + 10\lambda + 21 = 0 \Rightarrow (\lambda+3)(\lambda+7) = 0$$
$$\lambda = -3, -7$$

Then apply final M1 A1 as in the original scheme. | ... M1 A1

Notes for Question 8 Continued

8. (b)

(You need to be convinced that a candidate is applying this method before you apply the Mark Scheme for Way 4).

Way 4: Using the dot product formula between $\overrightarrow{PA}$ and $\overrightarrow{PB}$, ie: $\cos 45^\circ = \dfrac{\overrightarrow{PA} \bullet \overrightarrow{PB}}{|\overrightarrow{PA}|.|\overrightarrow{PB}|}$.

$$\overrightarrow{PA} \bullet \overrightarrow{PB} = \begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix} \bullet \begin{pmatrix} 14 + 2\lambda \\ 8 + 2\lambda \\ -1 - \lambda \end{pmatrix} = 56 + 8\lambda - 16 - 4\lambda - 4 - 4\lambda = 36$$

$$\{\cos 45^\circ =\}\ \frac{1}{\sqrt{2}} = \frac{36}{6\ \sqrt{9\lambda^2 + 90\lambda + 261}}$$

For finding $|\overrightarrow{PA}|$ as before. — M1

$\sqrt{36}$ or 6 — A1 **cao**

$|\overrightarrow{PB}| = \sqrt{9\lambda^2 + 90\lambda + 261}$ — B1 oe

$$\frac{1}{2} = \frac{36}{9\lambda^2 + 90\lambda + 261}$$

$$9\lambda^2 + 90\lambda + 261 = 72 \Rightarrow 9\lambda^2 + 90\lambda + 189 = 0$$

$$\lambda^2 + 10\lambda + 21 = 0 \Rightarrow (\lambda + 3)(\lambda + 7) = 0$$

$$\lambda = -3, -7$$

Then apply final M1 A1 as in the original scheme. — ... M1 A1

8. (b)

(You need to be convinced that a candidate is applying this method before you apply the Mark Scheme for Way 5).

Way 5: Using the dot product formula between $\overrightarrow{AB}$ and $\overrightarrow{PB}$, ie: $\cos 45^\circ = \dfrac{\overrightarrow{AB} \bullet \overrightarrow{PB}}{|\overrightarrow{AB}|.|\overrightarrow{PB}|}$

$$\cos 45^\circ = \frac{1}{\sqrt{2}} = \frac{\begin{pmatrix} 10 + 2\lambda \\ 10 + 2\lambda \\ -5 - \lambda \end{pmatrix} \bullet \begin{pmatrix} 14 + 2\lambda \\ 8 + 2\lambda \\ -1 - \lambda \end{pmatrix}}{\sqrt{9\lambda^2 + 90\lambda + 225}\ \sqrt{9\lambda^2 + 90\lambda + 261}}$$

Attempts the dot product formula between $\overrightarrow{AB}$ and $\overrightarrow{PB}$. — M1

Correct statement with $|\overrightarrow{AB}|$ and $|\overrightarrow{PB}|$ simplified as shown. — A1

Either $|\overrightarrow{AB}| = \sqrt{9\lambda^2 + 90\lambda + 225}$ or $|\overrightarrow{PB}| = \sqrt{9\lambda^2 + 90\lambda + 261}$ — B1

$$\{\cos 45^\circ =\}\ \frac{1}{\sqrt{2}} = \frac{140 + 20\lambda + 28\lambda + 4\lambda^2 + 80 + 20\lambda + 16\lambda + 4\lambda^2 + 5 + 5\lambda + \lambda + \lambda^2}{\sqrt{9\lambda^2 + 90\lambda + 225}\ \sqrt{9\lambda^2 + 90\lambda + 261}}$$

$$\{\cos 45^\circ =\}\ \frac{1}{\sqrt{2}} = \frac{9\lambda^2 + 90\lambda + 225}{\sqrt{9\lambda^2 + 90\lambda + 225}\ \sqrt{9\lambda^2 + 90\lambda + 261}}$$

$$\frac{1}{2} = \frac{(9\lambda^2 + 90\lambda + 225)^2}{(9\lambda^2 + 90\lambda + 225)(9\lambda^2 + 90\lambda + 261)}$$

$$\frac{1}{2} = \frac{(9\lambda^2 + 90\lambda + 225)}{(9\lambda^2 + 90\lambda + 261)}$$

$$9\lambda^2 + 90\lambda + 261 = 2(9\lambda^2 + 90\lambda + 225) \Rightarrow 9\lambda^2 + 90\lambda + 189 = 0$$

$$\lambda^2 + 10\lambda + 21 = 0 \Rightarrow (\lambda + 3)(\lambda + 7) = 0$$

$$\lambda = -3, -7$$

Then apply final M1 A1 as in the original scheme. — ... M1 A1

Notes for Question 8 Continued	
8. (b)	**Way 6:** $\overrightarrow{PA} = \begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix} = 2\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$ and direction vector of l is $\mathbf{d} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$ So, $\lvert\overrightarrow{PA}\rvert = 2\,\lvert\mathbf{d}\rvert$ or $PA = 2\,\lvert\mathbf{d}\rvert$ — A correct statement relating these distances (and not vectors) — M1 A1 B1 Apply final M1 A1 as in the original scheme. — ... M1 A1 **Note:** $\overrightarrow{PA} = 2\mathbf{d}$ with no other creditable working is M0A0B0... **Note:** $\overrightarrow{PA} = 2\mathbf{d}$, followed by $\overrightarrow{OB} = \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix} \pm 2\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$ is M1A1B1M1 and the final A1 mark is for both sets of correct coordinates.